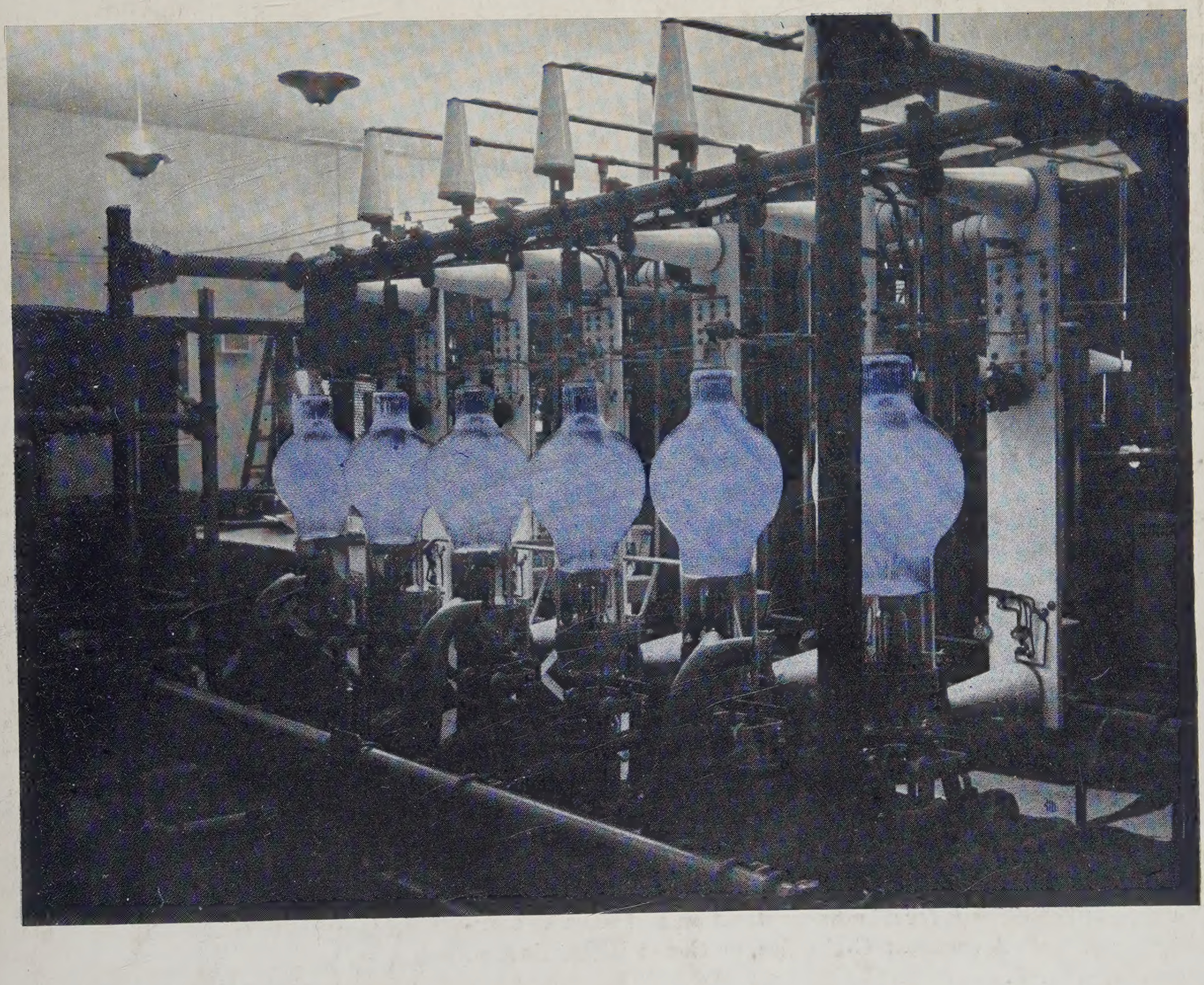
Electrical Engineering

October
1935



Published Monthly by the
American Institute of Electrical Engineers

Published Monthly by

American Institute of Electrical Engineers

(Founded May 13, 1884)

Electrical Engineering

Registered U. S. Patent Office

October 1935
Volume 54
No. 10

The Official Monthly Journal and Transactions of the A.I.E.E.

PUBLICATION OFFICE, 20th and Northampton Streets, Easton, Pa.

EDITORIAL AND ADVERTISING OFFICES, 33 West 39th Street, New York, N. Y.

MAIL should be sent to New York address

ENTERED as second class matter at the Post Office, Easton, Pa., April 20, 1932, under the Act of Congress March 3, 1879. Accepted for mailing at special postage rates provided for in Section 1103, Act of October 3, 1917, authorized on August 3, 1918.

SUBSCRIPTION RATES—$12 per year to United States, Mexico, Cuba, Porto Rico, Hawaii, and the Philippine Islands, Central America, South America, Haiti, Spain, and Spanish Colonies, $13 to Canada, $14 to all other countries. Single copy $1.50.

CHANGE OF ADDRESS—requests must be received by the fifteenth of the month to be effective with the succeeding issue. Copies undelivered because of incorrect address cannot be replaced without charge. Be sure to specify both old and new addresses and any change in business affiliation.

ADVERTISING COPY—changes must be received by the fifteenth of the month to be effective for the issue of the month succeeding.

ELECTRICAL ENGINEERING is indexed annually by the Institute, weekly and monthly by *Engineering Index,* and monthly by *Industrial Arts Index;* abstracted monthly by *Science Abstracts* (London).

Printed in the United States of America. Number of copies this issue— 17,100

Front Cover

17,000-volt mercury-vapor rectifier tubes glowing with purple light under full load as they supply plate current for the 50,000 watt transmitter of station WOR at Carteret, N. J. Western Electric Co. Photo

Special Articles

A.I.E.E. Papers

—Turn to next page

Discussions

Communication

Electrical Machinery

Instruments and Measurements

Insulation

Power Generation

News 1122

In This Issue—

USEFULNESS of the cathode ray oscillograph has been extended greatly by development of means for using it as a multielement instrument. Two types of equipment for this purpose are described in 2 papers in this issue: One describes an electron tube circuit which enables 2 or more waves having frequencies in simple multiple relationship to each other to be portrayed (*pages 1095–1100*); the other describes mechanical auxiliaries for use in circuits that permit of the repeated application of the waves to be observed (*pages 1045–7*).

HEAVISIDE'S operational method of circuit analysis is achieving greater importance as a tool for regular use in engineering problems. For readers who have had ordinary mathematical training, but who are not familiar with the operational method, the fundamental characteristics of this method are outlined in this issue, operational equations are developed for a few typical circuits, and methods of solving these equations are indicated (*pages 1037–45*).

TO REDUCE radio interference from pin insulators, low resistance conducting coatings commonly are applied to the heads. By extending the coating to cover the entire head and by using a coating of higher resistance, the voltage at which interference begins can be raised considerably without materially lowering the flashover voltage of the insulator (*pages 1084–7*).

THE method of symmetrical components may be applied in determining the optimum values of external resistance and reactance which should be used for split phase starting of 3 phase induction motors which are to be operated from single phase lines. The calculated results obtained by this method are found to be quite accurate (*pages 1068–72*).

HARMONICS in the input current to a network containing any number of linear and nonlinear resistances when a sinusoidal voltage is applied may be determined by a semigraphical method. Such a method, restricted to nonlinear resistances having the same d-c volt-ampere characteristic regardless of direction of current, is presented in this issue (*pages 1055–7*).

TESTS on unbleached, unsized linen paper show that the chemical changes caused by overheating of cellulose insulation in service may result in failure of the insulation even within a range of temperatures not generally associated with dangerous physical effects (*pages 1088–94*).

SEATTLE, Wash., was host to a highly successful Pacific Coast convention of the Institute held in that city August 27–30, 1935. A complete report of this convention including a number of interesting snapshots taken during its progress are contained in this issue (*pages 1122–4*).

RATE of wear of electrical brushes is shown by recent tests not to depend entirely on mechanical forces, but to be related intimately to the conduction of current across the contact between brush and rotating part (*pages 1050–4*).

EXPERIMENTAL models through which electric current is passed may be used to map magnetic and dielectric flux lines. Two methods, both using materials of relatively high resistance, are available (*pages 1032–6*).

A STATIC frequency changer using thermionic tubes for transforming from 60 to $25^5/_7$ cycles and rated at 300 kva has been constructed. It is designed so that it later can be operated as a rectifier (*pages 1063–7*).

IN AN effort to meet demands for better performance in watt-hour meters and for adaptability to changing service requirements, a meter embodying several new features has been developed (*pages 1073–84*).

ELECTROLYTIC capacitors for alternating current are coming into increased use, but further refinements are necessary before many potential uses of these devices may be made satisfactorily (*pages 1058–63*).

ELECTRIC furnace developments which have taken place in Europe during the last year or so have been summarized by a leading authority on the electrometallurgical industry in Europe (*pages 1048–50*).

STANDARD wave shapes are desirable for performing impulse tests on electrical apparatus. Methods of producing waves having predetermined time constants have been developed (*pages 1100–04*).

LAST CALL for the meeting of the Institute's Great Lakes District to be held at Purdue University, West Lafayette, Ind., Oct. 24–25, 1935, is being issued (*page 1125*).

The Engineer and the Modern World

A Message From the President—

SHARING THE WEALTH has been a topic of wide discussion in recent months, but, strangely enough, practically nothing has been said of the production of wealth.

Wealth, in its broadest sense is something for which people exchange their own labor and services. To my mind engineers have done more to increase the production of wealth than any other group of our citizens. For his efforts along these lines the engineer has received little credit and a great deal of criticism. In the popular mind he is held responsible for so complicating our civilization as to cause a disorganization of the entire economic system.

During the course of recorded history the engineer has completely reversed his rôle in society. Originally there were none but military engineers whose job was to cause destruction of enemy fortifications and the greater the destruction the higher was the engineer's reputation among his associates. It is true that his work involved such construction as earth works, towers, and military engines, but such work was merely incidental to the larger program of destruction which is the basic object of war.

The history of modern engineering is generally considered to have begun in 1829 when Stephenson produced the first successful steam locomotive, but I always consider that it began in the work of Newcomen and Watt in the development of the automatic steam engine. Neither of these men had any very lofty purpose in starting their investigations. They had in mind the necessity for cutting operating cost of pumping water from coal mines by substituting steam for the muscles of men and animals. However, whatever their original object was, it is generally agreed that their work in perfecting the steam engine liberated the human race from the drudgery which had been its lot from the beginning of time.

Prior to the application of power to the tasks of agriculture and industry the common man was only a little better off than an animal. His daylight hours were consumed almost entirely in an individual struggle for subsistence.

I think that if we are honest with ourselves we will admit that practically none of us is either mentally or physically able to make a good living or even a bare existence by our own unaided efforts. The close cooperation of individuals, each carrying on his specialized work, has raised our scale of living to the point where the poor man today has infinitely more in the way of comforts and luxuries than the rich nobles of past ages. For this condition the engineer is largely responsible because he has multiplied a millionfold the potential effectiveness of each individual in his capacity to produce wealth and has opened for each of us almost boundless horizons.

Those who quarrel with the machine age blame the engineer and say that he has made man a slave to his machines, but I cannot subscribe to this view. The work of tending the automatic machines in use by industry today is not one-tenth as monotonous or exhausting as the heavy manual labor which had to be performed by our forebears less than 2 centuries ago. What appears to have been overlooked is the enormous increase in the productive capacity of the individual and the fact that the increased production in each trade and industry makes it possible to exchange this larger output for more necessaries and luxuries produced by other groups.

Not only has the engineer improved the material condition of his fellows, but by releasing men from long hours of drudgery has had a profound effect on the culture of the world. Perhaps with some justice the engineer can be criticised for stopping there and not taking a mo e important and commanding part in the actual management of the world so largely created by him. This situation cannot be remedied instantaneously, or by decree, but it can be corrected to the benefit of society as a whole.

In the first place, I believe that the fundamental training of our present and future engineering students should cover a field much broader than mathematics, physics, chemistry, and allied technical subjects. The engineer should have a thorough grounding in economic subjects, and in his education should be given sufficient cultural background to enable him in after years readily to recognize, understand, and evaluate what is going on in the world.

In the second place, I would urge each engineer to take an active interest in things outside engineering which, after all, covers only a relatively small part of the field of human activity.

Progress today is made not by a single genius but by a common effort. In professional bodies, such as the Institute, the idea of co-operative effort has proved itself. As a group having certain general and technical interests in common we tend to act more effectively and with greater devotion to the common good than we could as segregated individuals. The future of society in general appears to depend more and more upon the vitality of such functional groups. One of our greatest problems may well be to extend professional ethics and ideals to the areas of industrial and public life.

The Institute has sought to reconcile scientific progress with compassion for humanity, and the highest stimulation of individual achievement with the widest distribution of social benefits. A realization exists that social and economic aspects of our profession must be taken into greater account. We cannot limit ourselves entirely to purely technical matters.

The engineers and scientists who have provided present-day society with its astounding equipment and conveniences, have given relatively little thought

to the deeper effects of their achievements upon the lives and characters of men, communities, and peoples. The ability to discover and develop more and better goods and services and the capacity to produce them have been achieved. But how to divide the benefits of engineering progress between the spiritual and material has not yet been learned well enough by business to maintain without disastrous fluctuations the steady flow of enjoyment through well ordered production, distribution, and consumption.

The dissemination of sound information on these subjects to society as a whole with its capacity to follow thought with action, surely will clarify the present confused situation and lead to workable solutions. Society does not relish fog with its dangerous, unpleasant impediment of the journey and its doleful sirens; it loves sunshine and full speed ahead. The steady sunshine of sound prosperity is all around us ready to be enjoyed as soon as wise leadership and intelligent following will break through this man-made mist of stupidity.

As professional men we cannot assume a *laissez-faire* attitude with the vast social and political experiments being conducted during the present period. Although it is neither recommended nor desirable that we abandon our professional activities to deal with social problems, it is extremely necessary, particularly so at this time, that we study the application of scientific thought to our social and political structure. Engineers are urged to apply their characteristic methods of reasoning to these problems in the same way as they deal with all types of natural phenomena.

It is a matter of common observation that there is much room for improvement in the relationship between individuals. The very complexities of modern life may be partly responsible for the independent and individualistic attitude that is so prevalent today. It seems almost certain that little or no social improvement is to be expected unless real stress is laid on the improvement of man's attitude toward other men, individually and collectively. It is for that reason that I particularly stress the importance of the true spirit of helpfulness of our profession and the necessity for a proper individual attitude.

Our work in the Institute is the biggest and best thing that comes to us and it can produce from within us the biggest and best of all we personally contain. Fundamentally we are working together in a common interest which lies in everything that makes for the greater development of the American Institute of Electrical Engineers.

We can be justly proud of the great engineering progress made, particularly during the past few years, and the future will depend entirely on how we, individually, carry on the sound policies established by the Institute and through which it has grown and expanded to its present position of preeminence.

E. B. Meyer

Two Methods of Mapping Flux Lines

Two methods of magnetic and dielectric flux mapping by means of experimental models are described in this paper. Both methods use resistance models, through which current is passed; electrical conductivities of model parts are adjusted to correspond to magnetic or dielectric permeabilities of corresponding parts of the original design, and paths of electric current flow and electric equipotential lines in the model correspond to magnetic or dielectric flux lines and equipotential lines in the original design. Low melting point waxes with carbon or graphite inclusions are particularly suitable for model making purposes due to easy working properties and ready control of the electrical conductivity.

By
FRANK W. GODSEY, JR. Sprague Specialties Co.,
ASSOCIATE A.I.E.E. North Adams, Mass.

PLOTTING flux maps by the method of curvilinear squares is at best a tedious method of estimating flux distribution; and in the case of magnetic and electrostatic circuits containing elements of different permeabilities and dielectric constants, accuracy is difficult to adhere to. Unfortunately, the most commonly utilized magnetic and dielectric structures in engineering practice do not lend themselves to mathematical solutions of flux distributions, and accuracy in design depends in large measure upon trial and error methods.

The 2 methods of flux mapping described in this paper require experimental models and measurements, but they greatly increase the accuracy of flux mapping. For both methods, resistance models are used, and either method may be used for mapping magnetic flux or dielectric flux lines. Electrical conductivities of model parts are adjusted to correspond to magnetic or dielectric permeabilities of corresponding parts of the original design, and alternating or direct current is passed through the model. If magnetic flux lines are being studied, the

A paper recommended for publication by the A.I.E.E. committee on electrophysics, and scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted Jan. 9, 1935; released for publication Apr. 9, 1935.

paths of electric current flow and electric equipotential lines in the model correspond respectively to magnetic flux lines and magnetic equipotential lines in the original design. If dielectric flux lines are being considered, the paths of electric current flow and electrice quipotential lines in the model correspond respectively to dielectric flux lines and dielectric equipotential lines in the original design.

One method described in this paper involves, for the determination of magnetic flux lines, the making of electrical *conductivities* of model parts proportional to magnetic permeabilities of design parts; current is passed through the model between 2 contact strips, each of which is placed on a section of model surface which is known to be a *magnetic equipotential surface*. Electric equipotential lines in the model then correspond to magnetic equipotential lines. This method may be designated hereinafter as the electromotive force-magnetomotive force method.

The other method described involves, for the determination of magnetic flux lines, the making of electrical *resistivities* of model parts proportional to magnetic permeabilities of design parts; current is passed through the model between contact strips which are placed, not along magnetic equipotential surfaces, but along 2 known *lines of magnetic flux*. Electric equipotential lines in the model then correspond to magnetic flux lines. This method may be designated as the electromotive force-flux method.

These 2 experimental methods applicable to magnetic flux determination also are suitable for use in dielectric flux problems, it being necessary only to substitute potential difference for magnetomotive force, dielectric constant for magnetic permeability, and dielectric flux for magnetic flux. For the sake of simplicity, only magnetic flux cases will be considered in the following problems.

A Typical Example

As an example, part of a cross section through a motor or generator armature and an opposing field pole is shown in figure 1. One full tooth is shown in the drawing and parts of the adjacent 2 teeth are shown. The part of the field pole piece opposite the armature teeth is shown, and the cross sectioning is carried well below the tooth roots and well up into the pole piece.

Magnetic flux lines flow down through the pole piece and across the air gap, entering the armature teeth and flowing through to the section below the tooth roots. Lines A-A and A_1-A_1 are lines of magnetic flux, and it is noticeable that these particular lines chosen are straight and located on the centerlines of tooth and armature slot, respectively. These 2 lines therefore lie along planes of symmetry in the armature and field pole as established by the design, and advantage is taken of this circumstance, as will be shown later.

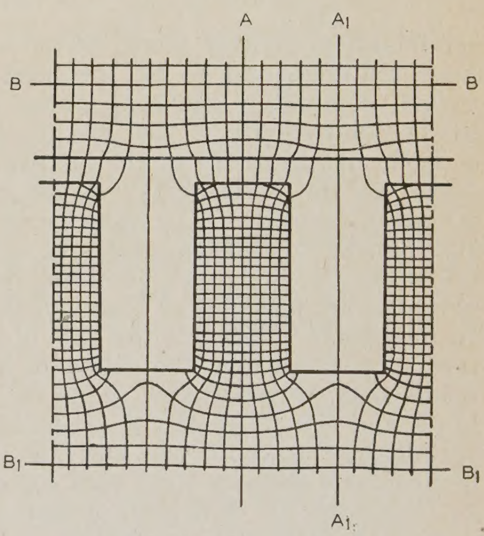
Magnetic equipotential lines are drawn in as the horizontal lines (except in the air gap). B-B and B_1-B_1 are magnetic equipotential lines, and it is noticeable that a short distance above the pole face and a short distance below the tooth roots, these lines are almost straight and parallel to the pole face. Thus by going to a depth below the tooth roots and above the pole face only slightly greater than the slot width of the armature, lines may be drawn parallel to the pole face and will substantially coincide with the magnetic equipotential lines.

Simple magnetic flux problems involving flux lines between equipotential surfaces separated by a homogeneous medium are commonly solved on an experimental basis by resistance models in which voltage difference represents magnetomotive force and current flow paths magnetic lines of force. However, such solutions have been limited to equipotential surfaces separated by a single medium.

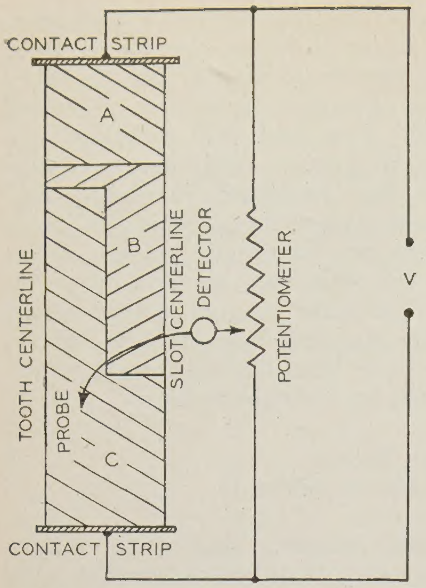
The Electromotive Force-Magnetomotive Force Method

In any design where magnetic lines of force may be present, advantage may be taken of the similarity of the law $B = \mu H$ (where B is the magnetic flux density, μ is the permeability of the medium and H is the intensity of the inducing force) to $I = GE$ (where I is the current density, G is electrical conductivity and E is the electric field intensity). In a scale model of a magnetic circuit, it is therefore necessary to substitute materials with the same ratios of electrical conductivities to each other that the original design exhibits in ratios of magnetic permeabilities of the corresponding parts to each other. Substituting electromotive force for magnetomotive force will then result in equipotential electric surfaces similar to magnetic equipotential surfaces in the original, and electric current flow lines similar in distribution to the magnetic flux lines in the original design. Designs which may be represented effectively by flat models in 2 dimensions such as in figure 1 are most readily solved.

Fig. 1. Flux map of section of armature and pole piece



Thus, to obtain the distribution of magnetic flux in a field pole piece of a given shape with a known air gap and an armature of given design as of figure 1, it is only necessary to make a plane model of the pole piece from a sheet of material of suitable electrical



Pole piece A, air gap B, armature C

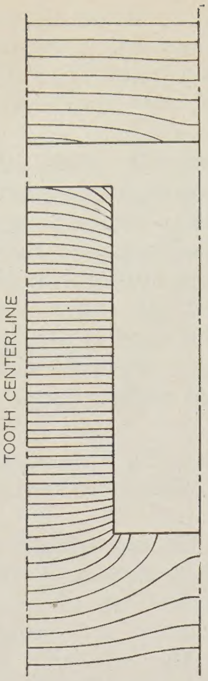


Fig. 2 (left). Resistance model for electromotive force-magnetomotive force method

Fig. 3 (right). Magnetic equipotential line plot by the method of figure 2

Armature and pole piece bounded by tooth and slot center lines

Air gap conductivity 0.005 mhos per cubic centimeter, armature and pole piece conductivity 0.5 mhos per cubic centimeter

conductivity, another model of the armature of the same material, and to fill the intervening air gap with a conducting material whose conductivity is to the conductivity of the pole piece and armature as is the permeability of the air gap in the original design to the permeability of the pole pieces and armature material. The model may be terminated and limited by known planes of symmetry established by the design in question, both as to equipotential lines such as B-B and B_1-B_1 and flux lines. In the smallest detail of the case of a slotted armature somewhere near the center of a field pole, the model may be terminated along the center lines of tooth and slot (A-A and A_1-A_1 in figure 1), and by parallel lines to tooth root and pole face surfaces removed some distance from such surfaces as B-B and B_1-B_1 in figure 1. When this is done, a model such as is shown in figure 2 is obtained.

Voltage is then applied to contact strips fastened along equipotential lines as in figure 2, and the model is explored by means of a probe connected to a voltmeter or a potentiometer to establish experimentally the contours of the electric equipotential lines existing in the model, which correspond to magnetic equipotential lines. The experimentally obtained lines are then transferred to paper on which the design under investigation is sketched as in figure 3. With the equipotential lines established, it is comparative y simple to sketch in the flux lines by the method of curvilinear squares.

The equipotential lines in figure 3 were plotted from data obtained from a model such as the one shown in figure 2. The conductivity ratio selected was 100/1, conductivity being proportional to permeability. The resistance materials used were wax compositions described later, whose characteristics are shown in figure 7.

The Electromotive Force-Flux Method

Where definite planes of symmetry are established by a design, and when the model may be reduced to a plane figure without materially altering flux distribution, a faster and more accurate method of determining flux distribution may be employed.

The equation $I = GE$ may be rewritten in the form $E = I/G$. In this form E corresponds to B in the equation $B = \mu H$, I to H, and $1/G$ to μ. Since $1/G$ is the volume resistivity of the model material, then the equation may again be rewritten $E = \rho I$ where ρ is the volume resistivity and is analogous to the permeability μ in the magnetic case. And by changing model conductivity ratio to the reciprocal of the ratio used in figure 2 as demanded by this new relation, lines B-B and B_1-B_1 of figure 1 become lines of electric current flow and lines A-A and A_1-A_1 become electric equipotential lines corresponding to magnetic flux lines.

Therefore, if it is possible to establish 2 magnetic equipotential lines as B-B and B_1-B_1, and the paths of 2 magnetic flux lines as A-A and A_1-A_1 in figure 1 to bound the region to be investigated, this new relation may be applied. In this instance voltage is not applied to the model in the same manner as before.

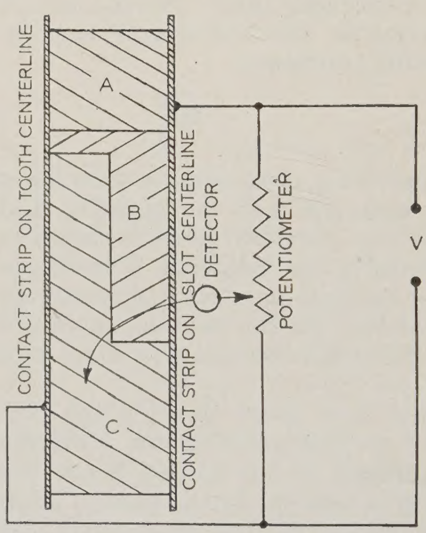


Fig. 4. Resistance model for electromotive force-flux method

Pole piece A, air gap B, armature C

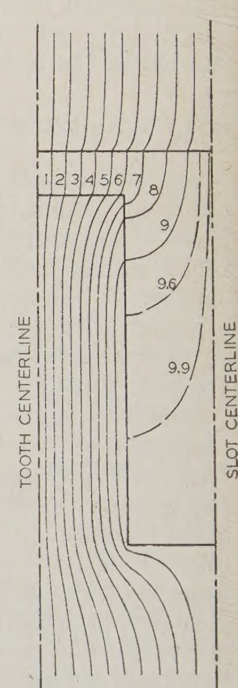


Fig. 5 (right). Magnetic flux line plot by the method of figure 4

Armature and pole piece bounded by tooth and slot center lines

Air gap resistivity 2 ohms per cubic centimeter, armature and pole piece resistivity 200 ohms per cubic centimeter

Here electric equipotential lines are correlated to magnetic flux lines, and paths of electric current flow are correlated to magnetic equipotential lines. Likewise, in the model the resistivities instead of the specific conductivities of the various parts are now proportional to the magnetic permeabilities of the corresponding parts in the original. Contact strips are placed along flux lines established by symmetry as in figure 4. The model is then explored with a probe and a voltmeter or potentiometer as was done in figure 2. The experimentally obtained electric equipotential lines are then plotted on a drawing of the parts under investigation, and correspond to the magnetic flux lines as required by the above relations.

A model with the same proportions as the model used to obtain data for figure 3 was used to obtain the data of figure 5. However, conductivity ratios were reversed as noted above using wax compositions in the model, and magnetic flux lines were plotted instead of magnetic equipotential lines.

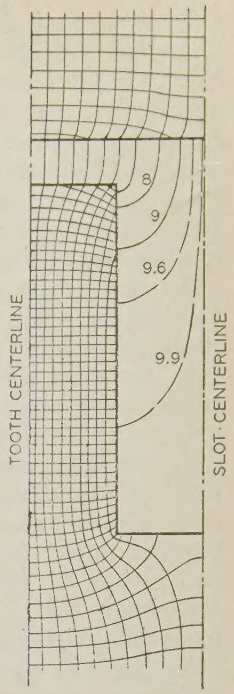
By superimposing figure 3 upon figure 5, the flux map of figure 6 is obtained, and it is seen by inspection that very good results are obtained. Most of the curvilinear squares approach true squares and flux line and equipotential line crossings are in the main right angle crossings. By using larger models and with greater care, quite accurate results are obtainable. The models used here were about 2 by 10 inches in size and somewhat small for the work.

Fig. 6. Magnetic flux map of armature and pole piece

Flux map obtained by combining figures 3 and 5

Ratio of iron to air gap permeabilities, 100

Model terminated on armature tooth and slot center lines



Experimental Difficulties to Be Avoided

In utilizing either the electromotive force-magnetomotive force or the electromotive force-flux method there are a number of experimental difficulties to be avoided. The first consideration is the question of conductivity or resistivity ratios in different model parts. In representing designs in which iron or steel magnetic paths are used in combination with air gaps, permeability ratios may vary from about 100/1 to several thousand/1.

From an inspection of figure 4 it is apparent that for the case of a slotted armature and a smooth pole face, the flux distributions at the interfaces of armature and air gap, and pole face and air gap are determined almost entirely by the shape of the air gap part of the model since this is the low resistance path in the model and its edges are in parallel with the relatively high resistance paths corresponding to the iron circuit. It is evident that large changes in resistivity ratios may be experienced with but small effect upon air gap flux distribution. As an extreme case, the air gap distribution may be obtained with armature and pole piece parts removed from the model. The armature and pole piece model parts may then be placed in position and if the resistivity ratio is assumed to be 100/1, the variation in flux distribution in the air gap at a given point will be of the order of one per cent or less. Thus, for ratios between $\infty/1$ and 100/1, the variations are probably less than one per cent.

The flux distribution in the armature and pole piece model parts is determined by the flux distribution at the air gap interfaces and by the angles at which flux lines enter the interfaces. The law of magnetic refraction, $\tan a / \tan b = \mu_a/\mu_b$, where the tangents are those of the angles of departure from the normal to the point of incidence of the magnetic line with the plane of separation of regions of magnetic permeability μ_a and μ_b, completely determines the paths of flux lines entering armature and pole face with a given interface distribution.

With a permeability ratio of 1,000/1 a flux line at 45 degrees to the normal in the iron would have a divergence of 3.4 minutes from the norma in air after traversing the interface. A change in permeability ratio or conductivity ratio from 1,000/1 to 100/1 would result in the same case with a divergence angle in the air gap of approximately 0.5 degrees. In this case also, it is seen that relatively large changes in ratio will exert but small influence upon flux distribution when ratios are large to start with.

Examining figure 2 and imposing the approximate air gap distribution obtained from the examination of figure 3, it is apparent that the high conductivity armature and pole piece parts will accept the interface flux distribution imposed by the air gap shape, and that potential drops in these parts will be attained with but negligible effect upon the potential drops across the air gap since they are of different orders of magnitude in a series circuit.

Thus, it is evident that air gap shape controls approximate air gap flux distribution, and flux distribution in the interfaces controls flux distribution and potential drops in armature and pole piece. Also, when conductivity ratios are large in the model, of the order of 100/1 or larger, comparatively large changes in ratio may take place with but small effect upon flux distribution.

The above does not hold true when saturation phenomena take place, such as magnetic saturation in iron parts. Unless saturation is carried in iron parts to high values, air gap flux distribution is but little affected; but when magnetic saturation is con-

siderable, flux distribution in iron parts may be changed appreciably due to permeability changes. Such changes usually are not serious until parts of the design are worked well above the knee of the saturation curve. Unless resistance materials are used in the model which match the magnetic saturation curve of the iron with similar conductivity or resistivity versus voltage curves, absolute accuracy cannot be obtained.

In the electromotive force-magnetomotive force method of flux mapping, there are at present no materials which may be interchanged easily for the iron parts in the model to exhibit such saturation phenomena since they would have to show an increase of resistance for an increase in voltage. However, in the case of the electromotive force-flux method, nonlinear resistance materials are available whose characteristic curves of voltage versus current may be made to approximate magnetic saturation curves due to a falling resistance characteristic with increases in voltage. Resistance materials with inclusions of silicon carbide, of which "thyrite" is an example, exhibit such nonlinear resistance characteristics. In the majority of cases it is not necessary to add the refinement of matching magnetic saturation curves and ordinary resistance materials are sufficient.

The selection of suitable workable resistance materials for the construction of models may be a source of trouble in obtaining resistance ratios of the order of 1,000/1. Thin metal sheets may be used in very simple models, but such models are low resistance

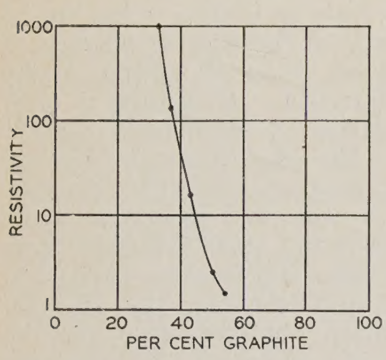


Fig. 7. Model resistance material

Resistivity in ohms per cubic centimeter of wax and graphite mixtures at 25 degrees centigrade

Superla wax and flake graphite

models and contact resistances and contact differences of potential introduce large errors. Soldering of contact surfaces introduces surplus material and causes errors; equipotential contact strips are also difficult to place in low resistance models without contact resistance difficulties. Thermoelectric effects also cause errors in low resistance models with d-c excitation.

The use of carbon or graphite plates cut to the proper shapes with mercury as the high conductivity medium results in quite large errors due to contact resistances.

Electrolytic models are in general unsatisfactory due to polarization and diffusion capacity effects. For instance, using alternating current to excite the model and a potentiometer with a pair of telephone receivers as the detector, a null point cannot be found on the model unless the potentiometer arms are corrected to the proper phase angle for each point. Electrolytic polarization introduces harmonics of the fundamental excitation voltage in the model in addition to phase angle rotation of the fundamental.

Wax Models

High resistance models may be made using wax with graphite, carbon, silicon carbide, etc., inclusions. Figure 7 is a characteristic resistivity curve of a wax with varying proportions of graphite added. Quite wide resistivity ratios may be obtained by controlling the amount of graphite, etc. The wax is melted, the resistance material added and well mixed in.

In making models, the melted compositions are poured into shallow forms or molds at a temperature just above the melting point of the wax. Too high a pouring temperature will result in settling out of the graphite. After the wax has set, the forms are removed and the remainder of the model is poured using the mixture of the other resistivity value. Metal contact strips along equipotential lines should be placed in position before pouring the wax. If hot contact strips are placed in cold wax, high resistance joints result.

Cross section lines are readily ruled on the wax surfaces, and for photographing, equipotential lines may be plotted directly on the wax with a stylus.

After use, such resistance materials may be broken up and remelted for further use. Model parts may also be cut from previously cast flat plates.

Three dimensional models may be made in soft wax and explored by means of an insulated probe with an exposed tip. Potential values at various depths and at various locations may be obtained without much difficulty. Such models are particularly well adapted to dielectric flux mapping in insulators which do not yield to other methods of analysis.

In obtaining potential values for plotting equipotential lines or flux lines, the probe may be moved about on the model until the desired balance point is located. A voltmeter may be used to indicate the potential of a point, although this is not recommended since the current required to operate a voltmeter is usually large compared to currents normally flowing under the area covered by the tip of the probe. A vacuum tube voltmeter would be satisfactory in most cases however.

The most satisfactory method of obtaining potential measurements is by means of a potentiometer and detector as shown in figures 2 and 4. Either alternating current with a galvanometer or telephone receivers, or a d-c source of power with a galvanometer detector is quite satisfactory and independent of variations in voltage or frequency of the power supply for low frequency models.

Flux maps shown in this paper were obtained from models with a potentiometer, telephone receivers, a low voltage vibrator as a source of power at about 200 cycles per second, and wax-graphite resistance materials with resistivities of 2 and 200 ohms per cubic centimeter.

Operational Method of Circuit Analysis

By B. L. ROBERTSON, Member A.I.E.E.
University of California, Berkeley

For that broad group of electrical engineers not familiar with Heaviside's operational method and its value in the analysis of electrical circuits, this article presents briefly some of the fundamental characteristics of the method and explains how it is applied; its application is illustrated by the formulation of operational equations for a few typical circuits. Although intended primarily to impart only the fundamentals of the subject to an engineer who has had ordinary mathematical training, it is hoped that the article also may serve as an introduction and guide to further study.

APPLICATION of operational methods to circuit studies has been growing rapidly within recent years, and more and more this form of analysis is achieving greater importance as a tool for regular use in handling engineering problems. The operational calculus is found to be of invaluable service in the solution of transients and in the treatment of dynamical systems, whether of an electrical or a mechanical nature. The steady advance made in the general knowledge and use of the Heaviside operational method is shown by the increasing number of technical papers that employ it.

This article has been written because of the spreading interest definitely indicated in operational circuit analysis, and the desire on the part of many who are unfamiliar with it to understand something of the principles of its application. The intention here is not to treat with the mathematics as such or with recent developments in the operational calculus. Merely, it is the aim to present what to the writer are some of the fundamental characteristics of the operational method, to show the formulation of the operational equations for a few circuits, and to discuss briefly some of the methods by which the solutions to these equations may be obtained. Stress is given the physical reasoning behind the steps.

Nature of Transients

Studies in engineering and physics give rise to problems on simple circuits, networks, and systems, these problems relating to any one of the major fields of electricity, mechanics, heat flow, sound, etc. Those of particular interest here are on transients in electrical circuits. The divisions of the field of electrical engineering in which transient analysis is of importance are many, as indicated by the applications Gardner has listed.[1]

In general, the study of electrical transients is the investigation of the response of such systems to disturbances produced in them. Specifically, the analysis of transients in electrical circuits involves the determination of the variation of charge, current, voltage, power, or energy with time while the system is undergoing a readjustment. It is inherent in all physical systems that even though the disturbing force may be applied instantaneously, time is required for the system to pass from one steady state condition to another. Time, therefore, is the independent variable, the quantity under investigation the dependent variable, to be expressed in the end as a function of time.

Heaviside's Objective

Briefly stated, the objective of Heaviside in analyzing circuits was to express the solutions of the problems in terms of functions of his operators, and then to assign such significance to these relations that their interpretations would be the *correct* solutions to his problems. The solutions were to be obtained as directly as possible, and were to give answers without further adjustment by way of any consideration of constants or functions of integration to include boundary conditions, as must be done in solving differential equations. The great majority of problems with which Heaviside treated involved systems initially at rest, the disturbing forces thus being zero for all values of time less than zero, and taking on their normal variations after zero time. Heaviside took advantage of this fact and sought to obtain solutions that automatically applied to such boundary conditions, his developments leading to this form of operational mathematics.

The operational equation, which is the initial expression for a problem in terms of operators and the circuit constants, must contain, therefore, all information necessary to solve the problem completely. The solution to the operational equation is to be the same as that for the differential equation after initial conditions have been included, and is to yield at once the total result, which for circuit transients is the transient plus the steady state variations of the quantity studied.

Heaviside's own method of interpreting his operational functions, and in converting them into explicit functions of time, appears to be one almost entirely experimental. It would seem that he must have applied his method to problems having known solutions, and in this way determined how well his general rules worked and what he could or could not do. No doubt such steps provided him with the body of rules which he used in his analyses but on which he gave little or no discussion in his technical

Written especially for ELECTRICAL ENGINEERING, based upon an oral presentation before the A.I.E.E. San Francisco Section, San Francisco, Calif., Feb. 8, 1935.

1. For all numbered references see list at end of article.

papers. Because he studied circuits of the linear type on which a direct voltage was impressed at time $t = 0$, he had as a guide the solutions of the differential equations for those circuits. The solutions of simple problems were not difficult to obtain. His aim was to secure the same results by means of his operators without having to make recourse to differential equations or to problem boundary conditions, once the operational equations were written. By comparing operational forms for the same problems with the solutions obtained otherwise, the significance of these forms was indicated.

It may be well to state at this point that once an operational function is found to be related definitely to a given form of solution, the same operational expression met in any other problem will carry the same interpretation. Its translation in terms of a time function may be written immediately. The operational expressions with their time function equivalents, as a table, therefore become fundamental to the working use of the operational calculus as the integral table is to the ordinary calculus.

The correlation existing between the operational calculus and classical mathematics has been brought about in relatively recent years, and the mathematical material presented today in support of operational methods was not touched upon by Heaviside. Very much of it, of course, did not exist then, and mathematical rigor as a defense for his developments did not appeal to Heaviside, particularly in view of the fact that he was obtaining correct results for complicated problems. He was concerned primarily that his mathematics give him accurate answers in a quick and satisfactory way. Guided greatly by intuition and his wealth of knowledge on the physics behind his circuit studies, he developed the operational calculus now ascribed to his name.

Formulation of the Problem

The first step toward the solution of a problem is the derivation, from the fundamental *physical* relations of the system, of the *analytical* equations that adequately are to represent the system. The mathematical expressions thus obtained become the shorthand statements of the conditions surrounding the problem, and are to convey the same thoughts that otherwise would be stated in a longer way by words. That is, the mathematical formulation of engineering problems is nothing else than the symbolizing of physical ideas. The effort required to symbolize a problem depends upon the end desired, whether steady state conditions only are to be considered, or partial studies of the aspects of transient phenomena are to be carried out, or the complete solution including both transient and steady state relations is to be found.

Not all problems can be formulated in this manner because of the complexities encountered, and the difficulty in considering all the variables and the way in which they possibly should be introduced. The phenomena under treatment in this discussion are those that may be described by linear differential equations or the operational calculus.

Methods of Solution

The second step in the solution of a problem consists of carrying out the formal mathematical operations by which the initial circuit equations may be said to be solved. The resulting system equations usually will be expressed also in symbolic form, consequent calculations giving numerical results. Fundamentally, the methods of solving engineering problems fall into 2 major divisions. The first of these is qualitative, based upon reasoning from the physics of the problem together with reasoning from past experiences. The second division is quantitative, under which experimental schemes and mechanical devices may be employed as well as mathematics. Because of its power as a tool in the analysis of circuits, the operational calculus is becoming more prominent and is supplanting other forms of solution in many instances.

The Operator and the Unit Function

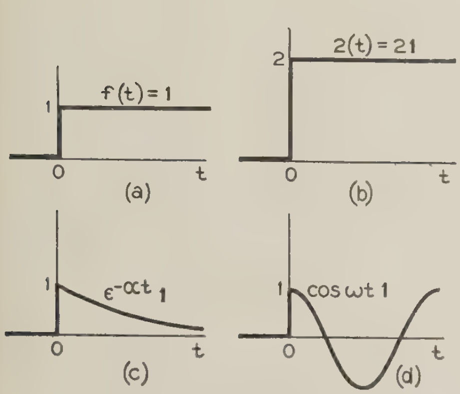
The operational calculus generally is typified by 2 symbols, the operator p, and the unit function **1**. The operator in its use probably is more mathematical than physical, the unit function more physical than mathematical. Operators are symbolic quantities indicating certain steps to be followed, or calling attention to given interpretations to be recognized. Operators are not new and engineers are familiar with many of them, for example, the trigonometric symbols "sin," "cos," "tan," etc.; the logarithmic and exponential notations; the operators j, and a (the latter met in studies on symmetrical components); and others such as D ($= d/dx, d/dt$, etc.). These quantities point to definite processes to be performed, although they are not always considered as operators.

The operator p in the Heaviside calculus [2,3] initially is to represent the time differentiator d/dt. Further, it is desired that this operator bear the reciprocal relation such that $1/p$ denote an integration. To make the operator as useful as possible it also should obey the ordinary rules of algebra. In addition to p indicating an operation, those who are acquainted with the solutions to linear differential equations will observe that it is the variable in the characteristic equation. The symbol p as used in the Heaviside operational calculus thus has several different interpretations.

If the purposes Heaviside had in mind are to be achieved, the operator p may not be defined presumptively. Its interpretation depends upon the physical nature of the problem at hand, and its meaning will change with transformations of the operational equations that contain it. The operational equation may be obtained from the corresponding differential equation by making the substitutions for the time differentiators, i. e., $p = d/dt$, ... $p^n = d^n/dt^n$, with the accompanying reciprocal relations $1/p = \int dt$, etc. In problems on transmission lines and in heat flow appear expressions containing p to half powers, i. e., $p^{1/2}$, $p^{3/2}$, etc. In no sense, though, does $p^{1/2}$ imply $(d/dt)^{1/2}$. Such a relation does not exist, and no significance may be

attached to it. Fractional powers of p must be interpreted as they are met, and in view of the problem itself. The operator p, as it appears in its various manners, is not subject to any single definition and may not be characterized completely as yet.

The unit function,[3, 4] **1**, is a discontinuous function of time, and is defined as the function plotted in figure 1*a*; it is a function that is continuously zero until the time $t = 0$, and continuously unity thereafter. It thus displays the discontinuous form of the disturbing force, as illustrated further by the 3 other graphs of figure 1. The unit function is not



$f(t) = 0$ from $t = -\infty$ to $t = 0$ and $f(t) = f(t)$ from $t = 0$ to $t = +\infty$

For simplicity, the complete description of these conditions for the disturbing force is given in the form $f(t)$**1**, the unit function to show the discontinuity

Fig. 1. Significance of the unit function

mentioned explicitly by all writers on operational calculus, but even when omitted in discussion or in their mathematics it usually is implied. Its principal use is to avoid a great many unnecessary words, for once it has been defined only $E(t)\cdot$**1** need be written to indicate an electromotive force that is applied at time $t = 0$ and thereafter has the value $E(t)$. Without the notation it would be necessary to explain that this is what is meant.

The following characteristics thus represent essentials in the operational method:

1. $p = d/dt$, $p^2 = d^2/dt^2$, ... $p^n = d^n/dt^n$; p is the time differentiator.

2. $\frac{1}{p}K\mathbf{1} = \int_0^t K\mathbf{1}dt = Kt$ (which may be written $Kt\mathbf{1}$, if desired); the definite integral from zero time to time t

   $\frac{1}{p^2}K\mathbf{1} = \int_0^t\int_0^t K\mathbf{1}dt^2 = \frac{Kt^2}{2}, \ldots\ldots$

   $\frac{1}{p^n}K\mathbf{1} = \frac{Kt^n}{n!}$, by repeated applications of $\frac{1}{p}K\mathbf{1}$; the inverse process of differentiation, but a definite integration.[5]

3. $p\frac{1}{p}K\mathbf{1} = K\mathbf{1}$; p obeys the ordinary rules of algebra.

Formulation of Operational Equations

With the foregoing discussions kept in mind, one turns next to the formulation of the operational equations for several simple circuits in order to investigate the manner in which they are expressed. Figure 2 shows a few typical circuit diagrams, these having been chosen purposely to illustrate certain fundamental features in deriving the operational equation. Space does not permit the inclusion of merited discussions on other circuit conditions of importance; these must be left to the reader to study.

Figure 2*a* represents a circuit of resistance and inductance to which is applied a direct voltage of E volts at $t = 0$, the current response being desired. From Kirchhoff's second law that, when taken with the proper algebraic signs, the sum of the voltages around a closed loop is zero, the differential equation for the circuit becomes

$$-Ri - L\frac{di}{dt} + E = 0$$

or

$$L\frac{di}{dt} + Ri = E \qquad (1)$$

The 2 terms involving the current carry negative signs with respect to the impressed electromotive force because these 2 voltages are in opposition to it.

Replacing the time derivative by the operator[6] p and introducing the unit function to call attention to the form of the applied voltage, the operational equation for the circuit becomes

$$Lpi + Ri = E\mathbf{1} \qquad (2)$$

Treating p algebraically, the operational solution for the current is

$$i = \frac{1}{Lp + R}E\mathbf{1} = \frac{E}{L}\frac{1}{p + \frac{R}{L}}\mathbf{1}$$

$$= \frac{E}{L}\frac{1}{p + \alpha}\mathbf{1} \qquad (3)$$

where $\alpha = R/L$. The operational impedance function for the circuit is designated by $Z(p) = (Lp + R)$. Solving the problem means finding a correct interpretation of $1/(p + \alpha)$ operating upon unit function.

For figure 2*b* there results (all capacitances in the examples are assumed initially uncharged)

$$Ri + \frac{\int idt}{C} = E \qquad (4)$$

which in operational symbols becomes

$$Ri + \frac{1}{C}\frac{1}{p}i = E\mathbf{1} \qquad (5)$$

and from which

$$i = \frac{1}{R + \frac{1}{pC}}E\mathbf{1}$$

$$= \frac{E}{R}\frac{p}{p + \alpha}\mathbf{1} \qquad (6)$$

where α now is $1/RC$. Just as a note of interest, those familiar with the elements of circuit theory will recognize in α, for both of these circuits, the reciprocal of the circuit time constants.

In like manner to the above developments, the

operational solution for the total current in the circuit given by figure $2c$ is

$$i = \frac{1}{Z(p)}E1 = \frac{1}{r + \dfrac{(R+pL)\left(\dfrac{1}{pC}\right)}{R + pL + \dfrac{1}{pC}}}E1$$

$$= E\frac{p^2LC + pCR + 1}{p^2LCr + p(CRr + L) + (R + r)}1 \qquad (7)$$

It may be observed that the operational impedance function as first written in terms of the circuit elements is identical in form to that obtained for the equivalent impedance of a series-parallel circuit when expressed in terms of complex quantities. The operator p replaces $j\omega$. One initially may handle circuits in this manner, later substituting the proper operational equivalents for the corresponding circuit elements and reducing the resulting algebraic equation to its simplest form.

The circuit of figure $2d$ assumes a current of constant magnitude i suddenly forced through a parallel combination of capacitance and inductance. The resulting circuit voltage, which now is desired, is

$$e = Z(p)i1 = \frac{pL\cdot\dfrac{1}{pC}}{pL + \dfrac{1}{pC}}i1 = \frac{pL}{p^2LC + 1}i1$$

$$= \frac{i}{C}\frac{p}{p^2 + \alpha^2}1 \text{ (as usually expressed)} \qquad (8)$$

where for this circuit $\alpha = 1/\sqrt{LC}$.

The circuit of figure $2e$ is the same as that of figure $2d$, but the current now is sinusoidal in form, the switch being closed at the instant the current is passing through zero and increasing positively. Since the impedance function for the circuit is the same as that just found, the voltage across the circuit elements becomes

$$e = Z(p)i1$$

$$= \frac{I}{C}\frac{p}{p^2 + \alpha^2}(\sin\omega t)1 \qquad (9)$$

The particular point to be emphasized here is that the operand is the entire part of the expression $(\sin\omega t)1$, and $p/(p^2 + \alpha^2)$ operates upon it as a whole. In other words, $\sin wt$ may not be treated simply as a multiplying factor, thus to retain its identity, nor may $p/(p^2 + \alpha^2)$ be evaluated by itself. The entire right-hand member, except for the constant term I/C, must be interpreted as a unit. This process is to be shown later.

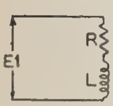
In order to handle properly the right hand member of equation 9 it is convenient to convert the time function $\sin\omega t$ to its operational equivalent which later is shown to be

$$\sin\omega t = \frac{p\omega}{p^2 + \omega^2}1 \qquad (10)$$

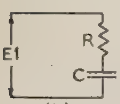
The operational equation for the voltage therefore is found to be

$$e = \frac{I}{C}\left(\frac{p}{p^2 + \alpha^2}\,\frac{p\omega}{p^2 + \omega^2}\right)1 \qquad (11)$$

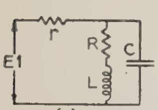
from which, with correct interpretation of the



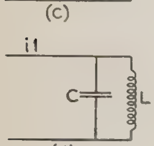
$$L\frac{di}{dt} + Ri = E \qquad Lpi + Ri = E1 \qquad i = \frac{E}{Lp + R}1 = \frac{E}{L}\frac{1}{p + \frac{R}{L}}1 = \frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right)$$



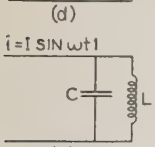
$$Ri + \frac{\int idt}{C} = E \quad Ri + pC\,i = E1 \qquad i = \frac{E}{R + \frac{1}{pC}}1 = \frac{E}{R}\frac{p}{p + \frac{1}{RC}}1 = \frac{E}{R}\epsilon^{-\frac{t}{RC}}$$



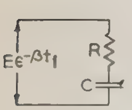
$$iZ(p) = E1 \qquad Z(p) = r + \frac{(R + pL)\left(\frac{1}{pC}\right)}{R + pL + \frac{1}{pC}} \qquad i = \frac{p^2LC + pCR + 1}{p^2LCr + p(CRr + L) + (R + r)}E1$$



$$e = Z(p)i1 \qquad Z(p) = \frac{pL\frac{1}{pC}}{pL + \frac{1}{pC}} \qquad e = \frac{pL}{p^2LC + 1}i1 = \frac{i}{C}\frac{p}{p^2 + \frac{1}{LC}}1 = i\sqrt{\frac{L}{C}}\sin\frac{t}{\sqrt{LC}}$$



$$e = Z(p)i1 = \frac{pL}{p^2LC + 1}I\sin\omega t1 \qquad e = \frac{I}{C}\left(\frac{p}{p^2 + \frac{1}{LC}}\,\frac{p\omega}{p^2 + \omega^2}\right)1 = \frac{IL\omega}{\omega^2LC - 1}\left(\cos\frac{t}{\sqrt{LC}} - \cos\omega t\right)$$



$$Ri + \frac{i}{pC} = E\epsilon^{-\beta t}1 \qquad i = \frac{1}{R + \frac{1}{pC}}E\epsilon^{-\beta t}1 \qquad i = \frac{E}{R}\frac{p}{p + \frac{1}{CR}}\frac{p}{p + \beta}1 = \frac{E}{1 - RC\beta}\left(\frac{1}{R}\epsilon^{-\frac{t}{RC}} - C\beta\epsilon^{-\beta t}\right)$$

Fig. 2. Circuit diagrams and associated equations illustrating the application of the operational method

quantity within the parentheses operating upon unit function, the solution is reached.

The last illustration, given by figure 2f, involves the same point made in the previous example. The expression for the current here is sought, whence

$$i = \frac{1}{R + \dfrac{1}{pC}} E\epsilon^{-\beta t} 1$$

$$= \frac{E}{R}\left(\frac{p}{p+\alpha}\,\frac{p}{p+\beta}\right)1 \qquad (12)$$

where $\alpha = 1/RC$. The same statements hold as before for the operation of the entire quantity within the parentheses on unit function. Both problems indicate the requirement of converting a time function, other than a constant, in the operand into its equivalent operational form before proceeding to the final solution. For the last circuit, however, there is another method, Heaviside's shifting theorem, which will remove the exponential term from the operand. There are several ways of treating this problem.

To summarize briefly, the operational equation is an expression in terms of constants of the circuit and functions of the operator p. It is simply the formulated statement, expressed in symbols, of the physical problem under consideration. For a single circuit it stands exactly for the differential equation with its surrounding conditions which otherwise might be written, and its initial form may be obtained directly from the differential equation if desired. For a system or a network of n loops, there results a system of n simultaneous operational equations just as there also may be derived a system of n simultaneous differential equations. The operational equations, properly interpreted, must give solutions identical to those found for the differential equations with their boundary conditions included. Operational equations hence may be said to represent in abbreviation the actual equations, differential or integral, for the problem, together with the special form of boundary conditions which implies that the system initially is at rest.

Solutions to the Operational Equations

It is necessary next to determine the interpretations for the operational equations, and, for value to the reader, to solve those developed in the preceding section of this article. Setting up the operational equation for a circuit is relatively simple, but to obtain its solution is an entirely different matter. Leaving aside any discussion of the so-called classical methods of solving circuits, i. e., by differential or integral equations, attention is restricted to purely operational modes of attack of which there are in general several for most problems.

Solutions to operational equations may be found in several ways, prominent ones of which are:

1. Interpretations of operational expressions from known solutions. The operational equation for a specific circuit must yield the same result as that found by other means, e. g., by differential equations. Heaviside interpreted the operator $p^{1/2}1$, for instance, as $(\pi t)^{-1/2}$ from the known solutions to heat flow problems.[7]

2. Long division of the operational equation to obtain a series, each term of which then is evaluated. The method includes binomial theorem expansions. Heaviside referred to this method as "algebrizing."[8]

3. Heaviside's expansion theorem.[9–17]

4. Partial fraction expansion of the operational equation into recognizable parts, a method identical to that employed in the integral calculus to assist in the evaluation of integrals, and a standard method in the theory of differential equations for many years.[18,19]

5. Heaviside's shifting theorem, to remove exponential time factors from the operand.[20–22]

6. Borel's theorem, relating to the product of 2 functions.[23–25]

7. Contour integration of the equivalent Fourier integral.[26–30]

8. Carson's integral equation.[31–35]

Some of these methods will be illustrated by application to the problems presented in figure 2.

Reasoning From Known Solutions

The solutions for the currents in the circuits of figures 2a and 2b are known readily—in fact, these 2 problems are perhaps the first ones discussed in any elementary study of transients. After the general solutions of the respective differential equations have been obtained, substitution of initial conditions gives the particular solutions. Determination of the constants of integration depends upon the physical relations that, in the first circuit $i = 0$ when $t = 0$, and in the second circuit, $i = E/R$ when $t = 0$. The currents are expressed explicitly as functions of time and are, respectively

$$i = \frac{E}{R}\left(1 - \epsilon^{-(R/L)t}\right) \qquad (13)$$

and

$$i = \frac{E}{R}\epsilon^{-t/RC} \qquad (14)$$

Since the solutions of the operational equations are to yield the same results, of necessity, then the interpretations of equations 3 and 6 follow immediately. Comparison of equations 3 and 13 gives

$$\frac{E}{L}\,\frac{1}{p+\alpha}1 = \frac{E}{R}\left(1 - \epsilon^{-(R/L)t}\right)$$

whence

$$\frac{1}{p+\alpha}1 = \frac{1}{\alpha}\left(1 - \epsilon^{-\alpha t}\right) \qquad (15)$$

as a general relation. Similarly, comparison of equations 6 and 14 gives

$$\frac{E}{R}\,\frac{p}{p+\alpha}1 = \frac{E}{R}\epsilon^{-t/(RC)}$$

whence

$$\frac{p}{p+\alpha}1 = \epsilon^{-\alpha t} \qquad (16)$$

It may be observed that whatever α may be

$$\frac{p}{p+\alpha}1 = p\left(\frac{1}{p+\alpha}1\right) = \frac{d}{dt}\left[\frac{1}{\alpha}\left(1 - \epsilon^{-\alpha t}\right)\right] \qquad (17)$$

In like manner

$$\frac{1}{p+\alpha}\,1 = \frac{1}{p}\left(\frac{p}{p+\alpha}\,1\right) = \int_0^t \epsilon^{-\alpha t}dt \qquad (18)$$

The time derivative or integral obviously is taken of the equivalent time function for the operational expression. It should be understood that the juxtaposition of 2 operational functions does not mean the product of their respective time functions. An operation is implied, and the entire quantity must so be treated.

Although these 2 examples are given for special and the simplest of circuits, the time function equivalents of the corresponding operational expressions are perfectly general and may be used on all other circuits that yield the same operational forms. There is only one time function that is related to a given operational expression.

To illustrate the value of what has just been presented, consider the operational equivalent of sin ωt (refer to equation 10):

$$\sin \omega t = \frac{1}{2j}\left(\epsilon^{j\omega t} - \epsilon^{-j\omega t}\right) = \frac{1}{2j}\left(\frac{p}{p-j\omega} - \frac{p}{p+j\omega}\right)1$$

$$= \frac{p\omega}{p^2+\omega^2}\,1 \qquad (19)$$

Similarly

$$\cos \omega t = \frac{p^2}{p^2+\omega^2}\,1 \qquad (20)$$

Both relations are of importance in operational circuit analysis.

Operational Division

The second manner of evaluating operational forms is that of long division, which for equations 3 and 6 gives the binomial expansions for those forms. To illustrate, when applied to equation 6 there results

$$\begin{aligned}\frac{p}{p+\alpha}\,1 &= p(p+\alpha)^{-1}1\\ &= p(p^{-1} - p^{-2}\alpha + p^{-3}\alpha^2 - p^{-4}\alpha^3 + \ldots\ldots)\,1\\ &= \left(1 - \frac{\alpha}{p} + \frac{\alpha^2}{p^2} - \frac{\alpha^3}{p^3} + \ldots\ldots\right)1\\ &= \left(1 - \alpha t + \frac{\alpha^2t^2}{2!} - \frac{\alpha^3t^3}{3!} + \ldots\ldots\right)\\ &= \epsilon^{-\alpha t}\end{aligned} \qquad (21)$$

The fourth step is obtained from the third by application of $\frac{1}{p^n}\,1 = \frac{t^n}{n!}$, and the resulting series is recognized as the expansion of $\epsilon^{-\alpha t}$. The same method of expansion holds for $\frac{1}{p+\alpha}\,1$.

The solutions thus obtained are complete; they include the transient and the steady state variations of the currents. In the method of operational division, or where p appears in the denominator as it does in the foregoing expressions, it is the added restriction of making the reciprocal of the operator p the definite integral from zero time to time t that has the effect of including the constants of integration in the solution. Other methods of solution, one of which has been illustrated in the preceding section, give the correct result without recourse to integration.

More complicated operational expressions may be treated similarly through some convenient form of expansion of the operator into a power series, often by formal division of the numerator by the denominator of the operational fraction. The objection to operational expansions of this kind, however, is that they lead to infinite series which at times are extremely difficult, if not impossible, to recognize and sum. Furthermore, they may be of no, or very limited, value for numerical computations, though they are sometimes very useful in obtaining approximate results.

One point should be presented before passing on. The process of "algebrizing" the operational equation will give 2 series, depending upon the manner of expansion. The binomial theorem, for example, always can be written in 2 ways $(p+\alpha)^{-1}$ or $(\alpha+p)^{-1}$. In the first instance p appears in the denominators of the consequent terms, and in the second instance in the numerators. Where p occurs in descending powers, as it does in equation 21, the time function may be evident in its relation to the correct problem solution. With p appearing in whole powers in the numerators, the time function being 1, a series of zeros results and correspondence with the true solution is not observed. In the example just given it has been necessary to expand in inverse powers of p as shown.

Series expansions of operators may lead to a convergent series, or to a divergent or asymptotic solution. One is satisfactory for numerical computation for small values of time, the other for large values of time. Failure, however, may occur with either series, the algebrizing process breaking down. In brief, Heaviside's method of series expansion of the operator will not always give correct results automatically; caution, therefore, must be exercised in obtaining such forms of solution. One should investigate the correctness of the methods employed and the completeness of his results.

The Expansion Theorem

With but little discussion, Heaviside gave, in volume 2 of Electromagnetic Theory, and volume 2 of Electrical Papers, his expansion theorem. The theorem is a formal means by which the solutions of operational equations may be found directly. It is applicable to most problems, including a great many on transmission lines and heat flow where the constants are distributed rather than lumped. When the theorem does apply it is probably the most convenient method to use.

No derivation of the expansion theorem is given here, merely the statement of it:

$$i = E\left[\frac{1}{Z(0)} + \sum_{p_1}^{p_n} \frac{\epsilon^{p_k t}}{p_k Z'(p_k)}\right]1 \qquad (22)$$

In brief, $Z(0)$ is the operational impedance function in which p and its powers have been replaced by zero; $p_1, p_2, \ldots\ldots p_n$ are the n roots of the equation $Z(p) = 0$; and $Z'(p_k)$ is $d/dp\ [Z(p)]$, after which the par-

ticular root p_k, for the kth term under consideration, is substituted for the general root p.

To show the steps in the use of the theorem, consider the R-L circuit of figure 2a. Here,

$$Z(p) = Lp + R = 0 \tag{23}$$

and only one root, $p = p_1 = -R/L$, exists; $Z(0) = R$ and $Z'(p) = L$, whence $p_1 Z'(p_1) = -R$. The solution for the current is

$$i = E\left[\frac{1}{R} - \frac{1}{R}\epsilon^{-(R/L)t}\right] \tag{24}$$

The circuit of figure 2c is solved more conveniently by the expansion theorem than by other means, although when written out completely in symbolic form the algebraic expressions are somewhat cumbersome to manipulate. With circuit data: E = 100 volts; r = 100 ohms; R = 150 ohms; L = 0.1 henry; and C = 80 microfarads, the solution for the current is

$$i = 0.40 + 0.6075\epsilon^{-223t} - 0.0075\epsilon^{-1402t} \tag{25}$$

which the reader may verify.

It may be seen that the term $Z(0)$ leads to the steady state current in the examples shown, the exponentials yielding the transient components. With an alternating voltage impressed on the circuits, the term $1/Z(0)$ disappears, and the steady state solutions come from the exponentials, those particular ones having imaginary indices $\pm j\omega t$ which lead to $\sin \omega t$ or $\cos \omega t$, the form of the applied voltage.

The expansion theorem often is easily handled, and, when such is the case, leads to solutions that readily can be evaluated numerically. The greatest difficulty encountered in its use enters in determining the roots of $Z(p) = 0$, particularly when $Z(p)$ is of degree in p higher than the fourth. This same difficulty, however, is met in factoring all algebraic equations and is not peculiar in any way to the expansion theorem. One of the restrictions on the expansion theorem is that in its development zero and repeated roots are excluded. This restriction is not one of much consequence because physical problems tend only to approach such conditions (as expressed in this mathematical manner), and that but very seldom.

Partial Fraction Expansion

Partial fraction expansion, the basic principle underlying the expansion theorem, is the operation of decomposing a given fraction into a group of simpler fractions. These partial fractions have denominators that are factors of the denominator of the given fraction, and hence the latter is converted into a sum of expressions each one of which can be treated individually and more conveniently. The given fraction usually is reduced to a proper fraction before being decomposed, unless the degree of the numerator is already less than that of the denominator. It is assumed that the reader will recall the method of partial fraction expansion.

This method applied to equation 8, which was derived for the circuit of figure 2d, gives for the voltage,

$$\begin{aligned} e &= \frac{i}{C}\frac{p}{p^2+\alpha^2}1 \\ &= \frac{i}{C}\left[\frac{1}{2(p+j\alpha)} + \frac{1}{2(p-j\alpha)}\right]1 \\ &= \frac{i}{C}\left[\frac{1}{2j\alpha}(1-\epsilon^{-j\alpha t}) - \frac{1}{2j\alpha}(1-\epsilon^{j\alpha t})\right] = \frac{i}{C\alpha}\frac{\epsilon^{j\alpha t} - \epsilon^{-j\alpha t}}{2j} \\ &= i\sqrt{\frac{L}{C}}\sin\frac{t}{\sqrt{LC}} \end{aligned} \tag{26}$$

Observe also, that

$$e = i\sqrt{\frac{L}{C}}\frac{p\alpha}{p^2+\alpha^2}1 = i\sqrt{\frac{L}{C}}\sin\frac{t}{\sqrt{LC}}$$

an application of the relation given by equation 19. This problem can be attacked equally as well by use of the expansion theorem.

Solving equation 11, for the circuit of figure 2e, the method of partial fraction expansion gives for the voltage

$$\begin{aligned} e &= I\sqrt{\frac{L}{C}}\left(\frac{\alpha\omega}{\omega^2-\alpha^2}\frac{p^2}{p^2+\alpha^2} - \frac{\alpha\omega}{\omega^2-\alpha^2}\frac{p^2}{p^2+\omega^2}\right)1 \\ &= \frac{IL\omega}{\omega^2 LC - 1}\left(\cos\frac{t}{\sqrt{LC}} - \cos\omega t\right) \end{aligned} \tag{27}$$

Heaviside's Shifting Theorem

To show the manner in which an exponential time function is removed from the operand of an operational equation, Heaviside's shifting theorem is employed. The theorem, not proved here, allows an exponential, such as $\epsilon^{-\beta t}$, to be shifted from the operand by the substitution of $(p-\beta)$ for all p's in the rest of the operational equation. That is,

$$\frac{p}{p+\alpha}\epsilon^{-\beta t}1 = \epsilon^{-\beta t}\frac{p-\beta}{p+\alpha-\beta}1 \tag{28}$$

The exponential time function now becomes a multiplying factor only.

Equation 28 is a second form of the operational equation representing the circuit of figure 2f, whence, by partial fraction expansion of the operand, or by use of the expansion theorem

$$i = \frac{E}{1-RC\beta}\left[\frac{1}{R}\epsilon^{-t/(RC)} - C\beta\epsilon^{-\beta t}\right] \tag{29}$$

Space does not permit further discussion of methods of solving operational equations, but the methods presented, though in brief, should give some idea of the operational attack on physical problems.

Fractional Powers of p

Because questions frequently are asked about the significance of fractional differentiation and integration, a word should be given before closing on that phase of operational calculus. In his investigations on telegraph cables, Heaviside obtained series solutions for many problems in which the operational equations contained the operator to half powers. Both ascending and descending series were found, and it was necessary to interpret these half-power operators.

The appearance of terms such as $p^{3/2}1$, $p^{5/2}1$, $p^{-1/2}1$, etc., called particularly for interpretation of $p^{1/2}1$. By comparing his operational mathematics with classical heat flow problems having known solutions, Heaviside determined that

$$p^{1/2}\,1 = \frac{1}{\sqrt{\pi t}} \tag{30}$$

Other half powers of p then became simply the derivatives or integrals of $1/\sqrt{\pi t}$, for example

$$p^{-1/2}\,1 = \frac{1}{p}\,p^{1/2}\,1 = \frac{2}{\sqrt{\pi}}\,t^{1/2}$$

$$p^{5/2}\,1 = \frac{d^2}{dt^2}\,\frac{1}{\sqrt{\pi}}\,t^{-1/2} = \frac{1}{\frac{2}{1}\cdot\frac{2}{3}\cdot\sqrt{\pi}}\,t^{-5/2}$$

Again may be noted the experimental manner by which the time function equivalent to an operational form had been obtained.

Whether the terminologies "fractional differentiation" or "fractional integration" have meaning depends perhaps upon the reader's viewpoint. The entire interpretation of the fractional power operator is based upon equation 30, after which the operations are in integral powers of p. In this sense, these terminologies have lost the significance that might be carried over to the operational calculus from the ordinary calculus.

Tables of Operational Equivalents

From what few problems have been discussed thus far in this article, the time function equivalents of several operational forms have been determined. The solution of a larger group of problems obviously would increase the number of related expressions. Further, by use of the described methods of solution it is possible to devise operational forms and to seek their time function equivalents without the necessity of having specific physical problems in view.

The advantage of equivalent expressions is that once they are obtained they need not again be derived, but the time function may be written immediately whenever its corresponding operational form is met, regardless of the problem giving rise to it. Tables of operational expressions with their related time function equivalents are given in the textbooks mentioned. In addition to these, attention also is called to the extensive table of Fourier transforms presented by Campbell and Foster.[36] Since the operational calculus readily can be interpreted in terms of the Fourier integral, as in the work of Bromwich and March, this table properly may be regarded as belonging to the class of operational equivalents. As stated by Bush, "Most of them (Fourier transforms) become immediately available as operational formulas by noting that the operand is $p1$ in our notation." One approaching the operational calculus can obtain familiarity with many of its fundamentals by deriving for himself operational formulas he will find in these several tables.

Quite frequently one finds equivalent expressions in which the unit function accompanies both members, e. g., $\frac{p}{p+\alpha}1 = \epsilon^{-\alpha t}1$. Whether or not one cares to retain the unit function with both members is perhaps optional. Unit function has exactly the same physical significance for both terms, and in the sense of being a signpost serves only to call attention to the nature of the impressed force, and to the fact that the ultimate solution may be computed using values of time beginning with $t = 0$. For that reason its retention is fully justifiable. One may argue, though, that, once having operated upon unit function to obtain the corresponding time function, unit function perhaps should disappear. One also may question the mathematical correctness of the statement "operating upon unit function." As the writer views the ideas underlying the operational method, $\frac{p}{p+\alpha} = {}^{-}\alpha^{\alpha t}$ whether or not the unit function is exhibited explicitly in either term. We always are left with a far greater physical significance of the unit function than with one that is mathematical.

As stated at the outset, this article has been directed toward the reader who has had little acquaintance with the Heaviside operational calculus but who has desired to have some of its fundamentals presented to him. It has been the aim primarily to discuss the physical ideas underlying the material given. The treatment has been specific, with the view that in definitely applying the operational method to particular circuits its elements may be grasped more readily.

The scope of the operational method is fairly large. In the great majority of circuits cause and effect are directly proportional to each other, and the coefficients of the terms involving differentials or derivatives in the problem statements are constant. As a consequence, the differential equations representing such a system are said to be linear with constant coefficients. To be precise, these statements are not strictly true, but for all practical purposes they are assumed to hold. The coefficients are the system parameters R, L, C, etc., (either singly or grouped) which usually are called the circuit constants.

It is to the linear circuit with constant parameters that the operational method may be applied. It is not restricted, however, only to that type of problem described by the ordinary differential equation; it includes also certain circuits that may be represented by partial differential equations, e. g., the transmission line, and some problems in heat flow. As yet the operational calculus has not been applied satisfactorily from an engineering point of view to systems that are nonlinear or that have variable "circuit constants."

Many phases of operational circuit analysis and its related features, of course, have not been touched upon. Listing several of such subjects of consideration, there are: circuits the past history of which must be taken into account, changes of circuit, coupled circuits, net-works and systems of equations, the theorem of superposition, the infinite integral equation, transmission lines and cables, heat flow, series solutions, transfer operators, traveling waves.

These topics are beyond the scope of this article and must be left to the reader to follow. It has been the hope, however, that in the treatment given here he will be led to a better understanding of the operational method of approach to the solution of engineering problems.

References

1. Operational Calculus, M. F. Gardner. Elec. Engg. (A.I.E.E. Trans.), v. 53, Oct. 1934, p. 1340–7.

2. Electrical Papers (a book), Oliver Heaviside, v. 2, p. 103, and p. 355 for some of his discussion on operators.

3. On Operators in Physical Mathematics, Oliver Heaviside. *Proc.*, Royal Soc. of London, v. 52, part 1, 1892–3, p. 504–29; v. 54, part II, 1893, p. 105–43.

4. Electrical Papers, Oliver Heaviside, v. 2, p. 446; Electromagnetic Theory (a book), Oliver Heaviside, v. 2, p. 36 and 286.

5. Electromagnetic Theory, Oliver Heaviside, v. 2, p. 15, 39, 130, and 289. The algebraic character of the operator p is shown on p. 288–9.

6. Electrical Papers, Oliver Heaviside, v. 2, p. 103, for example.

7. Electromagnetic Theory, Oliver Heaviside, v. 2, p. 35 and 286.

8. Electromagnetic Theory, Oliver Heaviside, v. 2, p. 15, 37, and 41.

9. Electrical Papers, Oliver Heaviside, v. 2, p. 226–7, 373, and 409.

10. Electromagnetic Theory, Oliver Heaviside, v. 2, p. 127 and 135.

11. Heaviside's Proof of His Expansion Theorem, M. S. Vallarta. A.I.E.E. Trans., v. 45, 1926, p. 429–34.

12. General Expansion Theorem for the Transient Oscillations of a Connected System, J. R. Carson. *Phys. Rev.*, v. 10, Sept. 1917, p 217–25.

13. Heaviside's Operational Calculus (a book), E. J. Berg. McGraw-Hill Book Company, N. Y., 1929, chapter 3.

14. Operational Circuit Analysis (a book), Vannevar Bush. John Wiley and Sons, N. Y., 1929, chapter 7.

15. Electric Circuit Theory and the Operational Calculus (a book), J. R. Carson. McGraw-Hill Book Company, N. Y., 1926, p. 32–5.

16. Heaviside's Expansion Theorem, W. O. Pennell. *Bell System Tech. Jl.*, July 1929, p. 482–92.

17. Heaviside's Electrical Circuit Theory (a book), L. Cohen. McGraw-Hill Book Company, N. Y., 1928, chapter 2.

18. Differential Equations (a book), D. A. Murray, p. 70.

19. Elementary Differential Equations (a book), T. C. Fry, p. 186.

20. Electrical Papers, Oliver Heaviside, v. 2, p. 427 and 446; Electromagnetic Theory, Oliver Heaviside, v. 2, p. 293–5.

21. Operational Circuit Analysis, Vannevar Bush, p. 130–5.

22. Heaviside's Operational Calculus, E. J. Berg, p. 61–6.

23. Lecons Sur Les Series Divergentes (a book), E. Borel, Gauthier-Villars, Paris, 1901 edition, p. 104.

24. Theory of Infinite Series (a book), T. J. I'A. Bromwich. Macmillan, London, 1908, p. 280–1.

25. Studies on Divergent Series and Summability (a book), W. B. Ford, Michigan Science Series, v. 2, MacMillan, N. Y., 1916, p. 93–4.

26. Normal Coordinates in Dynamical Systems, T. J. I'A. Bromwich. *Proc.*, London Math. Soc., series 2, v. 15, 1916, p. 401–43.

27. Symbolical Methods in the Theory of Conduction of Heat, T. J. I'A. Bromwich. *Proc.*, Cambridge Phil. Soc., v. 20, part 4, 1921, p. 411–27.

28. The Solution of Circuit Problems, T. C. Fry. *Phys. Rev.*, v. 14, 1919, p. 115–36.

29. The Application of Modern Theories of Integration to the Solution of Differential Equations, T. C. Fry. *Ann. Math.*, series 2, v. 22, March 1921, p. 182–211.

30. The Heaviside Operational Calculus, H. W. March. Amer. Math. Soc. *Bul.*, v. 33, May-June, 1927, p. 311–18.

31. Theory of Transient Oscillations of Electrical Networks, J. R. Carson. A.I.E.E. Trans., v. 38, part 1, 1919, p. 345–427.

32. The Heaviside Operational Calculus, J. R. Carson. *Bell System Tech. Jl.*, v. 1, 1922, p. 43–55.

33. The Heaviside Operational Calculus, J. R. Carson. Amer. Math. Soc. *Bul.*, v. 32, Jan.-Feb. 1926, p. 43–68.

34. Electric Circuit Theory and the Operational Calculus, J. R. Carson. McGraw-Hill Book Company, N. Y., 1926.

35. Notes on the Operational Calculus, J. R. Carson. *Bell System Tech. Jl.*, v. 9, Jan. 1930, p. 150–62.

36. Fourier Integrals for Practical Applications, G. A. Campbell and R. M. Foster. Bell Tel. System *Monograph B-584*, Sept. 1931.

The reader is referred also to the rather extensive bibliographies in the books by E. J. Berg and J. R. Carson.

Multielement Operation of the Cathode Ray Oscillograph

With the aid of specially designed auxiliary devices, the cathode ray oscillograph may be operated as a multielement instrument, in circuits that permit of the repeated application of the wave to be observed. The number of waves that may be traced with apparent simulataneity on the oscillograph screen is limited only by the complexity of the control circuit and the crowding of the various curves on the relatively small screen.

By
L. F. WOODRUFF
MEMBER A.I.E.E.

Massachusetts Institute of Technology, Cambridge

THE cathode ray oscillograph has undergone a very rapid development during the past few years, largely because of its promise as a receiver in television apparatus. Laboratory workers have benefited from this development, but its rapidity perhaps has resulted in an as yet incomplete realization of its potentialities.

For the observation and measurement of quick transients in electric circuits perhaps the only important disadvantage of the cathode ray oscillograph, in comparison with the bifilar type, is that it is designed to trace only a single curve. To overcome this disadvantage, there have been developed in the electrical engineering department at the Massachusetts Institute of Technology devices for making possible the operation of the instrument as a multielement oscillograph, in circuits that allow the repeated application of the waves to be measured. The waves may be either steady-state or transient.

Figure 1 shows the simple apparatus required, in addition to the standard oscillograph and sweep circuit, for handling 2 transients, and figure 2 a similar device developed to handle up to 4 transients simultaneously. In the apparatus of figure 1 a small synchronous motor drives 3 insulating disks bearing copper insets on portions of their peripheries. The function of the device in brief is to act as a commutator and to switch to the measuring plates of the oscillograph the various voltage waves to be traced. At the same time the device must trip the sweep circuit at the proper time so that the curves will remain stationary on the oscillograph screen.

Figure 3 indicates the layout of the 3 disks that

A paper recommended for publication by the A.I.E.E. committee on instruments and measurements. Manuscript submitted June 27, 1934; released for publication March 22, 1935.

Fig. 1. Synchronous-motor-driven commutating disks, with brush assembly

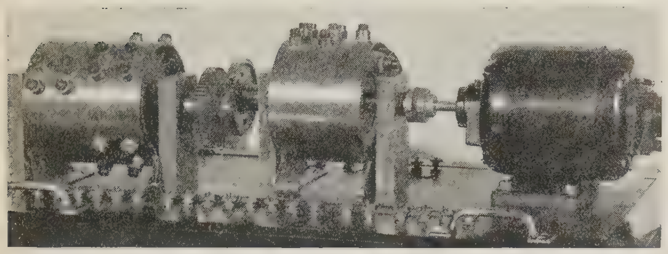


Fig. 2. Synchronous-motor-driven commutating disks, with change gears, designed to handle as many as 4 circuits simultaneously

form part of the device. For recording 2 voltage curves, such as, for example, on a transmission line transient, disks 2 and 3 only need be used and the circuit connected as indicated in figure 4. As disk 2 rotates, the conducting insets, traveling counter-clockwise, pass the 3 brushes marked 3, 4, and 5. The wire to the sweep-circuit trip contact is connected to the supply when the inset reaches brush 4 and 20 degrees later, when brush 5 is reached, the line to the sending end of the transmission line is energized. These contacts remain closed for almost a half revolution, and then after a brief open period are reclosed by the passage of the other metal inset. Turning on the same shaft, disk 3 contains only a single metal inset; 2 pairs of brushes, diametrically opposite as shown, bear on this disk. One pair of the opposite brushes is connected together to a common deflecting plate of the oscillograph. The other 2 brushes are connected to the 2 points at which the voltage variation curves are to be traced.

In operation, 2 transients are thrown on the line for each revolution of the disks. During alternate transients the voltage of one point is recorded, and during the other transients the voltage of the second point is recorded. These alternate curves are traced in such rapid succession that to the eye they both appear to be present continuously on the screen. The persistence of the willemite fluorescence contributes to this illusion.

Figure 5 shows a series of oscillograms taken with the aid of this device, representing the d-c transients on a 100-mile laboratory smooth transmission line. The line was open at the receiving end, and a constant direct voltage was applied intermittently at the sending end. Provision was made for draining the residual charge from the line between applications of voltage. On each oscillogram is traced the sending-end voltage, which jumps suddenly from zero to a constant value. The other curve is the voltage at another point along the line. Not only are the shapes and sizes of the various voltage waves and reflections clearly indicated, but the time required for the wave to reach the several positions on the line is indicated by the horizontal distance between the application of sending-end voltage and the initial arrival of the wave at the several points.

Figure 6 shows a series of oscillograms taken on the same line, but with alternating voltage impressed. The voltage was thrown on at different parts of the cycle in the various oscillograms. The driving motor of the commutator disks is of the synchronous type, which is necessary in order to hold steady the a-c transients. A nonsynchronous drive would result in the voltage being thrown on at different points of the cycle even though the locking feature of the oscillograph sweep circuit might keep it synchronized with the wave to be measured.

The device is not limited to voltage measurements but may be applied equally well to measurements of current. If one voltage and one current curve are to be traced, then all 3 disks must be used along with

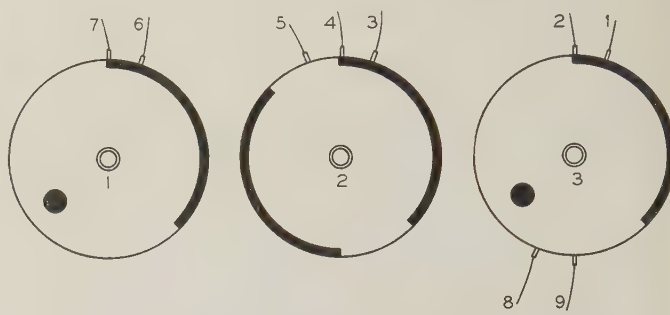


Fig. 3. Commutating disks and brushes for 2 element operation

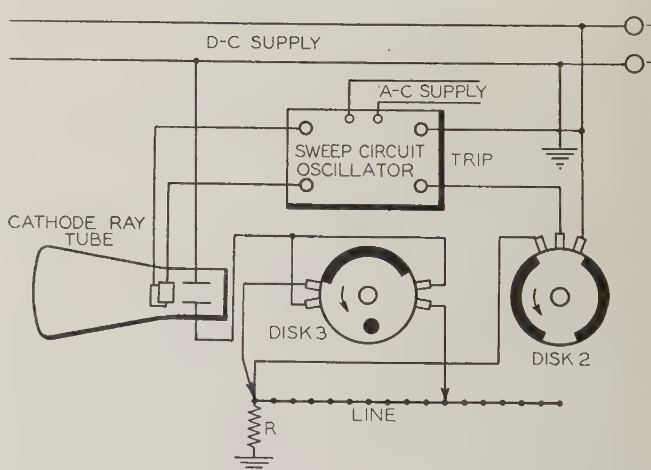


Fig. 4. Circuit used in recording 2 voltage transients simultaneously

the circuit shown in figure 7. It will be assumed that the oscillograph is equipped with magnetic coils for current operation. These must be connected in series with the section of the line in which the current is to be measured, but this connection is to be established only on alternate applications of the transient. This is accomplished by short-circuiting the coils on alternate transients by means of disk 1 on which bear a single pair of brushes.

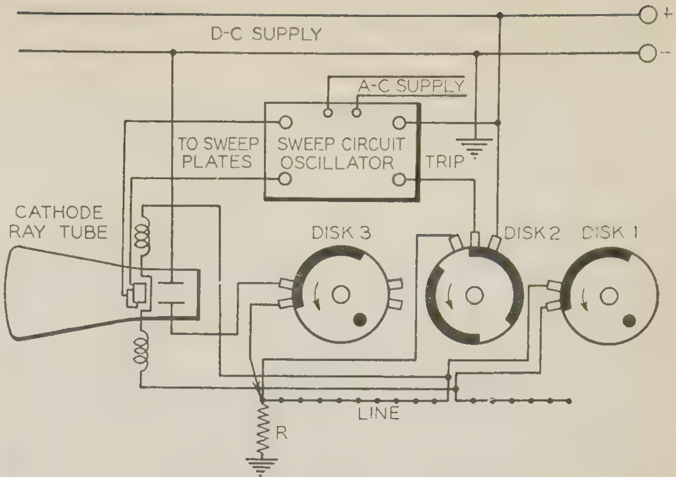


Fig. 7. Circuit used in recording one voltage and one current transient simultaneously

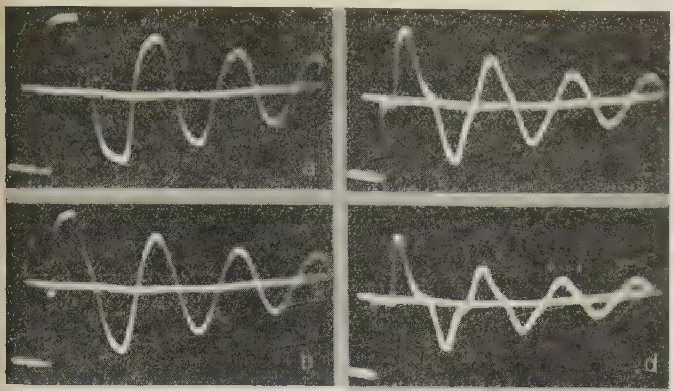
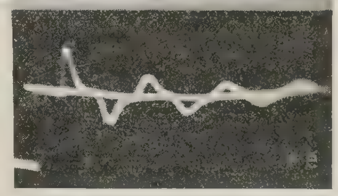
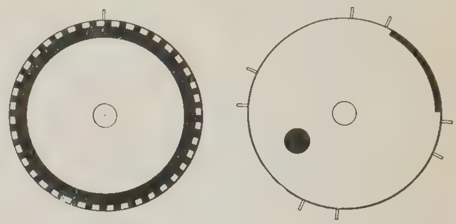



Fig. 5. D-c transients on a laboratory smooth line

Line was open at receiving end, and direct voltage was impressed suddenly at sending end. On each oscillogram one curve is the sending-end voltage; the other curve is voltage: (*a*) at receiving end; (*b*) $^1/_5$ of way back from receiving end; (*c*) $^2/_5$ of way back from receiving end; (*d*) $^3/_5$ of way back from receiving end; and (*e*) $^4/_5$ of way back from receiving end

There is no definite limit to the number of curves that may be traced with apparent simultaneity on the oscillograph screen. The practical limitation is only the complexity of the control circuit and the crowding of the various curves on the relatively small screen. If 4 voltage curves are to be traced, one of the disks should be of the design shown at the right in figure 8. The sweep trip and initiating voltages are thrown on 4 times per shaft revolution. During each quarter revolution the oscillograph plates are connected to 1 of the 4 points whose potential variations are to be recorded. If the voltages to be recorded are not all connected to a common neutral, then an additional disk is needed for the second lead from each voltage. If there be a common neutral, the lead from this neutral may be connected permanently to one of the deflection plates of the oscillograph. The left-hand disk of figure 8 is for the purpose of providing a convenient timing wave in the form of a series of rectangular steps.

Fig. 8. Timing disk and 4-wave switching disk



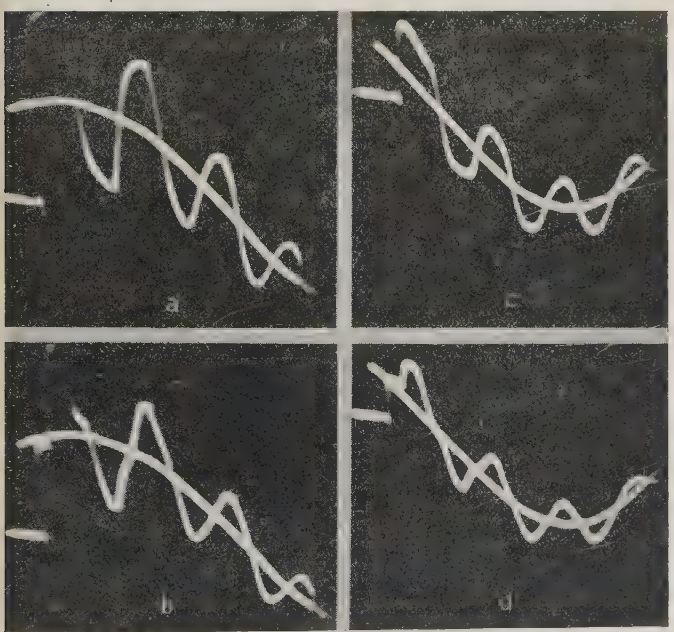


Fig. 6. A-c transients on a laboratory smooth line

Line was open at receiving end, and alternating voltage was impressed suddenly ats ending end. On each oscillogram one curve is the sending-end voltage; the other curve is voltage at: (*a*) receiving end; (*b*) center of line; (*c*) receiving end; and (*d*) center of line. In (*a*) and (*b*) the voltage was impressed near its positive peak; in (*c*) and (*d*) just before passing through zero

References

1. The Oscilloscope: A Stabilized Cathode-Ray Oscillograph With Linear Time Axis, Bedell and Reich. A.I.E.E. Trans., v. 46, 1927, p. 546–9.

2. The Transient Visualizer, H. M. Turner. A.I.E.E. Trans., v. 43, 1924, p. 805–13.

3. Improved Means for Obtaining the Polar Diagram of an Alternating Electromotive Force or Current, McGregor-Morris and Ramsey. British Patent, No. 198,765, June 7, 1923.

Recent Electric Furnace Developments in Europe

A summary of progress during 1934 and part of 1935 in electric furnace developments in Europe has been prepared by D. F. Campbell, a leading authority on the electrometallurgical industry in Europe, at the request of the Institute's committee on electrochemistry and electrometallurgy. The summary is presented herewith for the information of the Institute's membership.

By
DONALD F. CAMPBELL* Electric Furnace Co., Ltd., London, England

ELECTRIC furnace progress in Europe during the 12 months preceding April 1935, has been characterized by steady development and improvement of types of equipment that had previously reached a stage of industrial application.

In the iron and steel trade, metallurgical practice has been improved in the high frequency induction furnace, which is now being used on a large scale for quality refining, and the preparation of highly alloyed steels, in the principal steelworks of England, Sweden, France, and Belgium. Furnaces in general use vary in capacity from laboratory sizes to 5 and 6 ton furnaces operated for high frequency generators of 1,250 kw. The generators are now generally of the inductor type, the wound rotor method of constructing these machines having been discarded, except in Germany. English, French, and Swedish constructors are manufacturing motor-generator sets having over-all efficiencies of about 88 per cent for 1,250 kw sets, and 86 per cent for 400 kw sets, which shows a marked improvement on the older type of machines built formerly.

The largest high frequency melting shop is that in the steelworks of the Etabs. Jacob Holtzer, in France, where there are 2 sets of 650 kw, and 3 sets of 150 kw, with a number of furnaces of various sizes.

An Unusual Installation for Quality Refining

The most interesting installation is probably that of Samuel Fox and Co. Ltd., near Sheffield, England. Important metallurgical advances have been made in quality refining in this installation, the following description of which may be of interest.

Furnaces of 2 and 5 tons are arranged for melting cold scrap, or molten metal by a duplex process.

The motor-generator is placed in a separate substation 80 feet from the furnaces, the condenser plant being situated under the furnace platform. The alternator consists of 2 625-kw stators, with a common rotor. Each stator consists of 2 separate halves, and provision is made to prevent interaction between the magnetic circuit of one stator and the excitation winding of the other stator. It is thus possible to control the voltage of each of the circuits independently and operate 2 furnaces simultaneously. When desired, the stators can be connected in series so that 1,250 kw can be applied to one furnace.

The furnaces can be completely automatically, semiautomatically, or hand controlled, at will.

A number of protective devices have been incorporated in the equipment. Relays are provided to cut off the current if the water supply to the coils fails, and colored lamps indicate any failure of the water cooling supply to the machine bearings, or any undue heating of the equipment.

The furnace bodies are arranged to be normally flush with the platform, and the provision of a special charging cylinder permits very rapid charging. In a full day's run, the current was only switched off for an average of 6 minutes per heat, which includes the time required for pouring and recharging the 2 ton furnace.

In a recently published article, Doctor Swinden, chief metallurgist of United Steel Companies Ltd., of which Samuel Fox and Co. Ltd. is a member, states that in this plant sulphur can be removed more rapidly than in normal arc furnace practice, owing to the intense movement of the metal. This movement also provides an opportunity for microscopic particles to coalesce and rise in the liquid steel. In this way, the nonmetallic inclusions formed from the products of deoxidation can to a large extent be removed. Tests made on air-hardening nickel-chromium steel, nickel-chromium-molybdenum case-hardening steel, low-carbon chromium-molybdenum steels, and many others, show the very high quality of steel made by quality refining. Perfectly sound stainless steel, with only 0.05 per cent carbon, is being cast, and it is found that this steel withstands weld decay tests under conditions many times as severe as those stipulated in the government specifications.

High Frequency Generators

The output of high frequency generators used for steelmaking in Europe is now between 30,000 and 35,000 kw.

Larger furnaces have involved the use of higher voltages, owing to the large kilovolt-ampere rating of the inductor coils, and 2,500 volts has now become within the limits of standard practice. For small furnaces up to $^1/_2$ ton, frequencies of 500 cycles per second have been found to give conditions unsuitable for the best steelmaking practice, and higher frequencies are now specified for these small units.

Especially prepared for ELECTRICAL ENGINEERING under auspices of the A.I.E.E committee on electrochemistry and electrometallurgy. Manuscript submitted April 1935; released for publication May 15, 1935.

* Member, Institution of Electrical Engineers, and Institution of Mining and Metallurgy, both of Great Britain.

For furnaces of 5 tons, periodicities of 800 to 1,000 cycles per second have been selected.

Induction Furnaces

Interesting attempts have been made to build induction furnaces with controlled movement of metal and slag. Extensive work was done at Hanau by Doctor Rohn, who has been responsible for great advances in the vacuum melting of metals and the working of heat resisting alloys of the nickel-chrome-molybdenum, etc., type. The induction furnace on which he has been working is a normal frequency, 3 or 6 phase, hemispherical furnace giving an intense and controlled movement of the metal and slag, which opens interesting possibilities in metallurgical practice. Electrical power input is limited by constructional difficulties, and the speed of melting and heat control is accordingly strictly limited, and has retarded the commercial development of this interesting design of furnace, and, until these limitations are overcome, its commercial value cannot be considered established.

In Sweden interesting work is being done with dual frequency furnace construction, in which normal frequency induction coils are used to control the movement of metal, while normal high frequency induction coils are used to maintain the heat and energy input that is required for rapid operation. It is expected that metallurgical reactions may be controlled better by this means, but the additional cost and the complication of the electrical apparatus are serious factors. The problem of maintaining the refractory linings for a sufficient period will, however, probably prove the determining factor in the success or failure of these new types of ironless induction furnaces with intense electrically-controlled movement of the charge.

Arc Furnace

Arc furnaces continue to find wide application, and many are now being built. The removable roof for rapid charging is being generally adopted, and greatly speeds up the charging time. There are no other substantial advances, and little improvement has been made in recent years in power consumption. Furnaces of large capacity, of about 30 tons, are used for refining liquid metal, but the difficulty of skimming these furnaces clean of slag makes the smaller units of about 10 tons generally preferred for the manufacture of the highest grades of steel from cold stock, and larger units are generally considered of little advantage for this class of work.

In the nonferrous trades, the Ajax-Wyatt, or furnaces of similar design, have become standard practice for melting in brass mills, and large units of 2,500 pounds and 360 kva capacity have been developed, and give marked economy in labor and power, and some metallurgical advantages, owing to the mixer effect of the larger unit, when melting miscellaneous scrap. These furnaces are of the Ajax-Wyatt type, and are operated by 2 phase current obtained from standard 3 phase systems by a Scott connection on the furnaces. It is noteworthy that this important improvement and economy does not seem to have been adopted in the United States.

Resistance Furnaces

A rocking resistance furnace has recently been introduced in France for the melting of bronzes and cast iron in small quantities for the manufacture of piston rings and similar special products. The construction of the furnace is generally similar to that used for the well-known horizontal rocking arc furnace, but, instead of heating by an arc, a graphite resistor is used and this is loaded sufficiently to heat the resistor to the required temperature. In furnaces of 1,000 pounds capacity, for example, the resistors of Acheson graphite are operated at 2,000 amperes per square inch. Special patented features are the terminals, arranged to prevent excessive loss of heat, and an arrangement whereby the power factor is maintained at a high value, even in the case of large furnaces.

This system of heating was first used for the manufacture of silica tubes, but it is now finding application in bronze foundries, where the Ajax-Wyatt furnace is unsuitable, owing to the continual changing of analysis of product and the small tonnages required. It is considered better than the horizontal arc furnace for certain applications, as there is no excessive heat to damage or volatilize the metals heated, and it is being installed by specialized iron foundries for small castings.

The use of resistance furnaces is developing rapidly, especially for the heat treatment of aluminium alloys within narrow limits at low temperature, for the bright annealing of copper in controlled atmospheres by a dry process which gives a perfect finish without the risk of water stain, and in the many other metallurgical processes to which this equipment has been previously applied. The greatest progress in Europe during the past year has been made in the finer control of atmosphere in electric furnaces for heat treatment.

Heat Resisting Alloys

Progress in the manufacture of heat resisting alloys for high temperatures, and to resist sulphur, has resulted in the production of metals which can be worked continuously at 2,350 to 2,400 degrees Fahrenheit. These are chrome-iron-aluminium-cobalt alloys, without nickel. The use of alloys of this type is increasing, both for high temperature work, and also for domestic applications, where the low intrinsic value of the metals contained makes its use competitive with the standard 80/20 nickel-chrome alloys.

In Europe, the metallurgical works of Great Britain and Sweden have worked generally to capacity, and considerable technical progress has been made. In France, industry, other than government contracts, has been very depressed, and little money has been available for technical developments. In Belgium, where nearly 80 per cent of metallurgical industry works for export, trade has been bad owing to import restrictions and currency questions, while

in Germany research work has had a serious setback, owing to the disturbance of works staff and the depression in many industries during the last year. The ironless induction furnace undoubtedly presents the greatest possibility of further development, and the experience now being obtained in Great Britain, Sweden, and Germany should result in a wider application of this interesting method of electric heating.

Electrical Brush Wear

An investigation of the effects of polarity and current density on the rate of wear of electrical brushes is reported in this paper, following a brief review of previous experimental work in this field. For carbon and copper impregnated carbon brushes the rate of wear of the positive brush is considerably less than that of the negative brush, while for metallic brushes the situation is reversed. The rate of wear of metallic brushes was found to be much greater than that of carbon brushes, under conditions of these tests. While no complete explanation of brush wear can be derived from the results, the rate of wear seems to be related intimately to the conduction of current across the contact.

By
V. P. HESSLER
ASSOCIATE A.I.E.E.

Iowa State College,
Ames, Iowa

A CAREFUL SURVEY of the literature on electrical brushes reveals little qualitative information concerning electrical brush wear. Practically all the investigations reported are concerned with the effects of various operating conditions upon contact potential drops.

Hunter-Brown[1] obtained 2 curves of electrical brush wear versus current density, which indicate that the negative brush (motor notation) wears at a greater rate than the positive brush, and that the rate of wear is approximately proportional to the current density. However, the points obtained are too few to justify a definite conclusion.

J. S. Dean[2] made an extensive survey of the life of carbon brushes in d-c railway service. In this type of service the actual end wear of the brush is small compared with all the other factors that bring about deterioration of brushes. He found that high grade graphitized brushes on commutating pole motors with undercut mica, when subjected to end wear alone, would have a probable life of 200,000 car-miles. With modern medium grade carbon brushes on non-commutating pole motors with flush mica, the probable life due to end wear is 20,000 car-miles. Dean concludes that burning action is responsible for most of the end wear. He gives 2 extensive tables of the factors causing brush wear in railway service. These tables indicate that if good contact is maintained between the brush and commutator at all times, the rate of brush wear will be low.

Under conditions of very low humidity the rate of wear of metallic brushes on high speed converter rings often becomes excessive. Bracken[3] states that whole sets of brushes sometimes are worn out in from 30 minutes to $1^1/_2$ hours. This effect never has appeared on low-speed 25-cycle machines. He finds that rapid wear is likely to start when the absolute humidity drops below 1.25 grains per cubic foot.

A series of tests was conducted by Baker[4] to determine the effect of a hydrogen atmosphere upon electrical brush wear and commutation. In these tests a d-c machine was operated both in air and in an atmosphere of hydrogen with varying qualities of commutation. Two runs were made to determine the effect of humidity in a hydrogen atmosphere with poor commutation. The following conclusions were drawn from the results of the tests:

1. A well designed commutator machine will operate satisfactorily and give good brush life in hydrogen.
2. If a brush must spark in hydrogen, the brush life may be increased many times by maintaining the relative humidity below 10 per cent.
3. Carbon and graphite brushes cannot be operated upon a tool steel ring in hydrogen. Particles of cementite (Fe_3C) are formed which immediately begin to score the ring and the brush face.

Perrier[5] found that the rate of brush wear is increased with the oxidation, sulfuration, or chlorination of the ring surface.

The foregoing items constitute most of the investigations of electrical brush wear that have been reported. The scarcity of quantitative data indicated a real need for further investigation of electrical brush wear. Information on brush wear should be of value to the brush manufacturers and to designing and operating engineers. It also should aid in extending or checking the present inadequate theories of the sliding contact. It is for the purpose of obtaining additional information of this character that

A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted Jan. 7, 1935; released for publication June 20, 1935.

The author wishes to acknowledge the assistance of P. D. Manbeck and J. A. Robinson of the National Carbon Company, which company supplied the brushes.

1. For all numbered references, see list at end of paper.

the investigation reported in this paper was undertaken.

Apparatus Used in Tests

Details of the mechanical arrangement of the apparatus used in the tests reported in this paper are shown clearly in figure 1. The rings were carried on a $1^{15}/_{16}$ inch shaft which rotated in self-aligning ball bearings. The rings were attached to the shaft by means of tapered sleeves with a sheet of 0.005 inch vulcanized fiber between the sleeve and the shaft for insulation. This arrangement held the rings securely to the shaft and gave sufficient insulation for all purposes of the tests. After completely assembling the rings, bearings, and bearing housings the complete unit was removed from the bearing hangers and mounted upon a lathe bed. The surfaces of the rings then were turned with the shaft rotating in its own bearings. By this method the eccentricity of the rings could be kept below 0.0005 inch. The translation of the brush caused by the eccentricity was measured with a dial micrometer attached to the vertical rod on the brush holder.

Rings. Hard drawn electrolytic copper bar was rolled and brazed to form the slip rings. A special low temperature brazing metal was applied to a V notch in the joint on the inside of the ring, which joint barely was visible on the outer surface of the rings. These copper rings were pressed upon cast aluminum disks. The surfaces of the rings were polished carefully with commutator dressing stones and burnished with a beechwood stick. They were repolished in the same manner at intervals throughout the period of the tests.

Brush Holders. The brush boxes consisted of 4 pieces of $^1/_8$ inch brass soldered and pinned together. Considerable care was taken with the internal dimensions to assure proper clearances between the brushes and the box; the boxes were one inch long. Details of the pressure arm mechanism are shown in

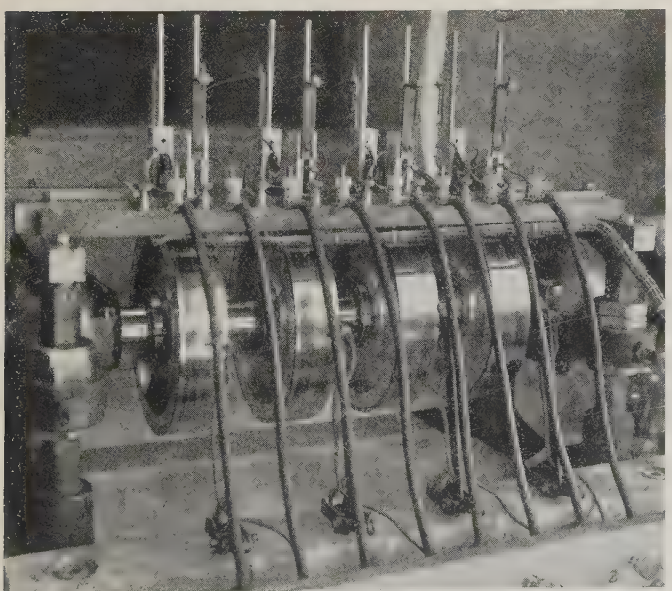


Fig. 1. Brush holder and ring mechanism

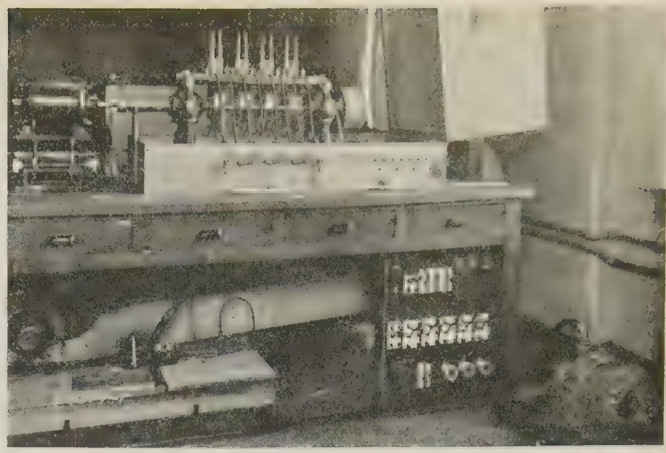


Fig. 2. View showing air conditioning equipment

figure 1. As the brushes wore away, the pressure arm was kept perpendicular to the brush by lowering the fulcrum. The brush pressures were adjusted with the aid of a balance and a pair of headphones, a click in the headphones indicating that the pressure arm had been lifted clear of the hammer plate. The reading of the balance always was corrected for the weight of the brush. The springs were continuously adjustable so that the brush pressure could be adjusted to any desired value.

Temperature and Humidity Control. The humidity control equipment is shown in figure 2. The box temperature was held constant by means of a thermostatic control. To maintain constant humidity air from the room was drawn into the blower at the lower left and was blown past a series of water spray nozzles in the tunnel, then through baffle plates to remove entrained moisture, next past a heater where the temperature was raised sufficiently to prevent condensation in the tunnel, and finally through a group of air nozzles into the box. The water was forced through the spray nozzles by the centrifugal pump at the lower right and flowed back into the supply tank. Humidity control consisted of maintaining the water temperature at the dew point corresponding to the relative humidity and air temperature desired.[6] The relative humidity was measured with a wet and dry bulb hygrometer.

Method of Measuring Wear. An attempt first was made to determine the wear by weighing the brushes. This method, however, was found to be impractical because of the variation of the absorbed moisture. The micrometer arrangement shown in figure 3 then was developed, which gave excellent results. The bed plate was of invar and had an effective expansion of less than 0.00002 inch for the maximum change in room temperature. To measure the length of a brush, it was placed in the V block as shown; the micrometer head then was screwed in until the hammer plate of the brush made contact with the insulated point, as indicated by a click in the headphones.

Method of Measuring Contact Drop. The positive and negative contact potential drops were measured by means of the auxiliary copper leaf brushes shown in figure 1, which were lowered to the ring whenever measurements were to be taken. They rode on

the middle of the ring between the positive and negative brush paths. Auxiliary potential leads were soldered to the hammer plate rivet to eliminate all possible *IR* drops from the measurements, and the readings were corrected for the drop through the brush material.

Brushes and Brush Circuit. The brushes were 1/2 inch square and 2 inches long. The hammer plates and shunts were attached with spun tubular rivets. Some of the physical constants and the types of material of the brushes are given in table I.

All brushes were connected in series, and the group was connected to a 110-volt d-c generator through a variable resistance. A low voltage generator was used originally, but it was impossible to hold the current constant because of the change in contact potential drop at the brushes.

Test Procedure

The brushes always were "run in" carefully before any measurements were taken. They first were sanded as nearly as possible to the curvature of the ring and then were run at approximately normal current density until the entire brush face made contact with the ring. Much less time was required and a better surface was obtained when the brushes were "run in" with current.

When a satisfactory brush surface had been obtained, the ring was repolished and the brush measured. The brush then was operated at a given current density for a sufficient time to give a measurable wear after which it was measured again. Readings were taken at each of several current densities. The rings were not polished during the time the metallic brush wear data were being taken. The positive and negative brushes were run on separate paths. Data for 4 different grades of brushes were obtained simultaneously upon the 4 rings.

Before proceeding with the current density tests a series of tests was made to ascertain whether the values chosen for brush pressure, speed, and humidity were critical as regards rate of wear. These tests indicated that any changes that were likely to occur in pressure, speed, or humidity would not have any appreciable effect upon the rate of wear.

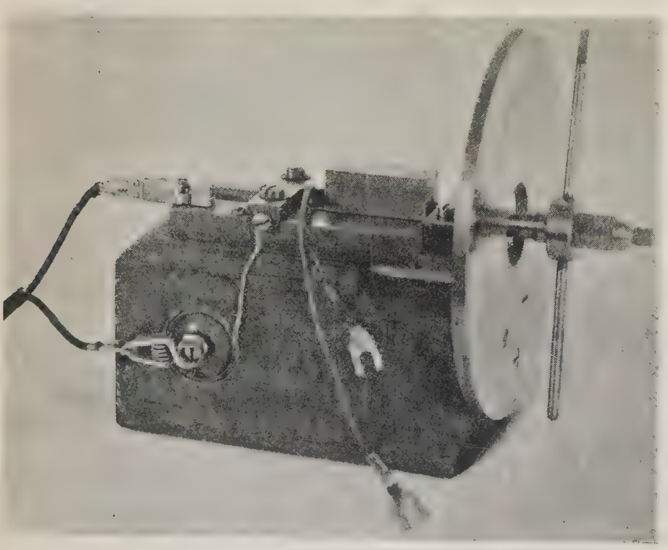


Fig. 3. Special micrometer for measuring brushes. Cord at left connects to headphones

Table I—Physical Characteristics of the Brush Materials

Brush	Resistance, Ohms per Inch Cube	Hardness (Scleroscope)	Transverse Strength, Lb. per Sq In.	Normal Current Density, Amp per Sq In.	Type of Material
A	0.0020	65	3,600	55	Electrographitic lampblack
B	0.00225	54	2,960	60	Electrographitic lampblack
C	0.00106	40	3,500	45	Carbon graphite
D	0.00142	40	3,500	45	Carbon graphite
F	0.00000242	5	9,700	150	Heavy metal graphite
G	0.000010	16	4,000	100	Metal graphite
H	0.0000064	20	4,900	115	Metal graphite
I	0.000214	35	6,000	85	Copper impregnated carbon

In the following discussion the brush in which the conventional current flow is from brush to ring is considered positive.

Carbon Brush Wear

The data on carbon brush wear and contact drop versus current density are presented in figures 4 to 7. Each of the experimental points of contact potential drop shown in the figures represents the average of from 12 to 15 readings taken at intervals throughout the tests at a given current density; thus each point represents the mean value of contact drop for a week of continuous operation. There can be no doubt that the contact drop did decrease with increased current density. It must be remembered that contact drop is affected by other factors as well as by current density.

A curve of contact potential drop versus current density obtained by increasing the current to its maximum value in a few minutes almost always will show an increase of contact drop with increased current; also any sudden increase in current always will result in an increased contact drop. If this were not true it would be impossible to operate brushes in parallel. For these instances of comparatively rapid change in current other factors affecting brush contact drop are constant and the drop is determined by the current density. When the current density is maintained at a given value for a week, the brush surface, ring surface, and ring temperature assume a condition more or less characteristic of that current density. It is not understood very clearly just how the surface conditions affect the contact drop, but increased ring temperature brings about a decided decrease.[7] Thus the decreased contact drop is the indirect effect of the increased current, which, unfortunately, cannot be indicated on a 2 dimension diagram.

Metallic Brush Wear

The data on metallic brush wear and contact drop versus current density are presented in figures 8 to 11. The rate of wear of metallic brushes is so much greater than for carbon brushes that a measurable

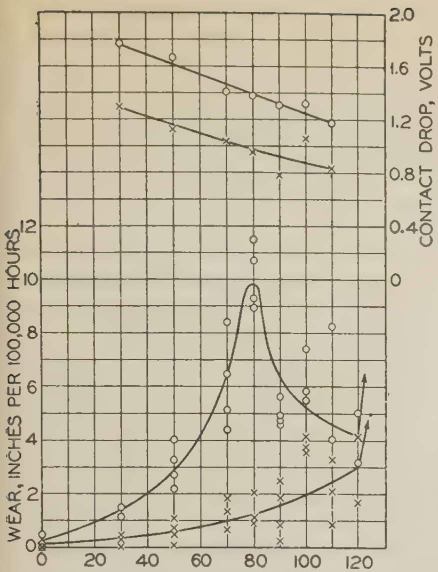


Fig. 4. Brush A

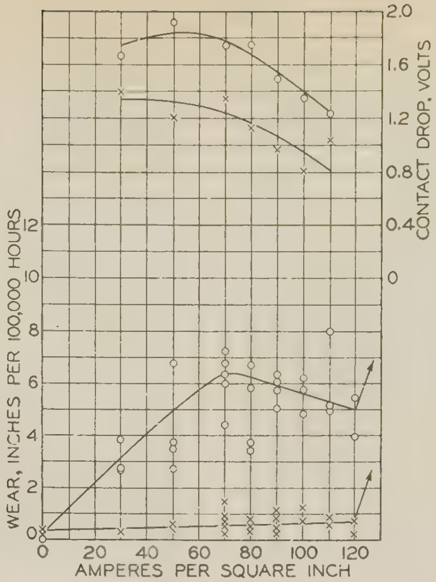


Fig. 5. Brush B

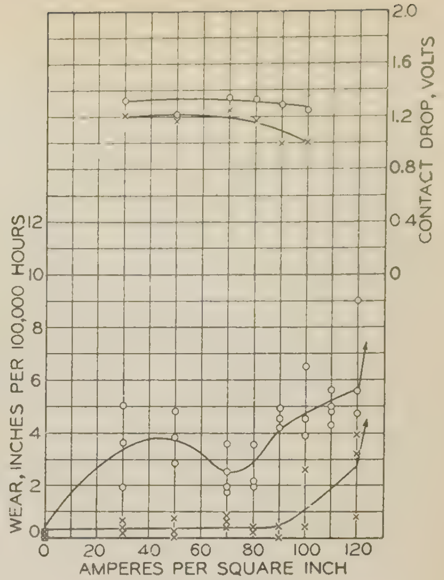


Fig. 6. Brush C

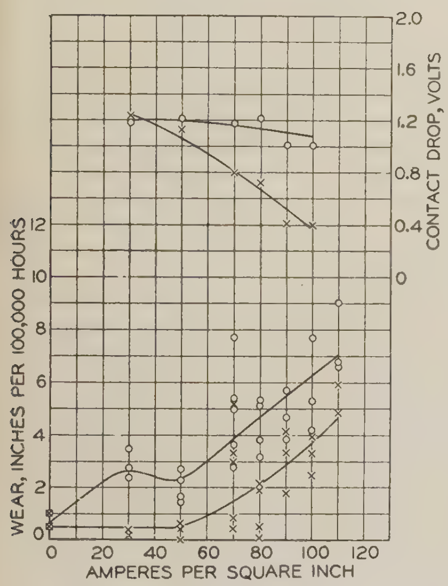


Fig. 7. Brush D

Figs. 4 to 7. Rate of wear of carbon brushes

Ambient temperature, 45 degrees centigrade
Relative humidity, 50 per cent
Brush circuit potential, 80 volts
Ring speed, 3,500 feet per minute
Brush pressure, 48 ounces per square inch
o—negative brush x—positive brush

Figs. 8 to 11. Rate of wear of metallic brushes

Ambient temperature, 45 degrees centigrade
Relative humidity, 50 per cent
Brush circuit potential, 90 volts
Ring speed, 3,760 feet per minute
Brush pressure, 48 ounces per square inch
o—negative brush x—positive brush

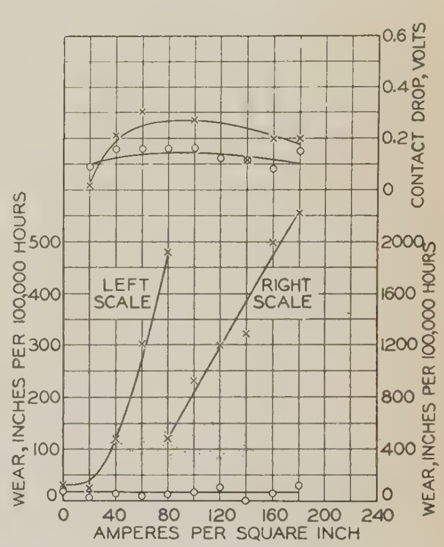


Fig. 8. Brush F

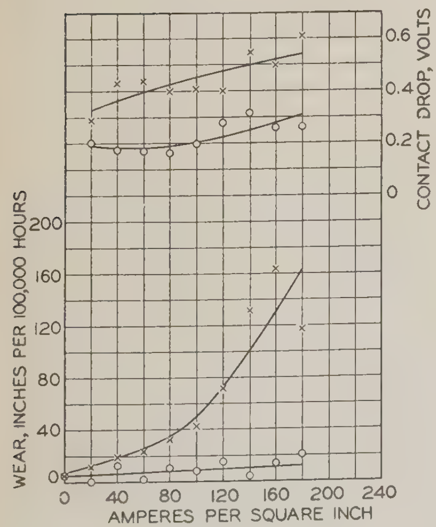


Fig. 9. Brush G

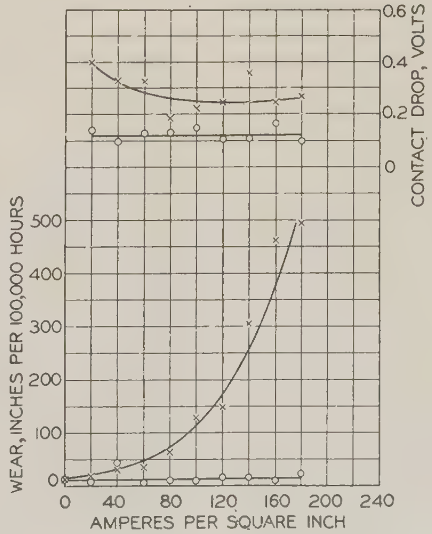


Fig. 10. Brush H

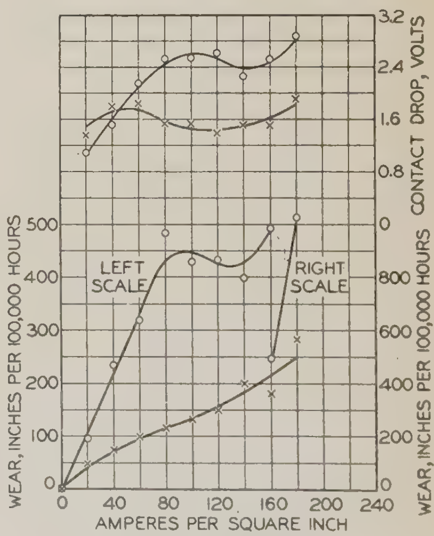


Fig. 11. Brush I

wear is obtained in a few hours. The heavy metal graphite brush F contains some lead. Shortly after the run was started the positive ring path assumed a grayish white appearance, which became more pronounced at the higher current densities. The brush also sparked badly at the higher current densities. This phenomenon did not appear at all at the negative brush, which indicates that there must be something akin to electrolytic action between the brush and the ring. Brush material I did not operate satisfactorily at either the positive or negative brush under the conditions of this test. Both ring paths blackened and the rate of wear was high. It has been learned since that this material cannot be used satisfactorily at the pressure used in this test. A few runs have been made at a higher pressure which indicate normal rates of wear. The curves are given here for purposes of comparison between carbon and metallic brushes.

Summary of Results

The rate of wear of positive carbon brushes is low and is independent of current density below a critical value which varies with different brush materials. The rate of wear of negative carbon brushes is greater than that of the positive and is proportional to current density below a certain value; above this value there is a decrease in the rate of wear with increasing current density, followed usually by a sudden increase in the rate of wear as the current density is increased to excessive values.

The rate of wear of positive metallic brushes is approximately proportional to current density, but for negative metallic brushes it is practically independent of current density to values considerably in excess of normal current density. The rate of wear of both positive and negative copper impregnated brushes is approximately proportional to current.

In general, there is no consistent relation between the contact potential drop and rate of wear of electrical brushes. The copper impregnated brush, how ever, is an exception; in all the tests made on it the configurations of the contact drop curve and the corresponding rate of wear curve were similar.

The fact that the rate of wear of positive metallic brushes is higher than that usually found in practice is ascribable to several causes: The normal current density of all of the brushes was exceeded. In some instances the ring speed was higher than the value recommended by the manufacturer for the particular grade of brush. All brushes were connected in series instead of being arranged in parallel groups as in practice; thus each brush carried the entire current continuously and could not shift the current to a neighboring brush during a moment of unfavorable contact. This last factor is known to affect the rate of wear, because a test with a brush circuit potential to 10 volts gave lower rates of wear than the test at 90 volts.

A rather thorough search of the literature revealed no theories regarding electrical brush wear. The variety of results obtained with different grades of brushes indicates that the physical characteristics of the brush materials have a decided effect upon the rates of wear. Some of the physical properties of the brush materials used in these tests are given in table I. In any attempt to correlate the rate of wear with these physical properties, the carbon group would have to be considered separately from the metallic group. There seems to be some consistent relation between the rate of wear of the negative carbon brush and the hardness and resistivity of the brush materials, but no correlation between the strength and rate of wear. The metallic brush with the greatest strength showed the greatest rate of wear, whereas the opposite result might have been expected. Apparently the strength of the brush does not materially affect its rate of wear. To form definite conclusions concerning the effect of the physical properties upon the rate of wear it will be necessary to obtain data for many different grades of brushes. Even so, it will be difficult to draw definite conclusions because of the many factors that vary simultaneously.

The pronounced polarity effect and the complex nature of the curves for the negative carbon brushes indicate that the rate of wear is connected intimately with the conduction of current across the contact. Material might be removed from the brush face by abrasion, combustion, or disintegration. The very low rates of wear at zero current show that ordinary abrasion does not account for more than a small portion of the total wear. Combustion might account for a small portion of the wear. It would be instructive to mount a ring and brush in an enclosed space and measure the amount of carbon monoxide and carbon dioxide produced. The quantity of these gases could not be very large, because one of the difficult problems connected with operating a brush in a closed space consists of removing the carbon dust from the atmosphere. Apparently the flow of current across the contact produces a disintegration of the brush surface. This disintegration might result from thermal expansion resulting from the high current density at the discrete points of contact between the brush and the ring. However, it is not apparent how this action would explain the difference between the rates of wear of positive and negative brushes. It is evident that much more quantitative data, involving other factors of brush wear will have to be obtained before the final and complete explanation of brush wear is found.

References

1. Carbon Brushes: Considered in Relation to the Design and Operation of Electrical Machinery, P. Hunter-Brown. Inst. of Elec. Engr. *Jl.*, v. 57, 1919, p. 193–224.

2. Carbon Brush Life on Direct Current Railway Motors, J. S. Dean. *Elec. Jl.*, v. 14, 1917, p. 416–19.

3. Humidity Control Prevents A-C Brush Disintegration, Ellis F. Bracken. *Elec. World*, v. 102, 1933, p. 410–11.

4. Commutation and Current Collection in Hydrogen, Robert M. Baker. A.I.E.E. Trans., v. 50, 1931, p. 714–17.

5. La chute de tension au contact des balais sur les bagues collectrices, M. Perrier. *Bulletin* de la Société Française des Electricien, v. 10, 1930, p. 903–17.

6. Air-Conditioning Apparatus—Principles Governing Its Application and Operation, W. H. Carrier and F. L. Busey. A.S.M.E. *Jl.*, v. 33, 1911, 1613–86.

7. Die Übergangs-Spannung von Kohlebürsten in Abhängigkeit von der Temperatur, E. Arnold and E. Pfiffner. *Elektrotechnische zeitschrift*, v. 12, 1907, p. 263–6.

Current Harmonics in Nonlinear Resistance Circuits

A semigraphical method of solving for harmonics in the input current to a network containing any number of linear and nonlinear resistances in any combination, when a sinusoidal voltage is applied across the network, is presented herewith. The treatment is restricted to nonlinear resistances in which the d-c volt-ampere characteristic is the same regardless of the direction of current. While the method has the errors inherent in any graphical solution, the results indicate good accuracy with little labor.

By
THURSTON D. OWENS
MEMBER A.I.E.E.
Case School of Applied Science, Cleveland, Ohio

METHODS of solving for alternating currents and voltages in various parts of a network containing only linear elements are well known, and calculations for such a circuit usually can be made without great difficulty. The problem of the circuit containing one or more elements that depend for their magnitude on the current flowing through them, is entirely another matter. Here mathematical expressions that characterize the nonlinear element or elements must be employed, which may cause the solution to become highly involved. Sometimes suitable mathematical expressions cannot be found and recourse then must be made to experimental means, or to approximations which may be quite unsatisfactory.

It is the purpose of this study to develop a semigraphical method of solving for the harmonics contained in the input current of a 2-terminal resistance network when a sinusoidal voltage is applied across its terminals. The network may contain resistances of the linear and nonlinear types in any number or combination. The nonlinear resistances, however, will be restricted to those having the same d-c volt-ampere characteristic regardless of the direction of current through them. The material known commercially as "thyrite" has a characteristic of this type.[1,2] A volt-ampere curve of a 2-terminal network containing a number of linear and nonlinear elements, subject to the foregoing restriction, will be identical in the first and third quadrants except for the signs of the current and voltage.

If a sinusoidal voltage wave be applied at the terminals of such a circuit, the current wave in general will not be a sine wave but will contain various harmonics. From inspection of the volt-ampere curve certain conclusions may be drawn regarding the nature of these harmonics. The current wave will pass through zero when the instantaneous voltage is zero. The positive half of the current wave will be symmetrical around the 90 degree ordinate, and the negative half symmetrical around the 270 degree ordinate. Furthermore, the positive and negative halves will be identical except for the signs of the ordinates. Such a wave may be represented by a Fourier series containing only sine terms of odd harmonics.[3]

The Basic Circuit

The circuit upon which the theory is based consists of a linear and a nonlinear resistance in series, with a sine wave voltage impressed across the terminals. The problem is to find the amplitudes of the current harmonics.

Let the curve OA, in figure 1 represent the volt-ampere curve of the nonlinear element. The linear resistance will have the magnitude R. For the instantaneous value of current OF, the voltage drop across the nonlinear resistance is FC. Adding the distance CD, which is the current multiplied by R to scale, gives the total instantaneous impressed voltage, FD or OB. If the line BC be drawn as shown, the tangent of the angle Φ is equal to R.

Similarly, for another value of current OF', the impressed voltage is OB'. The tangent of Φ' is also R; therefore Φ and Φ' are equal. Thus to find the current corresponding to any value of instantaneous voltage, it is necessary only to lay off the voltage on the vertical axis and from this point draw a line parallel to BC or $B'C'$. The intersection of this line and the curve OA marks a point which, if porjected to the current axis, determines the current. If the instantaneous voltage be a function of time, such as $E \sin \omega t$, then in a like manner any number of values of instantaneous current can be determined and plotted against time. This procedure actually is followed to obtain current waves for analysis by

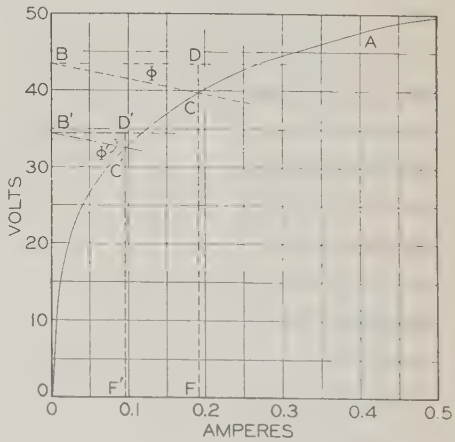


Fig. 1. Volt-ampere relations in a circuit containing a linear and nonlinear resistance in series

A paper recommended for publication by the A.I.E.E. committee on electrophysics, and scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted Feb. 25, 1935; released for publication May 17, 1935.

1. For all numbered references see list at end of paper.

means of the harmonic analyzer. The results of these analyses are used for comparison in the numerical examples. To avoid the labor of plotting an accurate current wave and then performing an analysis the following method was developed.

If a sinusoidal voltage, $E \sin \omega t$, be impressed across the 2 elements in series, the current can be expressed in the form

$$i = \Sigma I_m \sin m\omega t \tag{1}$$

where

$$m = 1, 3, 5, 7 \ldots\ldots\ldots$$

Equating the voltage drops,

$$E \sin \omega t = R\Sigma I_m \sin m\omega t + \Sigma A_m \sin m\omega t \tag{2}$$

where

E = maximum value of the impressed voltage
I_m = maximum value of the mth current harmonic
A_m = maximum value of the mth harmonic voltage across the nonlinear resistance
R = value of the linear resistance

At a time $\omega t = \theta_1$, equation 2 becomes

$$E \sin \theta_1 = R\Sigma I_m \sin m\theta_1 + \Sigma A_m \sin m\theta_1 \tag{3}$$

Similarly, when $\omega t = \theta_2$

$$E \sin \theta_2 = R\Sigma I_m \sin m\theta_2 + \Sigma A_m \sin m\theta_2 \tag{4}$$

Thus a set of equations can be written for the angle $\omega t = \theta_n$, where $n = 1,2,3,4,\ldots..$ Now if θ_2 be set equal to $2\theta_1$, θ_3 equal to $3\theta_1$, and in general θ_n equal to $n\theta_1$, the set of equations with some rearrangement becomes

$$\begin{aligned} E \sin \theta_1 - R\Sigma I_m \sin m\theta_1 &= \Sigma A_m \sin m\theta_1 \\ E \sin 2\theta_1 - R\Sigma I_m \sin 2m\theta_1 &= \Sigma A_m \sin 2m\theta_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ E \sin n\theta_1 - R\Sigma I_m \sin nm\theta_1 &= \Sigma A_m \sin nm\theta_1 \end{aligned} \tag{5}$$

The general term $\Sigma I_m \sin nm\theta_1$ is the instantaneous value of current in the circuit corresponding to the instantaneous value of impressed voltage, $E \sin n\theta_1$. The numerical value of the left hand member of any equation in group 5 is equal to the instantaneous voltage across the nonlinear resistance at a time when ωt is equal to $n\theta_1$. In general let

$$M_n = E \sin n\theta_1 - R\Sigma I_m \sin nm\theta_1 \tag{6}$$

The term M_n is the voltage drop across the nonlinear element when a voltage $E \sin n\theta_1$ is impressed across the entire circuit. Thus, in figure 1, if the terminal voltage, $E \sin 4\theta_1$, be represented by OB, then M_4 is represented by FC, and if OB' represents $E \sin 3\theta_1$ then $F'C'$ is the value of M_3. The other values of M_n are found in a similarly from the curve.

The value of θ_1 will be chosen as 15 degrees. As it is only necessary to work with $^1/_4$ cycle, it follows that 90 degrees, measured on the fundamental, will be covered by 6 equations. Substituting equation 6 in equations 5 and expanding,

$$\begin{aligned} M_1 &= A_1 \sin \theta_1 + A_3 \sin 3\theta_1 + \ldots\ldots A_{11} \sin 11\theta_1 \\ M_2 &= A_1 \sin 2\theta_1 + A_3 \sin 6\theta_1 + \ldots\ldots A_{11} \sin 22\theta_1 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ M_6 &= A_1 \sin 6\theta_1 + A_3 \sin 18\theta_1 + \ldots\ldots A_{11} \sin 66\theta_1 \end{aligned} \tag{7}$$

In these equations the 6 amplitudes, $A_1 \ldots\ldots A_{11}$, are the only terms that are unknown. Solving for A_1 by means of determinants,

$$A_1 = \frac{\begin{vmatrix} M_1 \sin 3\theta_1 \ldots\ldots\ldots\ldots \sin 11\theta_1 \\ M_2 \sin 6\theta_1 \ldots\ldots\ldots\ldots \sin 22\theta_1 \\ M_3 \sin 9\theta_1 \ldots\ldots\ldots\ldots \sin 33\theta_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ M_6 \sin 18\theta_1 \ldots\ldots\ldots\ldots \sin 66\theta_1 \end{vmatrix}}{\begin{vmatrix} \sin \theta_1 \sin 3\theta_1 \ldots\ldots\ldots\ldots \sin 11\theta_1 \\ \sin 2\theta_1 \sin 6\theta_1 \ldots\ldots\ldots\ldots \sin 22\theta_1 \\ \sin 3\theta_1 \sin 9\theta_1 \ldots\ldots\ldots\ldots \sin 33\theta_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \sin 6\theta_1 \sin 18\theta_1 \ldots\ldots\ldots\ldots \sin 66\theta_1 \end{vmatrix}}$$

Let the denominator of this expression be denoted by D; on expanding,

$$A_1 = M_1 \frac{\begin{vmatrix} \sin 6\theta_1 \ldots\ldots \sin 22\theta_1 \\ \sin 9\theta_1 \ldots\ldots \sin 33\theta_1 \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \\ \sin 18\theta_1 \ldots\ldots \sin 66\theta_1 \end{vmatrix}}{D} - M_2 \frac{\begin{vmatrix} \sin 3\theta_1 \ldots\ldots \sin 11\theta_1 \\ \sin 9\theta_1 \ldots\ldots \sin 33\theta_1 \\ \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots \\ \sin 18\theta_1 \ldots\ldots \sin 66\theta_1 \end{vmatrix}}{D}$$

In the foregoing expansion only the first 2 terms are shown. The remaining terms are similar in form. Since the angle θ_1 has been chosen as 15 degrees the determinants are constants and the multipliers of M_n will be denoted by K with suitable subscripts. The solution for the A terms then can be written

$$\begin{aligned} A_1 &= K_{11}M_1 + K_{12}M_2 + K_{13}M_3 + \ldots\ldots + K_{16}M_6 \\ A_3 &= K_{21}M_1 + K_{22}M_2 + K_{23}M_3 + \ldots\ldots + K_{26}M_6 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ A_{11} &= K_{61}M_1 + K_{62}M_2 + K_{63}M_3 + \ldots\ldots + K_{66}M_6 \end{aligned} \tag{8}$$

From these equations the amplitudes can be calculated by making use of the 6 scaled values from the curve and the coefficients. These coefficients are given in table I.

If equation 2 be expanded and terms of like frequency equated, the following equations can be written:

$$E = A_1 + RI_1; \quad 0 = A_3 + RI_3; \quad \ldots\ldots \quad 0 = A_{11} + RI_{11} \tag{9}$$

Since a sinusoidal voltage wave is impressed, the amplitudes of the higher harmonics are zero.

Solving equations 9 for the current amplitudes,

$$I_1 = \frac{E - A_1}{R}; \quad I_3 = \frac{-A_3}{R}; \quad \ldots\ldots \quad I_{11} = \frac{-A_{11}}{R} \tag{10}$$

Example. Let a voltage $e = 50 \sin \omega t$ be impressed across the nonlinear resistance, as represented in figure 1, in series with a linear resistance of 20 ohms.

Table I—Numerical Values of the Coefficients in Equations 8

n	K_{1n}	K_{2n}	K_{3n}	K_{4n}	K_{5n}	K_{6n}
1	0.0863	0.1667	0.2357	0.2887	0.3220	0.1667
2	0.2357	0.3333	0.2357	0.0000	−0.2357	−0.1667
3	0.3220	0.1667	−0.2357	−0.2887	0.0863	0.1667
4	0.3220	−0.1667	−0.2357	0.2887	0.0863	−0.1667
5	0.2357	−0.3333	0.2357	0.0000	−0.2357	0.1667
6	0.0863	−0.1667	0.2357	−0.2887	0.3220	−0.1667

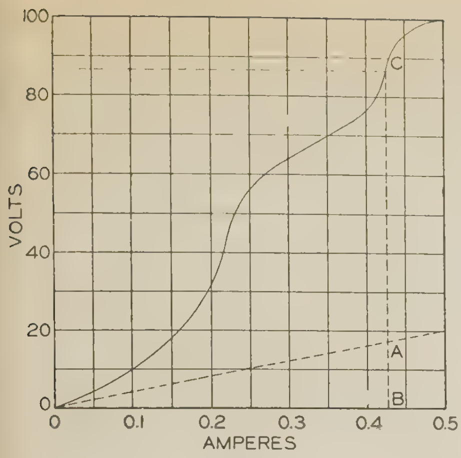


Fig. 2. Volt-ampere curve illustrating general application of method

From the curve the following voltages are found:

$M_1 = 12.8 \quad M_3 = 33.3 \quad M_5 = 43.2$
$M_2 = 24.3 \quad M_4 = 39.6 \quad M_6 = 44.2$

The amplitude of the first harmonic voltage across the nonlinear resistance is calculated from equation 8:

$$A_1 = 12.8K_{11} + 24.3K_{12} + 33.3K_{13} + 39.6K_{14} + 43.2K_{15} + 44.2K_{16}$$

Taking the values of the coefficients from table I and performing the operations indicated,

$$A_1 = 45.71$$

From equation 10

$$I_1 = \frac{50 - 45.71}{20} = 0.2195$$

In the same manner

$$A_3 = 12.8K_{21} + 24.3K_{22} + 33.3K_{23} + 39.6K_{24} + 43.2K_{25} + 44.2K_{26} = 1.42$$

$$I_3 = -\frac{1.42}{20} = -0.0710$$

Higher harmonics are found to be negligible. The current then may be expressed as follows:

$$i = 0.2195 \sin \omega t - 0.0710 \sin 3\omega t$$

The corresponding solution obtained by means of the harmonic analyzer gives the following series:

$$i = 0.2184 \sin \omega t - 0.0726 \sin 3\omega t$$

General Application of Method

The method outlined can be applied easily to more general circuits. While the restriction on the type of nonlinear resistances used in the circuit still must hold, there may be any number of linear and nonlinear resistances, connected in any manner, between the 2 input terminals. Since any resistance in the circuit will pass the same current irrespective of the polarity of a given voltage across it, the network as a whole will behave in a similar manner.

Let the volt-ampere curve OC in figure 2 represent the relation between the terminal voltage and input current of a complex network as found by experimental means. The curve supplies sufficient information for computing the current harmonics resulting from a sinusoidal impressed voltage, 100 $\sin \omega t$. The problem here consists of finding a linear resistance and a nonlinear resistance that, when connected in series, will be equivalent to the complex network. In other words, each of the 2 circuits will have the same volt-ampere curve. If the equivalent circuit is used for computing the current harmonics, the problem is identical to that described in the preceding section, and the same procedure may be followed.

A great many equivalent circuits are possible. Let the line OA in figure 2 be drawn at any convenient angle Φ. This line may be considered the volt-ampere curve of an imaginary linear resistance R, which is equal to the tangent of Φ. For a current OB, the vertical distance AC from the straight line to the curve represents the voltage drop across the nonlinear resistance. Thus if BC is some definite value of voltage, $E \sin 5\theta_1$ for example, then AC is the value of M_5. Other values of M_n may be found in a similar manner and the solution carried out as before.

Curve OC in figure 2 is an arbitrary one designed to introduce some of the higher harmonics in order to check the accuracy of the method. The expression for the current is found to be

$$i = 0.491 \sin \omega t + 0.014 \sin 3\omega t + 0.018 \sin 5\omega t + 0.015 \sin 7\omega t + 0.027 \sin 9\omega t$$

The corresponding solution obtained by means of the harmonic analyzer gives

$$i = 0.488 \sin \omega t + 0.0168 \sin 3\omega t + 0.0132 \sin 5\omega t + 0.0118 \sin 7\omega t + 0.0288 \sin 9\omega t$$

Accuracy of Method

While the method shown has the errors inherent in any graphical solution and also those resulting from the use of a limited number of points, the results indicate a reasonable accuracy with little labor. If greater accuracy be desired, θ_1 may be chosen smaller than 15 degrees, say 9 or 7.5 degrees, thereby including more points. In this event, however, the table of coefficients would have to be recomputed.

The amount of work involved in plotting a current wave against time and analyzing it is considerable, and if the values are obtained from the volt-ampere curve similar errors are involved. Experimentally, the curve may be obtained with an oscillograph and then analyzed. This method may be more accurate, but time and labor are large factors.

It should be noted that the application of the coefficients shown in table I is perfectly general. They refer to no particular curves or circuits. All nonlinear resistances, however, must have volt-ampere curves that are symmetrical in the first and third quadrants.

References

1. Thyrite: A New Material for Lightning Arrestors, K. B. McEachron. *General Electric Review*, v. 33, 1930, p. 92–9.

2. The Calculation of Circuits Containing Thyrite, Theodore Brownlee. *General Electric Review*, v. 37, 1934, p. 175–9 and 218–23.

3. Electric Circuit Analysis, M. G. Malti. A book, John Wiley and Sons, Inc., New York, N. Y.

The A-C Electrolytic Capacitor

The electrolytic capacitor for alternating current is coming into increased use, having recently been introduced for the starting of small single phase motors. Little is known of the film upon which its operation depends, but the general construction and characteristics of the capacitor are reviewed in this paper. The need for the development of testing procedure that will lead to an improved product is pointed out.

By
C. F. LOMONT
Membership Application Pending

F. S. DUNLEAVEY
Membership Application Pending

Both of The Magnavox Co.
Fort Wayne, Ind.

THE general principle of the electrolytic capacitor is not new. It was investigated in the laboratory nearly a century ago, but its practical development awaited a commercial demand which first presented itself in the a-c radio receiver and later in the single phase motor.

The principle of the electrolytic capacitor lies in the insulating film which may be formed by chemical or electrochemical means on the surface of various metals of which aluminum is the most commonly used. This film with a thickness of the order of the wave length of light, has both high insulating value and high dielectric constant, so from these qualities it approaches an ideal dielectric. It does, however, have certain limitations. In spite of its high insulating value it permits an appreciable leakage current to flow as compared with other high grade dielectrics. Moreover, the film must be operated in the presence of a suitable electrolyte. Attempts to place the film in direct contact with another metallic conductor, or to place 2 filmed surfaces together, result in prompt puncture of the film when voltage is applied. Hence in practice it is necessary to use some spacing material between the filmed surface and the other electrode, or—in a-c capacitors—between the 2 filmed surfaces. The spacers commonly employed commercially in dry capacitors are gauze and paper, both specially selected for physical properties and chemical purity.

A paper recommended for publication by the A.I.E.E. committee on electrochemistry and electrometallurgy, and scheduled for discussion at the A.I.E.E. Great Lakes District meeting, West Lafayette, Ind., October 24–25, 1935. Manuscript submitted July 29, 1935; released for publication August 29, 1935.

Formation of Film

The film is usually formed on the aluminum foil by impressing positive potential on the foil while passing it through a suitable electrolyte such as a water solution of borax and boric acid. The electrolyte is in a metal container which acts as the cathode. About the film itself little is known. Next to the aluminum surface it is probably a very dense deposit of aluminum oxide which degenerates upward into a spongy amorphous structure, perhaps an hydroxide of the metal.

The a-c electrolytic capacitor consists of 2 filmed aluminum strips placed together with spacers between, the entire assembly being rolled up concentrically. Suitable electrolyte is introduced either during or after the rolling. The film possesses the unique property of permitting charge, negative electrons, to flow only in the direction from metal to electrolyte. Hence by placing 2 filmed electrodes in series conduction current cannot flow through the 2 in either direction, because the permeable direction from metal to electrolyte for one film becomes the impermeable direction from electrolyte to metal for the other film. Thus a true a-c capacitor is formed since there is no flow of conduction current through it (neglecting a small leakage current).

At any given instant the charging current of the capacitor consists of the capacitance current of one filmed electrode only. The other filmed electrode acts merely as a conductor and could momentarily be replaced by an unfilmed electrode without in any way affecting the instantaneous performance of the capacitor as a whole. With the reversal of direction of current flow on the next half cycle, these functions of the 2 electrodes are reversed or interchanged.

Voltage Relations in A-C Electrolytic Capacitors

Since negative electrons flow freely through the film in the direction metal-electrolyte but not in the direction electrolyte-metal a very interesting condition of potential develops.[1] Starting with a completely discharged capacitor when all parts are at the same potential, an alternating voltage is applied to the electrodes. At the instant of application negative electrons flow into the negative electrode. The film being permeable to negative charges in the metal-electrolyte direction these negative charges pass freely through it into the electrolyte through which they are conducted ionically to the surface of the other electrode. There they are stopped by the film which is impermeable in the electrolyte-metal direction. This group of negative charges thus comprises the total charge Q of the capacitor. They constitute a bound charge at the surface of the positive film so long as the capacitor voltage is unchanged.

As the alterating voltage wave falls to zero, the charge is no longer bound to the surface of the film since both electrodes are now at the same potential. The charge in the electrolyte, being free, tends to dissipate itself in the external circuit. It is restrained from doing so, however, by the impermeability of

1. For numbered reference see end of paper.

both films in the electrolyte-metal direction. Thus, the electrolyte permanently retains the entire charge acquired during the first half cycle of voltage. (This entire discussion neglects the leakage current of the film which will eventually discharge the electrolyte.)

Since, as has been shown, the electrolyte retains its full charge at the zero point of the voltage wave it is obvious that an equal and opposite charge must appear on one or both electrodes. However, the potential difference between the 2 electrodes being zero, this charge must be equally distributed between them. The important point is that its total magnitude is the same as when the capacitor was fully charged.

As the impressed voltage on the second electrode now reaches its maximum negative value no additional electrons flow from the negative electrode into the electrolyte, because the latter already possesses a complete charge for the particular maximum value of voltage. At the same time all the negative charge in the electrolyte has accumulated at the surface of the opposite electrode, which has now acquired a full positive charge.

Thus it is seen that on alternating current the charge of the capacitor is pumped by the generator alternately from one electrode to the other and simultaneously the charge in the electrolyte which equals the charge Q of the capacitor is flowing back and

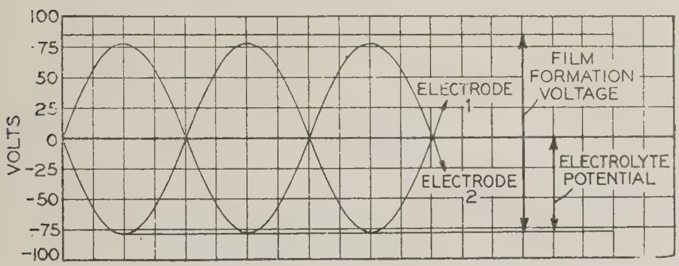


Fig. 1. Ideal voltage-time relationships in a-c electrolytic capacitor with 110-volt 60-cycle sine wave

forth through the electrolyte from one electrode to the other. The curves shown in figure 1 illustrate graphically this action. It may be observed that the electrolyte potential slides down along the first voltage wave to the latter's maximum negative value where it remains, and that neither electrode ever becomes negative as regards the electrolyte. This is a theoretical discussion only, and is not based upon actual voltage measurements which will vary somewhat from the curves. A grasp of the principle involved, however, is necessary to a clear conception of the various phenomena encountered in electrolytic capacitors.

Power Losses in Electrolytic Capacitors

Since as has been shown the entire charging current of the capacitor flows from plate to plate through the electrolyte at each reversal of potential, it would be expected that this flow would result in a power loss I^2R where R is the ohmic resistance of the electrolyte.

The current density in the electrodes or foil varies with its distance from the terminal, being the full charging current of the capacitor at the terminals and zero at the opposite end. By integrating the expression

$$P_E = 2\int_0^l \frac{4\pi^2 f^2 C_0^2 V^2 W R_0 (10^{-18}) X^2 dx}{t}$$

we can determine the power loss due to conduction in the foil, where

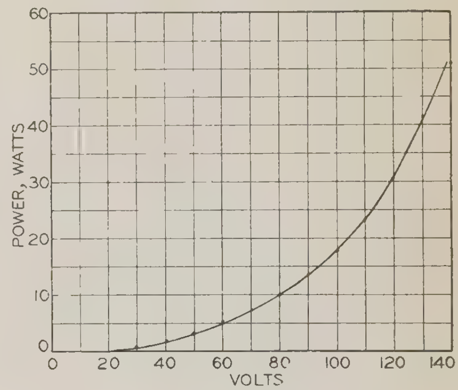
P_E = total losses in watts in both electrodes
F = frequency of voltage in cycles per second
C_0 = capacitance in microfarads per square inch of one electrode
V = root mean square value of impressed potential in volts
W = width of electrode in inches
R_0 = resistivity of electrode material in microhms per inch cube
l = length of a single electrode in inches
t = thickness of electrode in inches

For foil electrodes 0.002 inch thick, 3 inches wide, 153.3 inches long and possessing a capacitance of $^1/_4$ microfarad per square inch, these losses prove to be about 0.4 watt when 110 volt, 60 cycle potential is impressed across the capacitor terminals.

By a simple computation based upon an electrolyte resistivity of 1,000 ohms per centimeter cube, it is found when the foregoing potential is impressed that the power loss in the electrolyte of this capacitor is approximately 0.1 watt. This is based upon an electrolyte thickness of 0.004 inch, as would exist when 2 0.002-inch paper spacers are employed.

The sum of the total electrode and electrolyte losses is thus 0.5 watt. Assuming a film leakage at

Fig. 2. Variation of power loss with 60 cycle voltage in a 110-microfarad 110-volt paper-spacer capacitor at 25 degrees centigrade



110 volts of 9 milliamperes, which is a fairly high average value, one additional watt loss is allocated. Thus all the accountable losses aggregate less than 2 watts.

Other Losses

Actual tests show that the losses so computed comprise less than 10 per cent of the entire losses in the capacitor. The nature of the additional losses is not definitely known though their existence has long been recognized. The authors, however, discovered that these losses increased with the voltage applied to the capacitor more rapidly than a square law would predict. Also, there seemed to be a definite rule governing this increase. It was first ob-

served that doubling the voltage increased the losses in the capacitor about 5 times. These figures are all based on voltages not exceeding the formation voltage of the foil.

Comprehensive tests on foils formed at various voltages were conducted. Characteristic data are plotted on rectangular co-ordinate and logarithmic paper, respectively, in figures 2 and 3. The capacitance ratings given in the figures are actual values. In all these capacitors the effective area of each anode per microfarad is 9.2 square inches for 220 volt rating, 4.6 square inches for 110 volt, and 3.2 square inches for 75 volt.

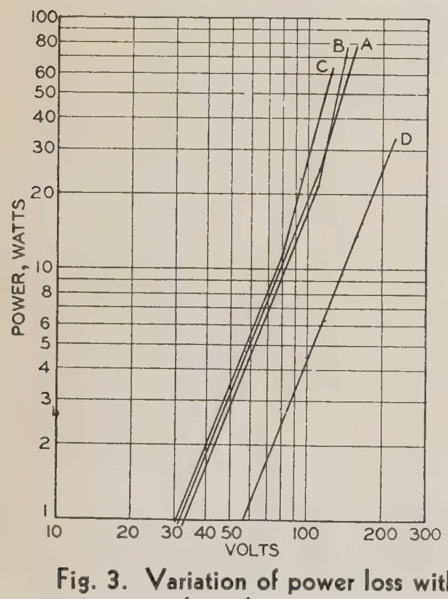
The foil in the 75 volt units is formed at 115 volts direct current, while the 110 volt and 220 volt units employ 165 and 330 volt formation, respectively. It will be noticed that the curves on logarithmic paper are straight lines, which indicate an equation with a definite exponent. Computation shows this exponent to be between 2.42 and 2.5 for various samples tested.

The expression for the losses in a capacitor at any voltage may then be written as

$$P = KE^{2.5}$$

where K is a constant for the particular capacitor depending upon its capacitance, power factor, and temperature.

Curves are shown for 3 different capacitors, A, B, and C in figure 3, all of which are parallel, indicating the same exponent. The horizontal displacement of



A. Paper spacer, standard electrolyte, 110 volts, and 110 microfarads

B. Gauze spacer, special electrolyte, 110 volts, and 119 microfarads

C. Paper spacer, standard electrolyte, 75 volts, and 124 microfarads

D. Gauze spacer, standard electrolyte, 220 volts, and 55 microfarads

Fig. 3. Variation of power loss with 60 cycle voltage in a-c electrolytic capacitors at room temperature

the several curves represents varying values of the constant K. These 3 curves are all at room temperature, about 25 degrees centigrade.

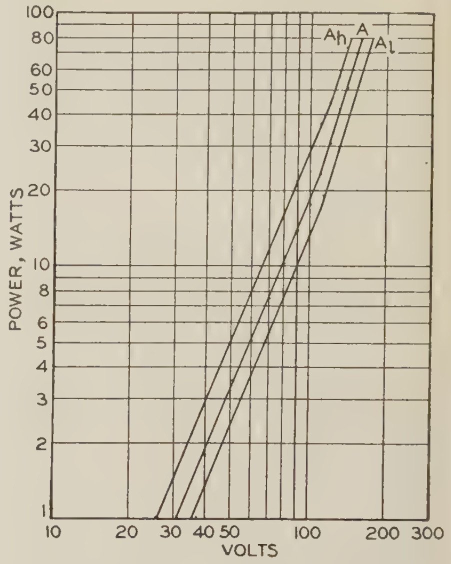
It was realized, however, that at different temperatures the exponent of the equation might change, or the curve might even depart from a straight line in which case the equation would be inapplicable. That K would change with temperature was a foregone conclusion. To cover these possibilities, the same capacitor that gave curve A, figure 3, was again tested at 60 degrees centigrade and at −18 degrees centigrade, and the respective curves are shown as A_h and A_l, in figure 4. To all intents and purposes these 3 curves are absolutely parallel. The exponent becomes 2.49 in every case within the limits of

Fig. 4. Variation of power loss with 60 cycle voltage in a 110-microfarad 110-volt paper-spacer capacitor at different temperatures

Curve A at 25 degrees centigrade

Curve A_l at —18 degrees centigrade

Curve A_h at 59 degrees centigrade



error of the graphical method employed. The constant K shows the expected variation. This constant, incidentally, can be read directly for various temperatures from a properly scaled power-temperature curve of the capacitor being investigated.

Capacitors rated from 75 volts to 220 volts were included in the foregoing studies, as well as samples with entirely different types of electrolytes, and in no case was the exponent found to vary outside the limits set forth. These different electrolytes were all of the so-called paste type as commonly used in dry electrolytics and employed an ammonium borate base but the solvents differed, resulting in a range of resistivity of from 200 to 1,000 ohms per centimeter cube.

As soon as the formation voltage of the foil is exceeded the curve takes a steeper slope upward showing that the losses increase more rapidly than the above exponent indicates. Whether this portion of the curve continues as a straight line is not known. Capacitors are not customarily operated above their formation voltage so it was not deemed of practical value to investigate this area too extensively. It is interesting to note, however, that in curves A and C, figure 3, the portions above the formation voltages are parallel. These 2 capacitors employ electrolyte of the same basic composition. Curve B assumes a definitely different slope above its formation voltage. This capacitor was built with an electrolyte in which another solvent was substituted for the ethylene glycol used in A and C. This solvent is not in the polyhydric alcohol group as is glycol. The resistivity of electrolyte B is far lower than that of A and C.

It is worthy of note that in this investigation of power loss at different voltages capacitors were built with radically different electrolytes; both

Table I—Temperature Distribution in Capacitors

Time, Hours	Temperature, Degrees Centigrade								
	With Gauze Spacer				With Paper Spacer				
	A	B	C	Can	A	B	C	Can	Room
0.0	34	33.5	34		34	33	32		35
0.5	42	40	41		42	41.5	40		34.8
1.0	46.5	45	43.5		47.5	46	45		36.8
1.5	48	47	47		50	47	49		35
2.0	53	52	50	39	53	52	49.5	41	35.4
18.0	60.5	59	57	42	62.5	59	57	45	31

Thermocouple *A* was in the inner turn of the capacitor winding, *B* in the middle of the winding, and *C* in the outer turn.

gauze and paper spacers were employed; electrodes formed over a wide range of voltages were used and all these units were tested at an extreme range of temperatures. Yet, in spite of varying all these factors the exponent of the equation remained fixed. It seems an unescapable conclusion, therefore, that this exponent is an inherent function of the electrolytic film itself and arises from the molecular structure and functioning of the latter.

Temperature Gradients in Electrolytic Capacitors

The life of an electrolytic capacitor is dependent definitely upon its internal operating temperature. This fact is recognized generally and is illustrated forcefully by the following experiment. A standard paper spacer electrolytic tested at room temperature with a certain number of line voltage applications per hour lasted for about 1,500 hours. Tested similarly in a 60 degree centigrade oven it lasted about 120 hours—approximately $^1/_{10}$ as long.

The importance of this internal temperature on the life of the capacitor has led to a study of the relative temperatures on gauze as compared with the more recent paper spacer capacitors. The argument has gained wide credence in the industry that since the paper type of a given capacitance occupies a smaller can than the gauze type it must, due to the smaller radiating area, run hotter inside. Obviously this is fallacious since radiating area is only one factor. Others equally important are thermal conductivity of the materials and the length of the conducting path. The latter is necessarily shorter in the smaller can. Conductivity of paper, however, may be less than of gauze.

Capacitors of both gauze and paper and of identical capacitances were built with thermocouples embedded in the inner and outer turns of the winding, and a third half way between them. Table I shows the actual temperatures as measured with the above arrangement. The capacitors were excited with rated alternating voltage for 3 seconds out of each 20 seconds until thermal equilibrium was attained. The greater part of the temperature rise occurs in the first 2 hours. The interesting point, however, is that the temperatures at the various points in the paper and gauze capacitors are practically identical; the slight discrepancy might easily be due to the difference between individual capacitors. The difference in can temperatures is to be expected since the same number of watts is being radiated from surfaces of different areas.

Capacitance-Temperature Studies

In certain applications capacitors are subjected to extreme temperature ranges. Used as a motor starter outdoors the capacitor should operate down to −40 degrees Fahrenheit, while in the summer on a similar installation the unit might be raised by the motor heat to as high as 125 or 150 degrees Fahrenheit. While the average electrolytic has behaved fairly well at the higher temperatures, its capacitance falls off very rapidly at low temperatures of about 0 degrees Fahrenheit and below.

The authors have attempted to develop a capacitor which would have a much flatter capacitance-temperature curve. The curve of figure 5 shows the results. Over a range of 125 degrees centigrade the reduction in capacity down to minus 50 degrees Fahrenheit was less than 25 per cent. This is only a fraction of the usual reduction in capacitance heretofore found in this type of capacitor. It is rather surprising to note that the points in figure 5 practically fall on a straight line, the maximum deviation being less than 4 per cent. The curves of similar temperature range previously published have approached hyperbolic shape. It is possible of course that these capacitors will exhibit a similar curve if the temperature is sufficiently reduced but the range explored more than covers the practical working range.

Based on the straight line function of figure 5, the change in capacitance per degree centigrade from

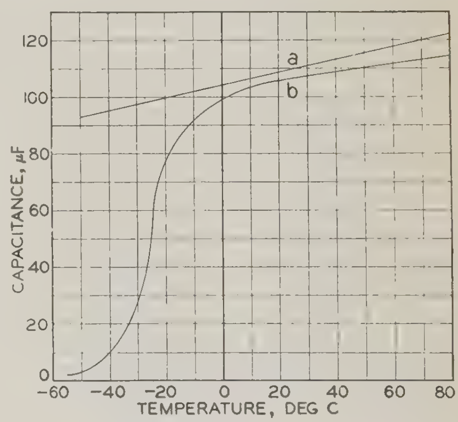


Fig. 5. Variation of capacitance with temperature

a. Improved electrolytic capacitor

b. Typical electrolytic capacitor

room temperature is 0.23 per cent or roughly 0.2 per cent. Then the following capacitance formula may be written:

$$C_t = C_T\,[1 + 0.0023(t - T)]$$

where

T = temperature at which the capacitance is measured
t = temperature at which the capacitance is desired
C_T and C_t are the capacitances at these respective temperatures

These tests were conducted at 2 widely separated times by different technicians, and 6 individual

capacitors were tested. As a final check the capacitors were placed overnight in a commercial cold storage room at −18 degrees centigrade. Results of all these tests coincided very closely.

Life Studies of Electrolytics

The average user probably lacks a complete realization of the vast variations in life of electrolytics resulting from almost uncontrollably small changes in materials or technique of design and manufacture.

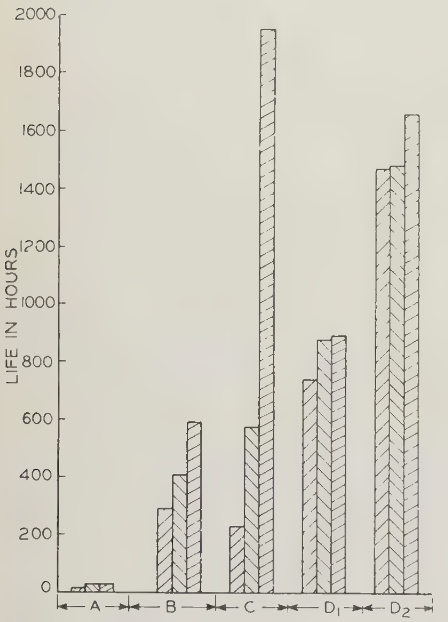


Fig. 6. Comparison of accelerated life tests of several makes of commercial electrolytic capacitors for motor starting service

All capacitors have paper spacers and are rated 110 volts and 100 microfarads

Even the best manufacturers in the industry find their ingenuity severely taxed to design capacitors so that they may be built with uniform life and then to control the manufacturing processes closely enough to secure this essential end.

It has been the experience of the authors based upon rather careful and searching investigation that correct elements in the design are vital to a uniformity of long life. If these design factors are lacking it is impossible to maintain materials and processes within sufficiently close limits to prevent wide variations in life of the final product. Obviously there is little advantage in having one third of a group of capacitors last 10 years in service if another third fails in one year. It is much better that they all run about the same length of time even though it may be less than this maximum.

For the purpose of illustrating these points a graphic presentation of life studies of various makes of commercial electrolytic capacitors is made in figure 6. Three capacitors of each of various leading makes were tested. All these capacitors employ practically identical materials in their fabrication. It may be argued that 3 capacitors are not enough to give an accurate picture; however, the authors believe this is not true in this case where it is the uniformity rather than the average length of life which is of primary importance. The life of make *A* was of such fleeting duration as to require no comments. Make *B* shows no particular merit either as to uniformity or length of life. Make *C* is highly interesting in that it contained one unit which displayed a materially longer life than any other of the 15 capacitors charted; yet another unit of this make failed earlier than any unit shown excepting make *A*.

This emphasizes the point of the preceding discussion as to the necessity of designing uniformity into the capacitors. Based upon his design this manufacturer finds it impossible to control his manufacture adequately to secure uniformity. If he could duplicate uniformly the best of these 3 capacitors the product would be outstanding.

Make *D* consists of 2 groups which differ slightly from each other in technique of manufacture. Although one group has a life about twice the other they are both good from this standpoint. They are, however, noteworthy from the angle of uniformity, since all units in each group failed within less than 10 per cent of the same time.

Standard Testing Procedure Needed

In spite of the value to the industry of some of the laws developed in this paper, the authors fully realize the insignificance of this contribution compared with the vast amount of fundamental investigation which remains to be done before engineers can profess any real knowledge of the subject of electrolytic capacitors.

While a product satisfactory for certain applications is available there are many potential uses which the manufacturer cannot satisfy today. Continuous a-c duty is one of the most common. It is hoped that before long the work being done by various manufacturers may realize this particular result.

Even on the question of accelerated life testing of the finished capacitor for a single type of application there is wide disagreement, which imposes hardships on both manufacturer and customer. The development of a satisfactory standard method of test would be a boon. It is the clear duty of the manufacturers to aim at such an objective.

Several large users of a-c electrolytic capacitors have voluntarily expressed their readiness to adopt any accelerated life test which might be agreed upon by the capacitor manufacturers. Unfortunately, the authors do not feel prepared at this time to suggest any such standard. While they have employed at various times and for various purposes several testing procedures, no one of these seems to possess every desirable feature of all of them.

One test commonly employed is the imposition of about 115 volts at 60 cycles for 3 seconds out of each 20 seconds. A variation of this test for use on motor starter capacitors is the introduction in series with the capacitor of an impedance duplicating that of the motor on which the capacitors are to be used. Other tests consist of one or 2 excitations per minute of one second duration each, either with or without impedance in series. Finally, the capacitor may be tested by actually using it to start a motor loaded or unloaded. In this case from one to 10 starts a minute may be employed. The disadvantage of the slower tests is that they require too long an elapsed

period to test the units to failure, and thereby obtain the desired answer.

It is the hope of the authors in presenting this paper that by giving to the profession the results of their research a better understanding of the underlying problems may be brought about. It is upon a fuller grasp of these basic facts that the fullest development of electrolytic capacitors for the entire range of present and future industrial applications must depend.

Reference

1. C. I. Zimmerman, Electrochem. Soc. *Trans.*, v. 5, 1904, p. 147–70.

A Static Thermionic Tube Frequency Changer

By
A. SCHMIDT, JR. ASSOCIATE A.I.E.E.
R. C. GRIFFITH Membership Application Pending

Both of General Electric Company, Schenectady, N. Y.

A 300 kva, synchronous type, 60–25 5/7 cycle static frequency changer, which was designed to be changed to a rectifier after the 25 cycle motor load has been replaced by 60 cycle motors, is described in this paper. The problems involved in the design of the control circuits are discussed in some detail, and the performance is described.

THE versatility of thermionic tubes in various types of service is illustrated by the static frequency changer described in this paper. An industrial plant in New Jersey (New York Shipbuilding Corp., Camden) generates its own power for a load consisting principally of 60 cycle and d-c motors and it was desired to replace by more efficient equipment an engine driven generator supplying power to a few hundred horsepower of 25 cycle motors. It was anticipated that during the next few years these motors would be replaced gradually by 60 cycle motors and there would be no further need for the new 25 cycle generating equipment. During certain periods of the week the demand for d-c power is considerably less than the capacity of the smallest generating unit in the plant so that an efficient source of a few hundred kilowatts of d-c power would return an appreciable saving. A static frequency changer was finally chosen which uses 12 grid-controlled mercury vapor tubes and delivers 300 kva at 220 volts, 2 phase, 25 5/7 cycles, from a 2,300-volt 3-phase 60-cycle source. When 25 cycle power is no longer required, the 60 cycle transformer windings will be reconnected for a lower secondary voltage, a portion of the control equipment will be disconnected, and the apparatus will be available as a rectifier with an output of 300 kw at 250 volts.

A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted Apr. 15, 1935; released for publication July 8, 1935.

Power Circuit

The frequency changer comprises 2 single phase conversion circuits whose outputs are displaced 90 electrical degrees. A network was chosen which affords maximum apparatus economy consistent with subsequent conversion to a rectifier circuit. Figure 1 shows the elements of one 3 phase to single phase power circuit. Assuming a resistance load, if groups *A* and *B* are fired alternately the output voltage will alternate and will have a frequency dependent upon the number of tubes fired consecutively

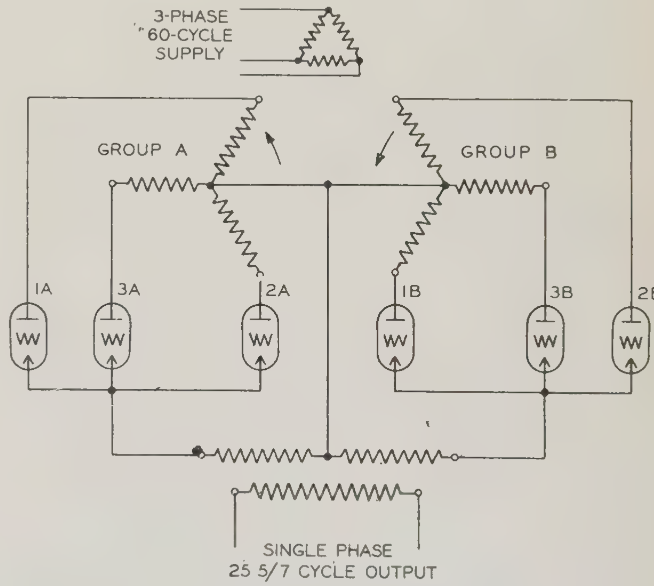


Fig. 1. Elementary diagram of a 3 phase to single phase frequency changer

in each group. In the present case 3 tubes are fired consecutively for each half cycle, resulting in the output voltage and current shown by figure 2*a*.

Next consider the case in which the load is purely reactive and, for simplicity, assume there are sufficient filters in the output circuit to permit only the fundamental current to flow, as shown by figure 2*b*. Tube 2*A* fires when its phase voltage is a maximum, conducts for 60 degrees (referred to the input frequency) and transfers the current to tube 3*A*. With a resistance load, tube 3*A* would conduct for 150 degrees after which tube 3*B* would fire. In this case, however, the current is a maximum at this time, and group *B* cannot conduct until the current in group *A* has become zero. The potential of phase 3*A* opposes the flow of current after the 150 degree point and gradually reduces the current, but at the end of another 150 degrees the current is still not zero, so the current is transferred to tube 1*A* which reduces the current to zero in 60 degrees.

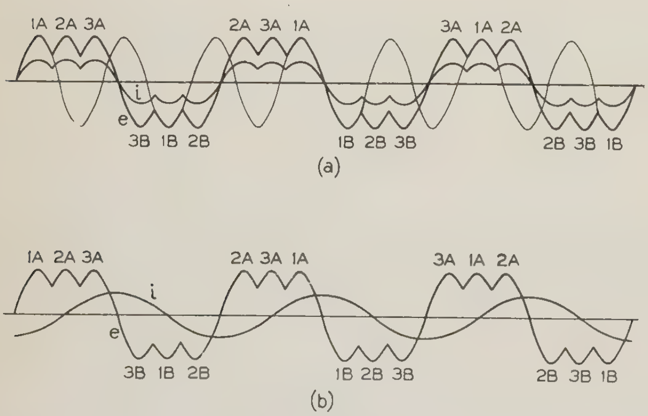


Fig. 2. Output currents and voltages

a—Output current and voltage of a frequency changer with a resistance load

b—Output current and voltage with reactive load and sufficient filter to prevent all but the fundamental current from flowing

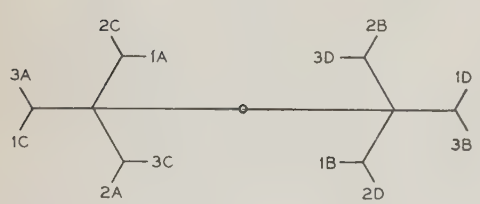


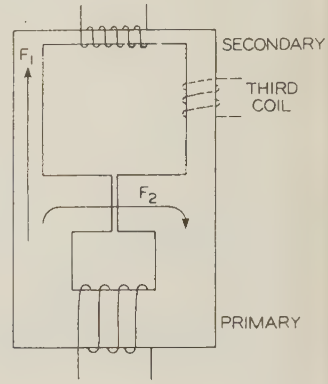
Fig. 3. Secondary windings of input transformer

It will be seen that the output voltage is the same whether tube 1*A* or tube 1*B* is conducting, so as soon as tube 1*A* ceases to conduct tube 1*B* is fired.

Current is transferred from tube 3*A* to tube 1*A*, in the above discussion, by phase commutation. Current must be transferred from 3*A* to 1*A* early enough for 3*A* to deionize before the anode again becomes positive. This time will increase with the load.

For any intermediate power factor and with current waves having higher harmonics the current transfer between groups *A* and *B* will occur in a similar manner: (*a*) the load current must be zero at the time of transfer; (*b*) the current always transfers to an anode 180 degrees out of phase with the last conducting anode; (*c*) when the power factor is very low a transfer of current by the method used in the phase commutated inverter is necessary.

Fig. 4. An impulse transformer



The actual windings of the input transformer secondary are shown by figure 3, where *C* and *D* refer to the second converter circuit.

Control Circuit

Four parts are necessary for the control circuit: (1) grid excitation for the tubes when supplying power; (2) grid excitation when regenerating; (3) a means for firing only 3 consecutive tubes in each group; (4) a means to prevent firing any tube in one group until the current has become zero in the other group. Of the several methods which have been proposed, a circuit using "impulse transformers" has been selected.

In this paper the term "impulse transformer" refers to a transformer which, when excited by a sinusoidal primary voltage, has a sharply peaked, nonsinusoidal secondary voltage. Figure 4 is a sketch of a simple impulse transformer with one secondary winding. The primary flux passes through 2 parallel paths, F_1 and F_2, when the density is low, and as the reluctance of F_1 is less than the reluctance of F_2 a relatively large secondary voltage is induced. After F_1 has saturated, however, most of the flux

Table I—Harmonic Composition of Output Voltage and Current

Quantity	Order of Harmonic 1	3	5	7	9
No load voltage	1.000	0.308	0.156	0.112	0.041
Voltage, 15% load 0.60 p. f.	1.000	0.275	0.166	0.108	0.060
Voltage, 100% load 0.76 p. f.	1.000	0.193	0.125	0.108	
Output current, 15% load 0.60 p. f.	1.000	1.223	0.464	0.169	0.096
Output current, 100% load 0.76 p. f.	1.000	0.521	0.266	0.142	

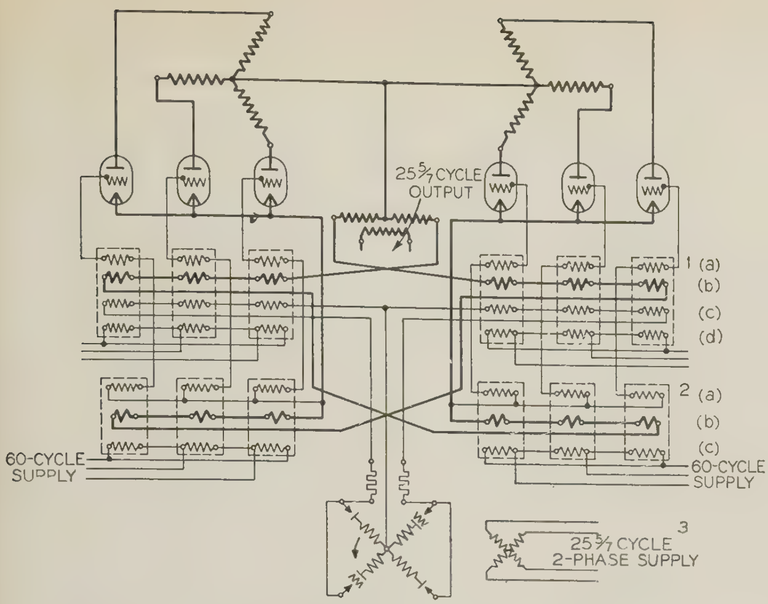


Fig. 5. Schematic diagram of power and control circuits

1*a*—Power control impulse transformer secondary
1*b*—Hold-off coil
1*c*—Frequency control coil
1*d*—Primary coil
2*a*—Regenerative control impulse transformer secondary
2*b*—Phase advancing coil
2*c*—Primary coil
3—Output frequency control circuit

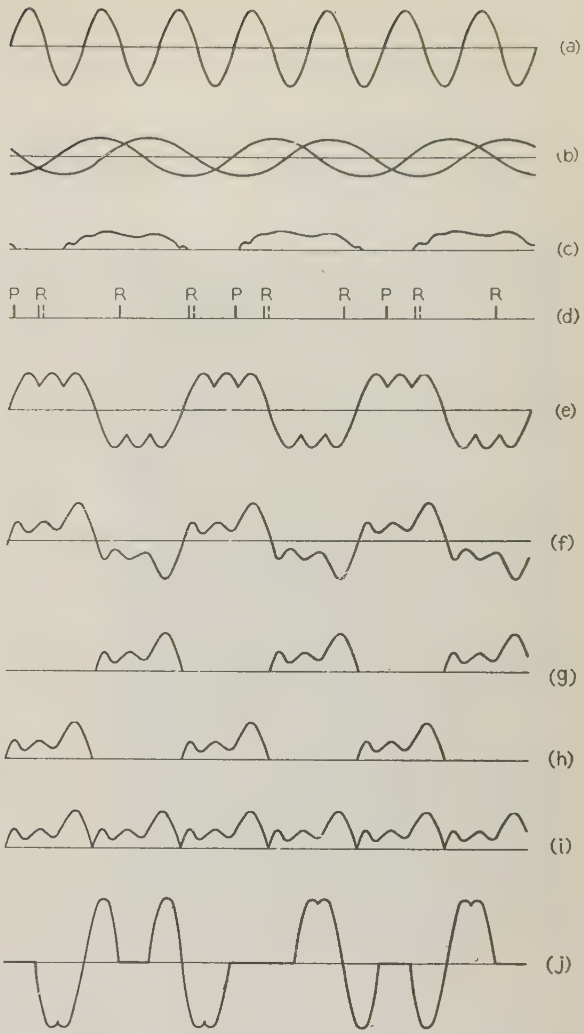


Fig. 6. Voltages and currents when supplying power to an induction motor

a—Phase voltage associated with tube 1*A*
b—Output frequency control voltage
c—Output frequency control current
d—Grid excitation. *P* is the power excitation and *R* is the regenerative excitation. The dotted lines show the position of regenerative excitation when unaffected by phase advancing currents
e—Output voltage
f—Output current to an induction motor load
g—Current in hold-off coil
h—Current in phase advancing coils
i—Current in common neutrals of groups *A* and *B*
j—Anode-cathode voltage of tube 1*A*

passes through F_2 and the rate of change of flux through F_1 is small. If a constant direct current is passed through another coil wound on F_1, the time at which the peak of voltage occurs will be different because the core density will be zero at a different time; by varying the magnitude and direction of this direct current the time at which the peak occurs can be changed ± 90 degrees.

If the number of ampere turns in the third coil is slightly greater than the number of primary ampere turns, no secondary voltage will be induced because the core will be saturated continually. Now assume that the current in this third coil varies in some regular manner. The flux through F_1 will be the sum of the fluxes induced by the primary and the third coil, and a peak of secondary voltage will occur when the resultant flux does not saturate F_1.

As it is necessary to supply excitation to each tube at 2 different times, 2 impulse transformers are necessary for each tube. The one supplying excitation for power control has a primary coil, a secondary coil, a frequency control coil, and a hold-off coil, the latter 2 surrounding F_1. The frequency control coil receives pulses of direct current at the required output frequency; the hold-off coil is in series with the output transformer primary winding associated with the group of tubes opposite to the group in which the tube under consideration is placed. The impulse transformer supplying excitation for regeneration has a primary coil, a secondary coil, and a phase advancing coil, the latter surrounding F_1. The phase advancing coil is in series with the half of the output transformer primary associated with the tube under consideration. Figure 5 is a schematic diagram of

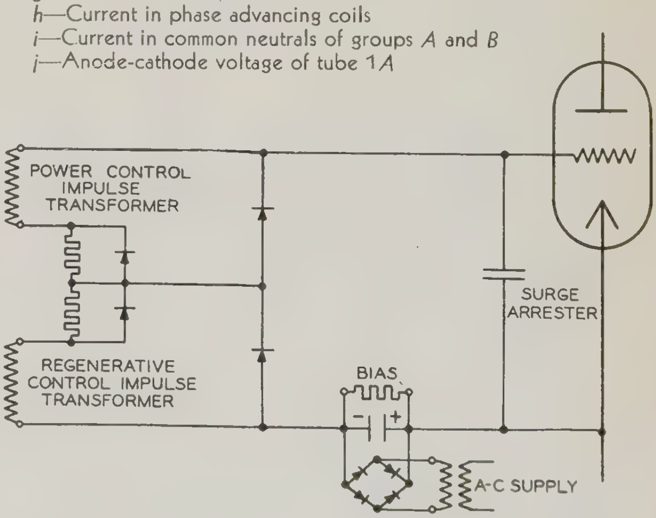


Fig. 7. Grid circuit used in the frequency changer

the power and control circuit, and figure 6 shows various voltages and currents in the several parts of the circuit when supplying power to an induction motor.

A small synchronous motor generator set having a frequency ratio of 7 to 3 furnishes power to the

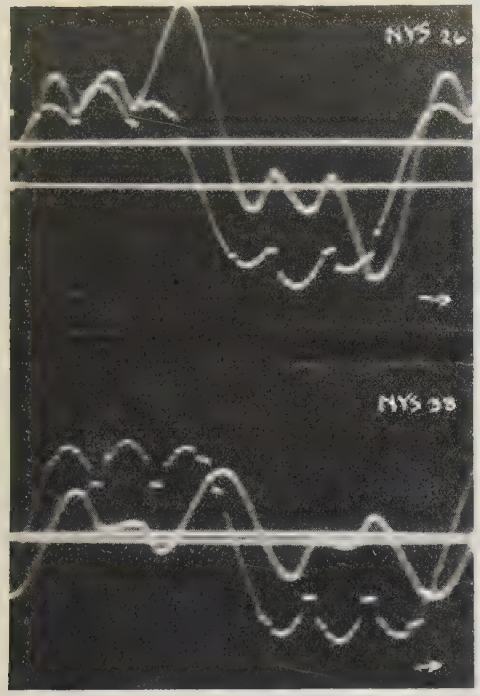


Fig. 8. Current and voltage in induction motor load

Upper oscillogram—100 per cent load, 0.76 power factor

Lower oscillogram—15 per cent load, 0.60 power factor

frequency control circuit which, in turn, supplies alternate pulses of direct current to the frequency control coils of groups *A* and *B*. By proper timing, just 3 consecutive tubes are fired in each group. With a unity power factor load the flux induced by the current in the hold-off coils adds to that induced

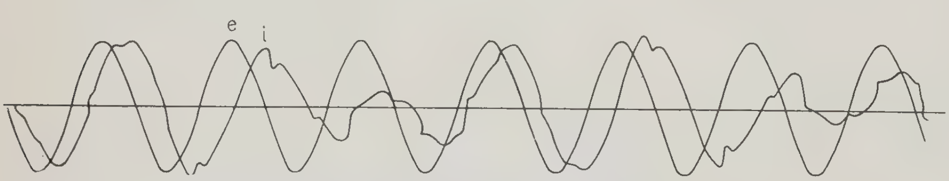


Fig. 9. Input current and voltage

by the current in the frequency control coils without further effect, but when the load is reactive the hold-off current maintains F_1 in a saturated state until the load current approaches zero.

As mentioned previously, the phase of the regenerative excitation must be advanced as the load increases. This is accomplished by the phase advancing coils.

Figure 7 shows the complete grid circuit for one tube. With this copper oxide rectifier combination, the impedance presented is low when the grid is excited, and the resistor limits the positive ion current to a value which does not reduce the bias voltage appreciably.

Output Voltage and Current, and Input Current

Table I shows the harmonic composition of the output voltage and current with various loads. The decrease in harmonic content of the current as the load increases is the result of a constant transformer inductance in series with a variable load impedance. Since the regulation of the harmonic voltages is greater than that of the fundamental, the harmonic voltages in the output decrease with load also. Oscillograms for the cases enumerated above are shown by figure 8.

Figure 9 shows the input current with an induction motor load. It will be noted from figure 2 that a given anode fires during a specified portion of the output cycle only once in 3 output cycles or 7 input cycles, hence the input current may be expected to contain fractional harmonics, namely, 1/7 and odd multiples thereof. Since these fractional harmonics increase the generator losses, their magnitude must be limited when the frequency changer output is a considerable portion of the generator rating. Reduction of harmonic input currents may be effected by the use of sufficient anodes and different anode potentials to secure an output voltage wave with small harmonic content. Resonant filters in the input transformer primary circuit may be used to reduce the harmonics in the generator windings.

Regulation

Regulation is due principally to 2 factors: (1) commutation from one anode to another, with a loss in voltage proportional to the load; and (2) reactance of the output transformer. In addition, there is some regulation due to resistance in the circuit and reactance of the control windings on the grid transformers. The measured regulation in terms of root mean square voltage was 14 per cent from no load

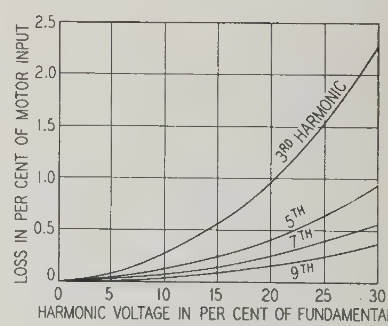


Fig. 10. Increase in losses of typical 2 phase induction motor due to harmonics in supply voltage

to full load. Since the harmonic voltages are more prominent at no load, the regulation of the fundamental component of voltage is less, the difference being 2 per cent.

Efficiency

A frequency changer of this type has the following circuit losses: (1) input transformer loss; (2) output transformer loss; (3) arc loss in tubes; and (4)

cathode heater and control circuit losses. The utility factors of the input and output transformer are 0.62 and 0.83, respectively. The total transformer loss is 8.6 per cent at full load and 0.7 power factor. The arc loss is 2.5 per cent. The loss in cathode heaters and a suitable control circuit totals 2.5 per cent. The apparent full load efficiency and the no load loss compare favorably with the values for a rotating set.

It should be noted that the output is not entirely useful. Figure 10 shows the additional losses in a typical 2 phase induction motor resulting from harmonics in the voltage wave. From table I, it can be shown that the harmonics in the no load voltage wave increase the losses in the motor by 2.8 per cent, while at full load on the frequency changer the losses are increased by 1.2 per cent. These losses should properly be charged to the frequency changer, decreasing its full load efficiency from 86.4 to 85.2 per cent.

In general, the harmonic voltages in the output may be reduced to any desired value if a sufficient number of anodes and transformer windings is available, and resonant shunt filters may be used in the output circuit to eliminate specific harmonics. In particular, by doubling the number of tubes, the third harmonic may be eliminated by making the potential of the center peak in figure 2, 1.88 times as great as the potential of the adjacent peaks.

Figure 11 shows the assembly of tubes and control transformers. Figure 12 shows a control panel containing control switches and means for insuring the proper heating time for the cathodes before voltage is applied to the anodes. During nonoperating periods, the cathodes are heated at reduced voltage

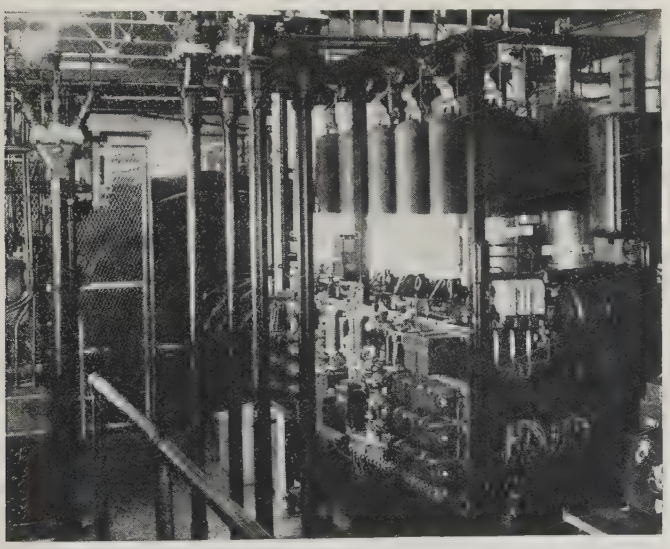
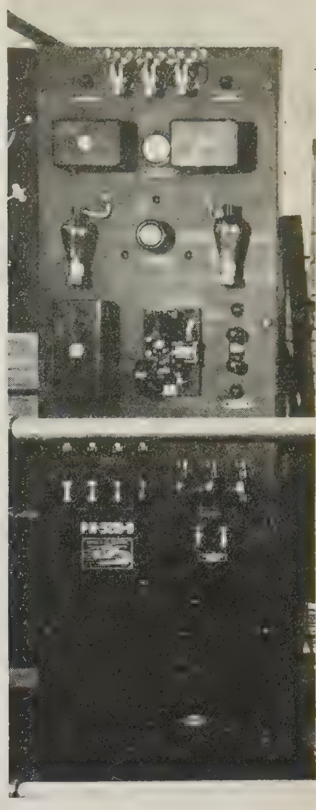


Fig. 11. Assembly of tubes and control transformers

to prevent mercury condensation on the anode or cathode. The control power switch is closed to apply full voltage to the cathodes, and after the proper heating period an indication is given for manual closing of the main power switch which energizes the input power transformer. A manually operated control switch is used to change transformer

Fig. 12. A control panel



taps under load, through a pair of contactors, to deliver the desired output voltage.

Figure 13 shows a typical load chart of operation. During the changes between the day shift and night shift, the voltage taps are changed to compensate for the regulation of the set. Normal operation is at or slightly under rated load. Starting peaks or operating peaks during the day have been observed up to nearly 200 per cent load.

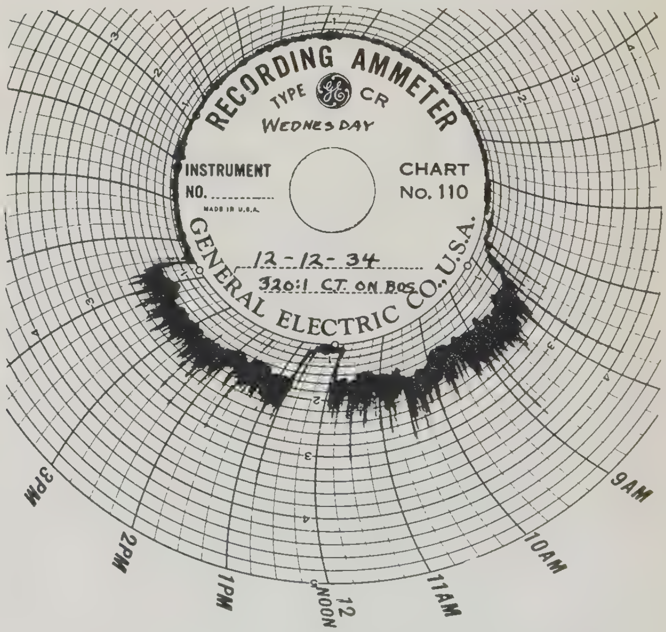


Fig. 13. Typical load chart during operation of the converter. Rated load = 2.1

Split Phase Starting of 3 Phase Motors

When 3 phase induction motors are to be operated from single phase lines, split phase starting is frequently used. The optimum values of external resistance and reactance for split phase starting may be determined by the method of symmetrical components. In the present paper, equations for determining these values for each of the 2 commonly used split phase starting schemes are derived. The result of calculations based upon this method are shown to compare favorably with experimental data.

By
GORDON F. TRACY
ASSOCIATE A.I.E.E.
Univ. of Wisconsin, Madison

WALTHER E. WYSS*
ASSOCIATE A.I.E.E.
General Elec. Co., Washington, D. C.

THE determination of the "best" values of external resistance and reactance to produce starting torque when standard 3 phase induction motors are operated from single phase sources of supply is a problem to which the method of symmetrical components is admirably adaptable. Two "split phase" schemes, commonly used to produce starting torque, are shown in figure 1, where R is an external resistor and X an external inductive reactor having as low a power factor as possible. In either case the resistor and reactor are cut out of the circuit as the speed of the machine approaches normal, and 2 terminals of the machine are connected directly to the line, the third being left open. By "best" values of R and X is meant that pair of values which will produce maximum starting torque for a given line voltage; or, if such values should be accompanied by an excessive line current, the "best" values would then be those which produce maximum torque at an arbitrarily fixed line current.

The use of the method of symmetrical components implies that the machine is considered to have 3 phase voltages, badly unbalanced, applied to its terminals during the starting period. These voltages are resolved into their positive and negative sequence components, and since each set of components consists of balanced 3 phase voltages, the standard expressions for torque and current under balanced conditions may be employed. The resultant torque and current may then be deduced. Finally, the conditions for maximum torque for each method are determined. The validity of the expressions for R and X so obtained has been amply verified by experiment, some of the experimental checks being shown in this paper.

The conclusions reached as the result of the study discussed in this paper may be summarized as follows. A comparison of the 2 methods shown in figure 1 for producing starting torque on single phase circuits reveals the fact that the torques obtainable by method 2 are larger than those of method 1, but that these greater torques are obtained at the expense of larger currents. The maximum torque obtainable by method 1 is in the neighborhood of 18 per cent of the 3 phase starting torque for the same voltage between lines, while the corresponding current in the line is in the neighborhood of 87 per cent of the 3 phase starting current. The torque obtainable by method 2 depends, as pointed out before, upon how much line current is permissible; but ranges, in general, from about 37 per cent of 3 phase starting torque at a line current of 200 per cent of 3 phase starting current to about 20 per cent at 110 per cent of 3 phase starting current. It is significant that the torque per ampere of line current is approximately the same for both methods. In choosing between the 2 methods for a particular case the deciding factor would be the upper limit on the starting current. If a line current of 110 per cent or more of 3 phase starting current at rated voltage is permissible, method 2 is preferable because of its higher starting torque. If the permissible starting current lies between about 75 per cent and 100 per cent of 3 phase starting current, method 1 is preferable, and can be used without a compensator. If the starting current must be kept below 75 per cent, a compensator must be used in conjunction with either method and there is little to choose between them, except that the "best" values of R and X are more directly obtained in method 1 than in method 2.

Theoretical Analysis

In deriving the expressions for R and X the following symbols are employed:

R = external resistance
X = external reactance, assumed to have zero power factor

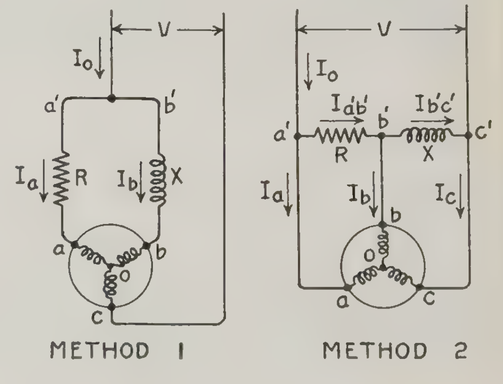


Fig. 1. Split phase connection

A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. Great Lakes District meeting, West Lafayette, Ind., Oct. 24–25, 1935. Manuscript submitted June 3, 1935; released for publication July 12, 1935.

*Formerly graduate student University of Wisconsin, Madison.

z = impedance per phase of motor at standstill, motor assumed star connected
r = resistance per phase of motor at standstill
x = reactance per phase of motor at standstill
V = voltage of single phase source
I_0 = single phase line current
a = 120 degree forward rotating operator ($a = -0.5 + j\ 0.866$),
a^2 = 240 degree forward rotating operator ($a^2 = -0.5 - j\ 0.866$),
T = torque in synchronous watts
Vector quantities are indicated by bold faced symbols, as $\mathbf{V}$, $\mathbf{I}$, and $\mathbf{z}$
Positive phase sequence quantities are indicated by $\check{\mathbf{V}}$, $\check{\mathbf{I}}$, and $\check{T}$
Negative phase sequence quantities are indicated by $\hat{\mathbf{V}}$, $\hat{\mathbf{I}}$, and $\hat{T}$

Method 1

The vector diagram for method 1 is shown in figure 2, in which the motor impressed voltages, $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$, considered as voltages of a 3 phase system,

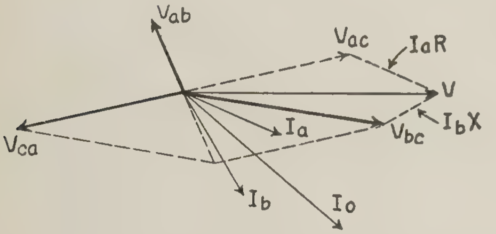


Fig. 2. Vector diagram for method 1

are shown in heavy lines. The expressions for these voltages, in terms of V, R, and X, are

$$\mathbf{V}_{bc} = \mathbf{V} - j\mathbf{I}_b X$$
$$\mathbf{V}_{ca} = -(\mathbf{V} - \mathbf{I}_a R) = \mathbf{I}_a R - \mathbf{V}$$
$$\mathbf{V}_{ab} = -(\mathbf{V}_{bc} + \mathbf{V}_{ca}) = -\mathbf{I}_a R + j\mathbf{I}_b X$$

The positive phase sequence component of $\mathbf{V}_{ab}$ is:

$$\check{\mathbf{V}}_{ab} = \frac{1}{3}\left[\mathbf{V}_{ab} + a\mathbf{V}_{bc} + a^2\mathbf{V}_{ca}\right]$$
$$= \frac{1}{3}\left[\mathbf{I}_a R(a^2 - 1) + j\mathbf{I}_b X(1 - a) + \mathbf{V}(a - a^2)\right] \qquad (1)$$

Similarly, the negative phase sequence component is:

$$\hat{\mathbf{V}}_{ab} = \frac{1}{3}\left[\mathbf{I}_a R(a - 1) + j\mathbf{I}_b X(1 - a^2) + \mathbf{V}(a^2 - a)\right] \qquad (2)$$

The zero phase sequence component is zero.

It is now convenient to make use of 2 important propositions in the theory of symmetrical components. The first relates the line-to-neutral voltage of each sequence to the corresponding line-to-line voltage. Defining the positive phase sequence to be the one in which b lags a, and $\mathbf{V}_{ao}$ to be the line-to-neutral voltage of phase a, these relations are:

$$\check{\mathbf{V}}_{ab} = \sqrt{3}\,\check{\mathbf{V}}_{ao}\angle 30° = (1 - a^2)\,\check{\mathbf{V}}_{ao} \qquad (3)$$
$$\hat{\mathbf{V}}_{ab} = \sqrt{3}\,\hat{\mathbf{V}}_{ao}\angle -30° = (1 - a)\,\hat{\mathbf{V}}_{ao} \qquad (4)$$

The second proposition states that when the 3 impedances of a star-connected load are balanced, the positive phase sequence voltage to neutral is simply equal to the product of the impedance per phase and the positive phase sequence current, and does not involve any cross products with the negative phase sequence current as it would if the impedances were unbalanced. Consequently:

$$\check{\mathbf{I}}_a = \check{\mathbf{V}}_{ao}/\mathbf{z} \qquad (5)$$
$$\hat{\mathbf{I}}_a = \hat{\mathbf{V}}_{ao}/\mathbf{z} \qquad (6)$$

Also

$$\mathbf{I}_a = \check{\mathbf{I}}_a + \hat{\mathbf{I}}_a \qquad (7)$$
$$\mathbf{I}_b = \check{\mathbf{I}}_b + \hat{\mathbf{I}}_b = a^2\check{\mathbf{I}}_a + a\hat{\mathbf{I}}_a \qquad (8)$$

Substituting equations 3 to 8 in equations 1 and 2

$$\check{\mathbf{V}}_{ao}\left[1 - a^2 + \frac{R(1 - a^2)}{3\mathbf{z}} - j\,\frac{a^2X(1 - a)}{3\mathbf{z}}\right] + \hat{\mathbf{V}}_{ao}\left[\frac{R(1 - a^2)}{3\mathbf{z}} - j\,\frac{aX(1 - a)}{3\mathbf{z}}\right] = \frac{1}{3}\mathbf{V}(a - a^2) \qquad (9)$$

$$\check{\mathbf{V}}_{ao}\left[\frac{R(1 - a)}{3\mathbf{z}} - j\,\frac{a^2X(1 - a^2)}{3\mathbf{z}}\right] + \hat{\mathbf{V}}_{ao}\left[1 - a + \frac{R(1 - a)}{3\mathbf{z}} - j\,\frac{aX\,(1 - a^2)}{3\mathbf{z}}\right] = \frac{1}{3}\mathbf{V}\,(a^2 - a) \qquad (10)$$

These 2 equations may now be solved simultaneously for $\check{\mathbf{V}}_{ao}$ and $\hat{\mathbf{V}}_{ao}$, yielding

$$\check{\mathbf{V}}_{ao} = \frac{\mathbf{V}}{\sqrt{3}}\left[\frac{aX + jR - \sqrt{3}\,a^2\mathbf{z}}{3\mathbf{z} + 2R + j2X + jRX/\mathbf{z}}\right] \qquad (11)$$

$$\hat{\mathbf{V}}_{ao} = \frac{\mathbf{V}}{\sqrt{3}}\left[\frac{-\,a^2X - jR - \sqrt{3}\,a\mathbf{z}}{3\mathbf{z} + 2R + j2X + jRX/\mathbf{z}}\right] \qquad (12)$$

The torque of a 3 phase induction motor under unbalanced 3 phase voltages is the difference between the torque corresponding to the positive phase sequence voltage and that corresponding to the negative phase sequence voltage.[1] Each component of torque, therefore, may be found by substituting equations 11 and 12 in the ordinary expression for the starting torque of an induction motor; *viz.*, $T = 3V_1^2r_2/z^2$, where r_2 is the rotor resistance (referred to the stator) and V_1 is the line-to-neutral voltage, and T is the torque in synchronous watts. The expression for the resultant torque under unbalanced voltages is

$$T = \check{T} - \hat{T} = 3\left[(\check{V}_{ao})^2 - ({}_{ao}\dot{V})^2\right]\frac{r_2}{z^2} \qquad (13)$$

Since all quantities in the expression for torque are scalar quantities, it is necessary to find the scalar values of the positive phase sequence and negative phase sequence voltages. The scalar values of the numerators of equations 11 and 12 are shown in equations 14 and 15, respectively:

$$V[(0.866r - 1.5x - 0.5X)^2 + (1.5r + 0.866x + R + 0.866X)^2]^{1/2} \qquad (14)$$
$$V[(0.866r + 1.5x + 0.5X)^2 + (-1.5r + 0.866x - R + 0.866X)^2]^{1/2} \qquad (15)$$

The scalar value of the denominator in each case is:

$$\sqrt{3}\left[\left(3r + 2R + RX\,\frac{x}{z^2}\right)^2 + \left(3x + 2X + RX\,\frac{r}{z^2}\right)^2\right]^{1/2} \qquad (16)$$

Substituting and simplifying equation 13 becomes:

$$T = \left(\frac{V^2r_2}{z^2}\right)\frac{2\sqrt{3}\,(Rx + Xr + RX)}{\left(3r + 2R + RX\,\dfrac{x}{z^2}\right)^2 + \left(3x + 2X + RX\,\dfrac{r}{z^2}\right)^2} \qquad (17)$$

1. See reference 1 at end of paper.

Table I—Calculated and Experimental Values of R and X for Typical Machines

Machine Number	Type	Rating H.P.	Rating Volts	Constants r	Constants x	Constants z	Method	Calculated Data R	Calculated Data X	Experimental Data R	Experimental Data X
1	High reactance	7.5	110	0.137	0.297	0.327	1	0.46	0.152	0.50	0.15
2	Single squirrel cage	3	110	0.584	0.628	0.858	1	0.94	0.713	0.88	0.713
3	Double squirrel cage	7.5	110	0.193	0.286	0.345	1	0.428	0.245	0.50	0.28
4	Single squirrel cage	5	220	0.86	1.51	1.74	1	2.32	1.00	2.40	0.75
5	Wound rotor	15	110	0.074	0.164	0.183	1	0.40	0.064	0.40	0.00
1	(See above)						2(C = 1.5)	0.48	0.48	0.42	0.42
2	(See above)						2(C = 1.5)	1.31	1.31	1.28	1.28
4	(See above)						2(C = 1.5)	2.59	2.59	2.52	2.52
4							2(C = 1.15)	5.13	5.13	5.10	5.10

$C = \dfrac{\text{Single phase starting current}}{\text{3-phase starting current}}$

All constants in ohms

The value of z occurring in equation 17 is the scalar value. Since the first term of equation 17 expresses the torque under balanced conditions for a between-line voltage equal to V, the remainder of the equation is the ratio of the split phase starting torque to the starting torque under normal 3 phase operation.

The line current I_0 is readily found as follows:

$$\begin{aligned} I_o &= I_a + I_b = (\check{I}_a + \hat{I}_a) + (a^2\check{I}_a + a\hat{I}_a) \\ &= (1 + a^2)\check{I}_a + (1 + a)\hat{I}_a = -a\check{I}_a - a^2\hat{I}_a \end{aligned} \tag{18}$$

Substituting equations 5 and 6 into 18, the vector expression for I_0 is

$$I_o = V\left(\frac{R + 2z + jX}{3z^2 + 2Rz + 2jXz + jRX}\right) \tag{19}$$

and the corresponding scalar expression is

$$I_o = \frac{V}{\sqrt{3}z}\left[\frac{3(R + 2r)^2 + 3(X + 2x)^2}{\left(3r + 2R + RX\dfrac{x}{z^2}\right)^2 + \left(3x + 2X + RX\dfrac{r}{z^2}\right)^2}\right]^{1/2} \tag{20}$$

The value of z in equation 20 is the scalar value. Since the first term expresses the line current for a balanced 3 phase voltage of V volts the remainder is the ratio between the single phase line current and the 3-phase starting current.

It is of interest to examine the manner in which the torque (equation 17) of a given machine varies as the external resistance and reactance are varied. For any fixed value of X the torque is relatively small when R is zero, but rises as R is increased, passes through a maximum value, then drops off approaching zero as R becomes very large. Such a curve is shown in curve A of figure 3, the data having been obtained experimentally on a 5-horsepower squirrel-cage ball-bearing induction motor. A different value of X yields another curve similar in shape to that of curve A, but having either a higher or a lower peak value depending on whether this second value of X is closer to, or farther away from the "best" value. A similar characteristic obtains when X is varied and R is kept constant, a typical curve being shown in curve B of figure 3. If a 3 dimensional diagram is built plotting torque as ordinates against R and X as mutually perpendicular abscissas, a sheet will result having a hump at the intersection of the best value of R with the best value of X. Such a diagram is shown in figure 4, for the machine described above.

In order to predetermine the best values of R and X, the partial derivative of equation 17 with respect to R is first taken and equated to zero. There results a quadratic in R of the form

$$AR^2 + BR + C = 0 \tag{21}$$

where

$$\begin{aligned} A &= (x + X)(4z^2 + 4Xx + X^2) \\ B &= 2rX(4z^2 + 4Xx + X^2) \\ C &= 4Xr^2(3z^2 + 3Xx + X^2) - z^2(x + X)(9z^2 + 12Xx + 4X^2) \end{aligned}$$

For any arbitrarily chosen value of X, the above equation yields the corresponding best value of R, i.e., that value corresponding to the peak of curve A in figure 3. The first choice of X, of course, would in general not be that which corresponds to the hump in figure 4, but this value could be found by assigning a few different values to X; solving for the corresponding value of R from equation 21 and then substituting the pairs of values of R and X so found into equation 17 until the optimum torque is found. A much shorter procedure, however, is to obtain a second equation similar to 21 by taking the partial derivative of 17 with respect to X and equating to zero. This is

$$DX^2 + EX + F = 0 \tag{22}$$

where

$$\begin{aligned} D &= (r + R)(4z^2 + 4Rr + R^2) \\ E &= 2xR(4z^2 + 4Rr + R^2) \\ F &= 4Rx^2(3z^2 + 3Rr + R^2) - z^2(r + R)(9z^2 + 12Rr + 4R^2) \end{aligned}$$

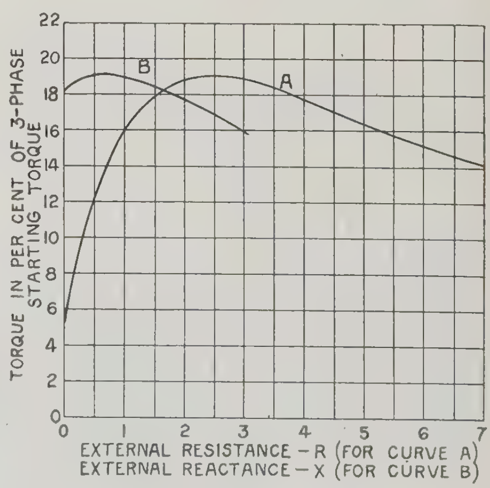


Fig. 3. Torque characteristics of method 1

Data pertains to machine 4, table I

Curve A: Reactance held constant at 1.0 ohm

Curve B: Resistance held constant at 2.0 ohms

R and X in ohms

Equations 21 and 22 are now used as follows. Any arbitrarily chosen value of X (preferably of the same order of magnitude as x) is substituted in equation 21 and the corresponding value of R found. This value of R is then used in equation 22 and a new value of X found, which is again substituted in equation 21. This process is continued until the values of R and X repeat themselves, which means that they are approaching the values corresponding to the hump of figure 4. Due to the symmetry of the equations and to the rapid convergence of R and X on their best values, a surprisingly small amount of labor is required in this process. Usually, only 4 or 5 of the above substitutions are necessary. For example, using the constants of machine 4, table I, and assuming $X = 1.5$ ohms as a first trial (suggested by the value $x = 1.51$), substitution into equation 21 yields a positive value of R of 2.27 ohms. Substituting this value of R, in turn, into equation 22 results in $X = 1.03$ ohms. Results of succeeding substitutions are $R = 2.305$, $X = 1.01$, $R = 2.32$, $X = 1.00$. Evidently the values of R and X have practically converged on their best values. The latter pair are therefore assumed to be the best values.

Method 2

An inspection of the circuit diagram of method 2, in figure 1, suggests the following relations:

$$\left.\begin{aligned} &\boldsymbol{I}_o = \boldsymbol{I}_a + \boldsymbol{I}_{a'b'} = \boldsymbol{I}_{b'c'} - \boldsymbol{I}_c \\ &\text{But } \boldsymbol{I}_{a'b'} = \frac{\boldsymbol{V}_{ab}}{R}, \text{ and } \boldsymbol{I}_{b'c'} = \frac{\boldsymbol{V}_{bc}}{jX} \end{aligned}\right\} \quad (23)$$

Expressing $\boldsymbol{V}_{ab}$ and $\boldsymbol{V}_{bc}$ in terms of the symmetrical components of the line-to-neutral voltage $\boldsymbol{V}_{ao}$:

$$\boldsymbol{V}_{ab} = \check{\boldsymbol{V}}_{ab} + \hat{\boldsymbol{V}}_{ab} = (1 - a^2)\check{\boldsymbol{V}}_{ao} + (1 - a)\hat{\boldsymbol{V}}_{ao}$$
$$\boldsymbol{V}_{bc} = a^2\check{\boldsymbol{V}}_{ab} + a\hat{\boldsymbol{V}}_{ab} = a^2(1 - a^2)\check{\boldsymbol{V}}_{ao} + a(1 - a)\hat{\boldsymbol{V}}_{ao}$$
$$= -j\sqrt{3}\,\check{\boldsymbol{V}}_{ao} + j\sqrt{3}\,\hat{\boldsymbol{V}}_{ao}$$

Upon substituting these relations, together with those expressed in equations 5 to 8 into equation 23 there results:

$$\left(\frac{1 + a}{\boldsymbol{z}} + \frac{1 - a^2}{R} + \frac{\sqrt{3}}{X}\right)\check{\boldsymbol{V}}_{ao} + \left(\frac{1 + a^2}{\boldsymbol{z}} + \frac{1 - a}{R} - \frac{\sqrt{3}}{X}\right)\hat{\boldsymbol{V}}_{ao} = 0 \quad (24)$$

Also, from figure 1,

$$\boldsymbol{V} = -\boldsymbol{V}_{ca} = -a\check{\boldsymbol{V}}_{ab} - a^2\hat{\boldsymbol{V}}_{ab} = -a(1 - a^2)\check{\boldsymbol{V}}_{ao} - a^2(1 - a)\hat{\boldsymbol{V}}_{ao}$$
$$= (1 - a)\check{\boldsymbol{V}}_{ao} + (1 - a^2)\hat{\boldsymbol{V}}_{ao} \quad (25)$$

Equations 24 and 25 may now be solved simultaneously for $\check{\boldsymbol{V}}_{ao}$ and $\hat{\boldsymbol{V}}_{ao}$, yielding

$$\check{\boldsymbol{V}}_{ao} = \left(\frac{V}{\sqrt{3}}\right)\frac{\sqrt{3}\,\boldsymbol{z}R - (1 - a)\boldsymbol{z}X - (1 + a^2)RX}{3\boldsymbol{z}R + j2RX + j3\boldsymbol{z}X} \quad (26)$$

$$\hat{\boldsymbol{V}}_{ao} = \left(\frac{V}{\sqrt{3}}\right)\frac{\sqrt{3}\,\boldsymbol{z}R + (1 - a^2)\boldsymbol{z}X + (1 + a)RX}{3\boldsymbol{z}R + j2RX + j3\boldsymbol{z}X} \quad (27)$$

As in the case of method 1, the next step is to find the scalar values of the above 2 equations and substitute them into equation 13 for torque. Passing over all intermediate steps the torque equation is

$$T = \left(\frac{V^2r_2}{z^2}\right)\frac{2\sqrt{3}\,RX(z^2 + Rr + Xx)}{(3Rr - 3Xx)^2 + (3Rx + 2RX + 3Xr)^2} \quad (28)$$

The value of z in the above equation is its scalar value. As in the corresponding equation for torque in method 1, the second term of equation 28 is the ratio of the split phase starting torque to the 3 phase starting torque.

The line current for method 2 is found from $\boldsymbol{I}_0 = \boldsymbol{I}_a + \boldsymbol{I}_{a'b'}$ (see figure 1); $\boldsymbol{I}_a$ and $\boldsymbol{I}_{a'b'}$ are expressed in terms of $\check{\boldsymbol{V}}_{ao}$ and $\hat{\boldsymbol{V}}_{ao}$ in equations 5 and 23, and $\check{\boldsymbol{V}}_{ao}$ and $\hat{\boldsymbol{V}}_{ao}$ are further expressed in terms of $\boldsymbol{V}$ in equations 26 and 27. The resulting vector expression for $\boldsymbol{I}_o$ is:

$$\boldsymbol{I}_o = \boldsymbol{V}\left(\frac{2\boldsymbol{z}R + 3\boldsymbol{z}^2 + j2\boldsymbol{z}X + jRX}{3\boldsymbol{z}^2R + j2\boldsymbol{z}RX + j3\boldsymbol{z}^2X}\right) \quad (29)$$

And the corresponding scalar expression is

$$I_o = \frac{V}{\sqrt{3}z}\left[\frac{3(2rR - 2xX + 3r^2 - 3x^2)^2 + 3(2xR + 2rX + RX + 6rx)^2}{(3rR - 3xX)^2 + (3xR + 2RX + 3rX)^2}\right]^{1/2} \quad (30)$$

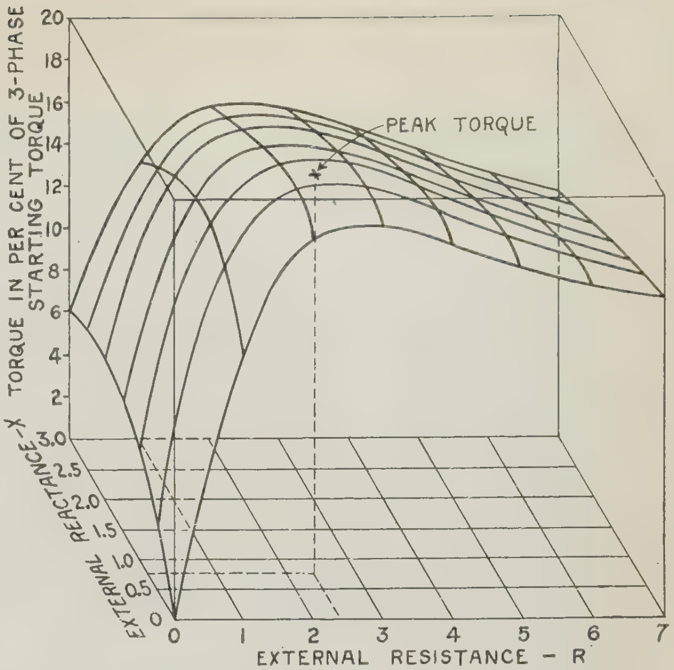


Fig. 4. Isometric graph of torque characteristics of method 1

Data pertain to machine 4, table I
R and X in ohms

As in the case of equation 20, the expression under the radical of equation 30 is the ratio of the single phase line current to the 3 phase starting current.

The manner in which the torque varies as either R or X alone is varied is much the same as is shown in figure 3. If torque, however, is plotted in a 3 dimensional diagram, the sheet so formed differs from that of figure 4 in that there is no hump at any positive values of R and X. The torque continues to increase as R and X are both reduced, and attains its greatest value (for positive values of R and X) as R and X approach zero. The torque sheet has a diagonal ridge which increases in height as it ap-

proaches the origin. Figure 5 is a 3 dimensional chart of the same machine as in figure 4 but connected as in method 2. The theoretical value of torque at the origin (when R and X are both zero) is $1/\sqrt{3}$ times the 3 phase starting torque. This value is found by dividing the numerator and denominator of equation 28 by RX, retaining only those terms which have appreciable magnitude when R and X are both very small, and finally assuming $R = X$ as an approximation. The current at the origin of figure 5, where R and X are zero, is infinitely large, and in the vicinity of the origin excessive current would be drawn from the source.

Since there is no hump or optimum value of the torque sheet in method 2 there is no object in taking derivatives of the torque equation to find the best values of R and X. Instead of understanding the best values to mean those values which yield optimum torque as in method 1, they are now understood to mean those values which yield the maximum torque at some *given line current*. Equation 30 for the line current may be written in the form

$$C^2 = 3\frac{(2rR-2xX+3r^2-3x^2)^2+(2xR+2rX+RX+6rx)^2}{(3rR-3xX)^2+(3xR+2RX+3rX)^2} \qquad (31)$$

in which C = ratio of single phase starting current to 3 phase starting current. Then if it were permissible, for example, to draw a starting current from the single phase source equal to 1.5 times the 3 phase starting current of the motor, the value 1.5 would be assigned to C and the procedure would be to assign a few arbitrary values to X in equation 31, solve for the corresponding values of R, and then substitute the pairs of values so found into equation 28. The best values of R and X would then be those corresponding to the largest value of torque.

A shorter but less exact method than that outlined above is to let $R = X$ in equation 31. The results of several tests and computations indicate that over a wide range of ratios of r to x the torque for any given line current when R is equal to X is within about 10 per cent of the maximum torque at the same current. Equation 31 may now be rearranged as a bi-quadratic in R (or X) of the form:

$$JR^4 + KR^3 + LR^2 + MR + N = 0$$

where

$$J = 3 - 4C^2$$
$$K = 12(1 - C^2)(r + x)$$
$$L = 6z^2(4 - 3C^2) + 36\,rx$$
$$M = 36z^2(r + x)$$
$$N = 27r^4 + 27x^4 + 54r^2x^2$$

This bi-quadratic may be solved directly for R by one of the standard methods that can be found in any handbook of mathematics. Horner's method was used to obtain the data in table I.

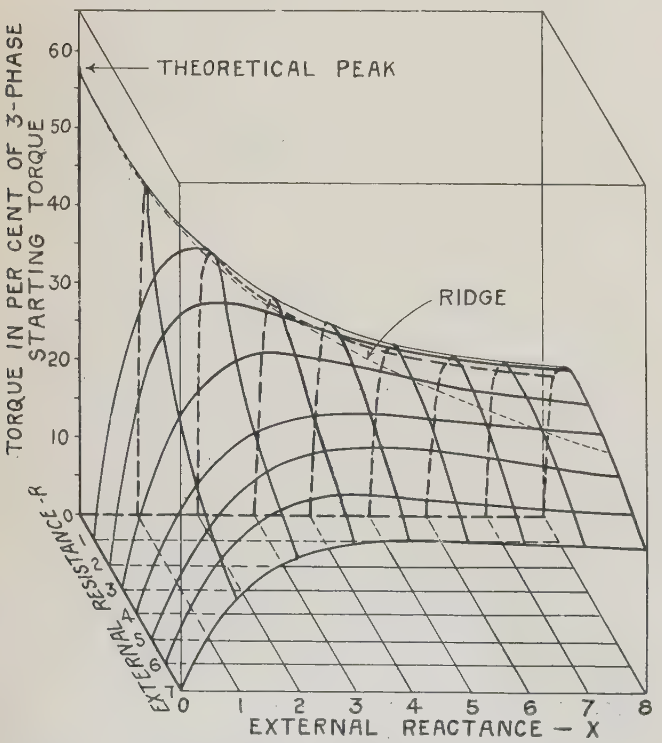


Fig. 5. Isometric graph of torque characteristics of method 2

Data pertain to machine 4, table I
R and X in ohms

Experimental Checks

The validity of the methods described above for computing the best values of R and X has been verified by experimental tests on a number of different induction motors. Most of the machines tested were ball bearing machines with skewed rotor slots, thus minimizing the possibilities of errors due to bearing friction and slot grabbing. The reactor used for X consisted of a rectangular laminated iron core with 2 $^1/_{32}$-inch air gaps. The winding was tapped in several places and heavy copper was used in order to minimize the resistance. Table I compares computed and experimental results.

Wound Rotor Machines

When wound rotor induction motors are used in conjunction with external resistors in the rotor circuits, the values of r, x, and z, when used in the above equations, should include the effect of the external resistance. In other words, if the constants of the machine are determined from the blocked rotor test, the same resistance should be connected in the rotor circuits during the test as will be used when the machine is started. A little consideration will show that the same value of rotor resistance that produces maximum starting torque under normal 3 phase conditions will also produce maximum starting torque when started split phase. Experimental results support this consideration.

References

1. Symmetrical Components (a book), C. F. Wagner and R. D. Evans. McGraw-Hill Book Company, New York, N. Y., 1933.

2. On the Current Locus of the Induction Motor Under Some Unbalanced Conditions, Yoshio Hasumi. Inst. of Elec. Engrs. of Japan, *Jl.*, v. 54, August 1934, p. 813–8. (English abstract, p. 96–8.)

3. Calculation of the Starting Torque of Single-Phase Induction Motors With Split-Phase Starting Devices, I. E. Hanssen. A.I.E.E. Trans., v. 27, 1908, p. 373–5.

A New Watt-Hour Meter

To meet the demand for sustained accuracy in watt-hour meters and for adaptability to service requirements, the meter described in this paper has been produced. It offers simplicity and interchangeability, light weight, high accuracy over a wide range of load, and convenience of adjustment, and features an improved overload compensator, a new form of full load adjustment, and a new form of combined open-potential-coil indicator and balance adjustment for polyphase meters.

By
STANLEY GREEN
MEMBER A.I.E.E.

Duncan Elec. Mfg. Co.,
Lafayette, Ind.

NEW watt-hour meter design has been necessitated by changing standards, which require a new line of meter elements designed especially to take advantage of an improved form of heavy load compensation. The space available for the element is determined closely by service installation requirements; at the same time new weather resistant nonferrous meter bases require an element substantially immune to stray field errors. The meter described in this paper secures this immunity by the use of a single potential pole and 2 current poles, which are constructed as a separable unit. Current windings are form wound and are interchangeable on a single lamination form for all ratings. The potential electromagnet has an unusually small mean length of turn with a minimum watts loss in the element, giving also a reduction in low power factor temperature errors and contributing to a light weight electromagnet without sacrifice in any performance detail.

Analysis of modern operating requirements resulted in the choice of a nominal full load torque of 46 millimeter-grams, and watt-hour constants in multiples of $^1/_3$ watt-hour per disk revolution. An improved overload compensator is used which gives substantially perfect registration at all loads between 5 and 300 per cent of nominal meter ratings, together with a 2 magnet damping unit of high coercive force and a new form of full load adjustment. Elements are used without change for polyphase meters, in which a new form of electromagnetic open-potential-coil indicator combines indication and balance adjustment in a single device. Accessibility for disassembly of all component parts is attained from the front in both single and multiple element meters.

The principal improvement embodied in the new meter, which is not brought out by the consideration of a single rating or size, lies in the adaptability of the new element to an entire line of meters with uniform performance and a maximum of simplicity and interchangeability both in manufacture and in use. Lighter weight, better accuracy, lower loss, and more convenience of adjustment are also among the advances. To those unfamiliar with the difficulties in design of the many models and types which have constituted the steps of meter progress in the past, these details may seem unworthy of mention. To this the answer is that simplifications are not obvious until after they are made, and even then serve only as steps to further improvement.

General

In the last decade, important changes in watt-hour meters have received attention in the technical press and in manufacturers' literature but mostly from the standpoint of performance and application. The user's point of view has been uppermost. Manufacturing and design features have received less attention, probably on the theory that the user is the one to be satisfied and that he is not particularly interested in *how* the manufacturer produces the product as long as the price and performance are satisfactory. This assumption is undesirable because among the users of watt-hour meters in the public utilities there is a large proportion of engineers who by training and interest are capable of appreciating design problems. Indeed, it is from this group that the manufacturer receives the current of constructive criticism which crystallizes into actual improvements.

Demands for better performance and adaptability to changing service requirements have been such that the meter of today is scarcely comparable with its forerunner of 10 years ago, and these have been met with little if any real increase in cost to the user. An outline of the causes of some of these improvements and of the design limitations involved should be interesting to the operator or user of meters, and, in addition, if it gives him any better understanding of the problems of the designer, should serve to stir up still further that responsive flow of constructive suggestion which has been the taproot of progress in the past.

For reasons of standardization, and to keep stocks of repair parts at a minimum, users of meters always have been loath to see frequent changes in types. In contrast to the extreme of annual models as indulged in by some industries, the meter manufacturer always has introduced a particular type with caution and deliberation, with the full realization that this type would be the fundamental "platform" upon which production would be based for a term of years, until the pressure of new requirements, better materials and processes, and accumulated improvements

A paper recommended for publication by the A.I.E.E. committee on instruments and measurements, and scheduled for discussion at the A.I.E.E. Great Lakes District meeting, West Lafayette, Ind., October 24–25, 1935. Manuscript submitted May 6, 1935; released for publication August 26, 1935.

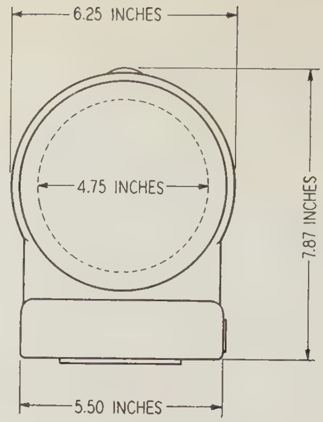
made it absolutely necessary to begin again with a clean slate.

The company with which the writer is associated recently made one of these infrequent but complete changes of model. A discussion of the salient features of its design and the reasons therefore may serve a useful purpose in indicating general design trend. Design of a line of meters cannot be accomplished by logical processes alone because it consists of interpreting the desires of the users, as well as drawing lessons from past experience, both in operation and manufacture. The resultant product is different from what might be produced if there were no restricting past practice or confining current opinion. Much of the process therefore will reflect the designer's opinion and interpretation of facts reported to him. Notwithstanding this, it should be possible to set down in a fairly orderly and logical form some of these factors and how they have affected the present form of the watt-hour meter, regardless of whether they be facts, theories, or plain preferences. To shorten the paper, it has been necessary to assume, on the part of the reader, a reasonably thorough knowledge of watt-hour meters and their performance and to dispense with such explanation or description of phenomena as is found in texts.[1]

Determination of Size

Figure 1 shows the over-all space relations for the ordinary bottom-connected single-phase meter. The case diameter is approximately 6.25 inches, and the width of the terminal chamber is 5.50 inches. Terminal chamber width has really evolved from practical experience in getting the proper number of wires of sufficient current-carrying capacity through the meter with the necessary insulation between them. Four principal wires are required and they may be called upon to carry 150 amperes continuously. The operating voltage between them may be a nominal 230 volts. A few years ago, at the request of the meter committee of the National Electric Light Association, the approximate terminal chamber width shown was standardized upon by all American manufacturers, and it is safe to say that it is not excessive, both insulation and copper being utilized to their maximum safe capacities under the above conditions. The user is benefited little in economy of wall space by reducing the enclosure diameter below the width of the terminal chamber; consequently a glass cover outside diameter of approximately 5.75 inches has also recently been standardized upon by all American manufacturers. The terminal chamber evolution has been a long process of trial and error in which the operating experience with meters in millions of installations has played a part. It is inextricably bound up with the standardization and operating practices of the utilities and to this extent is almost entirely out of the control of the designer except from the very long range point of view. At least for a term of years, it can be assumed that it has been determined for him by past co-operative efforts. The fact that it has been determined, however, does provide the fundamental factor of his design—the space available for the meter element.

1. For all numbered references see list at end of paper.



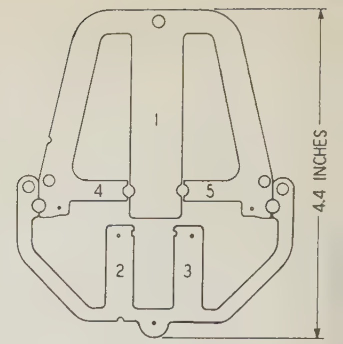


Fig. 2 (above). Outline of electromagnet lamination

Fig. 1 (left). Outline dimensions of typical terminal chamber type of single-phase meter

For economy of materials, a smaller meter diameter could be used, irrespective of the terminal chamber, but past practice has indicated that this is not desirable. Meters of smaller size have been tried and abandoned in favor of new types of approximately the present size. One plausible explanation of this is that there is a limit to the material economies obtained by size reduction where they are overshadowed by the extra manufacturing cost introduced by the crowding of parts and closer tolerances required to build a fine measuring instrument into the smaller space.

In the more recently developed socket type meter, shown in figure 14, it has been found that the socket diameter is largely determined by the required current carrying capacity and necessary spacing for contact clips and circuit wires. This diameter closely approximates the size already established by the former bottom-connected practice. The same fundamental size-fixing factors, therefore, apply to the socket form of meter.

Within the space of the meter cover shown in figure 1 is drawn a circle 4.75 inches in diameter. This is the approximate space which it is safe to utilize for the meter element itself. It will be noted that a fairly generous margin has been left between the element and the outside of the cover. This is to take care of such conditions as special registers requiring more room than usual, variations in cover dimensions and other tolerances, and finally for unforeseen future contingencies, because lack of space is the manufacturer's worst enemy.

Electromagnet Laminations

The shape of the lamination for the electromagnet is shown in figure 2. It is characterized by a single leg (1) for the potential pole and 2 legs (2 and 3) for current poles. A primary reason for this general structure is its resistance to the effects of stray a-c fields. Present practice is to make meter cases from die-cast aluminum or similar nonferrous alloy for weather resistance because of the increasing use of meters out of doors. The cast iron cases of older

meters acted as a rather effective shield against such stray fields, but with present bases the shielding effect is negligible.

The most damaging stray fields are those in which the magnetomotive force is in a direction to cross the gap and cut the disk. Consider the effect of such a stray field on the form of electromagnet shown in figure 2. The path of any flux as a result of this field would have to be through the current poles, across the gap, and through the potential pole. If the potential pole were unexcited by its normal magnetizing current I_m, the stray magnetomotive force would indeed cause flux to flow along such a path. However, with the coil connected across the potential of the circuit to be measured, the total flux through the pole is necessarily proportional to such potential. If any stray magnetomotive force causes a flux in the pole tending to change this proportionality, I_m immediately changes a compensating amount. Although this compensating action originates in the potential coil, its action is naturally effective to a considerable extent throughout the length of the path of the stray field flux, which also includes the length of the 2 current poles. Thus the accuracy of the meter is unaffected by the components of stray a-c fields which are in a direction perpendicular to the disk.

Finally, the yoke completely surrounding the outer ends of the poles and fastening them together in an integral structure acts to some extent as a shield against stray field, whatever its direction. The resultant electromagnet without any enclosing case of magnetic material can be classified as substantially immune to the effects of any stray fields which could be encountered in ordinary installation practices. It is interesting to note that all American makes of meters now embody the general scheme,

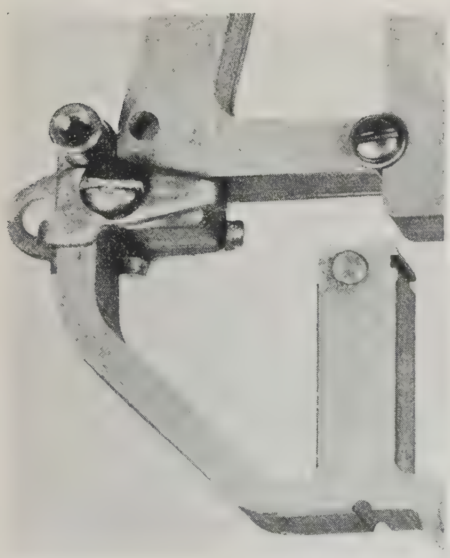


Fig. 3. Magnetic joint between separable current and potential electromagnet cores

which includes one single potential pole and 2 current poles, although the structural details to accomplish this in the various electromagnets naturally vary.

Another important advantage of the electromagnet form is that current and potential poles are held together in mutual relation in one integral structure without being clamped together for support upon some other part, such as a base, and thus are independent of the trueness and accuracy of this second member. Fortunately, it is not necessary to stamp the entire lamination in one piece to accomplish this. Such a structure would impose very great difficulties in applying the windings and especially the current coils, which are of various sizes of heavy wire. Instead, the potential and current parts are stamped separately and joined together. Figure 3 shows the details of this joint, which allows the parts to be detached conveniently at any time. The joint, nevertheless, is always effective in holding the parts in just as precise mutual position as though they were integral. This is important as it affects the length of the disk gap.

This detachability is advantageous to both manufacturer and user. For example, 6 current electromagnets cover the range of ordinary use for 5, 15, and 50 ampere meters in both 2 and 3 wire types. Only 2 potential electromagnets are necessary for the 115 volt and 230 volt ratings. Combinations of these produce any desired meter, thereby reducing parts stocks. Also, in the case of repairs, either the current or potential electromagnet can be replaced separately. The simplification mentioned has been merely for 2 voltage ratings and 3 current ratings in 2 and 3 wire at 60 cycles. If additional voltage, current, or frequency ratings are introduced, the advantages pyramid.

Electromagnet Windings

The starting requirement in the electromagnet design and one which has not always been met heretofore is to have a single form of electromagnet core of a fixed depth of stack which is capable of accommodating all ratings merely by putting on it windings of the proper number of turns. The ideal has nearly always been attained in transformer design where a relatively large number of turns may be employed on both primary and secondary; but for a meter element it is much more difficult, mainly because of the small number of turns which must be used on the current poles.

American practice requires that current windings be capable of carrying continuously 300 per cent of the nominal current rating on the basis of which nominal full load calibration is made. The extra 200 per cent is usually spoken of as "overload capacity," although it has really ceased to be that and modern meters may be momentarily loaded to far beyond the 300 per cent point. In the case of the 50 ampere rating, which is the largest present standard nominal rating, the current coils must be capable of carrying 150 amperes continuously. The ideal construction for such a coil is from relatively heavy copper bar or strap. It therefore is necessary in laying out the electromagnet to consider the entire range of current coils which may be used and preferably to start with the ones which present the greatest mechanical difficulty in applying them to the core.

Table I shows the current coil requirements throughout the necessary ratings. The figure of 12 turns per pole for the 5 ampere size is significant because of its divisibility by 2, 3, 4, and 6. Starting with this number, the required nominal ampere ratings are obtainable with identical cores and with identical torques through 15 amperes. The 25 and 50 ampere sizes are in reality 30 and 60 ampere elements calibrated at the nominal 25 and 50 ampere points. The so-called $^1/_2$ turn per pole in the table for the 50-ampere 3-wire coil is in reality one turn on each pole but with the turns in parallel so that the effect, at least, of a half turn is secured.

It is apparent that no number but 12 is suitable unless a full jump to 24 is made. Although possible, this would be very inconvenient and objectionable because the nominal 60 and actual 180 ampere turns per pole provided by the 12 turn base is already generously sufficient as compared to the potential coil ampere turns which it will be desired to use. Thus, it is a peculiarity of the design that for interchangeability and simplicity, the ampere turns per pole are definitely fixed at a nominal 60 with a base of 12 turns per pole for the 5 ampere size.

It is a simple matter to choose the proper copper cross section for the various coils on the basis of a 30 to 40 degree temperature rise. The lower rise applies to the coils of smaller capacities, while it is necessary to tolerate the higher rise in the case of the larger coils because of heat conveyed to the coil from the terminal chamber and line and load connections to the meter. Figures 4 and 5 show a 50-ampere 2-wire and a set of 50-ampere 3-wire coils which are chosen as the ones representing the most difficulties—mechanical, insulation, and space—for the line. It is to these that the electromagnet must really be "tailored" to provide ease of manufacture and reliability of insulation.

The distance between the 2 current poles is deliberately chosen so that these coils can be wound, insulated, and applied to the poles by simply dropping them in place. Similarly, the height of the current poles is made ample to take care of all possible contingencies for coil and insulation space, present or future. The roominess makes it possible to reverse coil sections from bottom to top on the respective poles for all 3 wire meter coils with the exception of the illustrated 50 ampere coil. This causes the annoying problem of coil balance on 3 wire meters to disappear. Even with the 50 ampere coil the space available makes perfect balance possible by proper shaping of the heavy leads. If space economies need be made, they should be at other points than here. Winding space and its convenient form are of paramount importance not only to the manufacturer but to the user, since restricted space is possibly the greatest contributor to difficult insulation problems and meager current carrying capacity.

Spacing apart the tips of the current poles so that they have no overhang and the coil slips on easily is a new feature and is accomplished at the expense of considerable torque reduction. However, the torque thus sacrificed is later restored by the overload compensator and indeed provides a basis for the working of this compensator. Consequently, an important structural advantage is secured with no loss and indeed with a performance gain, which will be understood fully when the overload compensation is discussed.

The current coil poles are made narrow to increase the clearance between coils and the yoke and to minimize the length of turns. Furthermore, a greater width than that shown adds almost nothing to the torque-producing ability of the element because the most effective portions of the current poles are near the tips of the potential pole. To really increase the torque, wider poles would require additional potential poles or counter poles above the disk. It is questionable whether the gain produced in this way would be worth the cost in space and the structural complications involved.

The potential pole is made as narrow as possible because over it must slip the potential coil, and it is necessary to keep down its mean length of turn. In the lamination shown in figure 2 this width is much less than usual for this style but is still sufficient for the outer tips of this pole to overlap the current pole faces by a slight amount. Thus the width of the potential pole is influenced by the initial choice of distance between current poles.

The primary cause of so-called class II temperature errors in watt-hour meters resides in the resistance of the potential coil, since this coil is of copper which has a large temperature coefficient. The most effective procedure, as well as the simplest, is to minimize this effect at its source, which means that the resistance drop I_mR of the potential coil must be kept low. To do this, the mean length of turn is kept short by using a coil of small diameter which hugs the narrow pole. The resistance of the coil is then further reduced by supplying a large winding cross section which, because of the small diameter, means a long coil. This requires a long potential pole. Here, again, is a place where false space economy should not be practiced and every available fraction of an inch has been utilized.

The 2 main factors in determining the over-all height of the completed lamination have been, of course, the height of the current poles and the length of the potential pole. In neither one of these has economy been exercised. Although in the single phase meter there is plenty of vertical room, the over-all height of the electromagnet enters directly into the height of 2 and 3 element meters and it is, therefore, desirable to keep it down. Notwithstanding all this, skimping on the essential pole dimensions is not justified in the electromagnet. It is possible to economize in over-all height in multiple-element meters in other places than within the

Table I—Current Coil Data

Ampere Rating		Turns per Pole	Turns per Coil Side	
Nominal	Actual		2 Wire	3 Wire
5	15	12	12	6
10	30	6	6	3
15	45	4	4	2
25	75	2	2	1
50	150	1	1	$^1/_2$

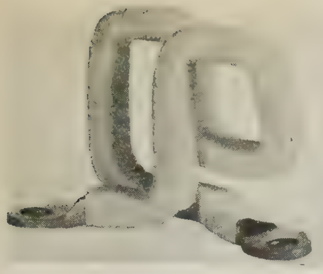


Fig. 4. A 50-ampere 2-wire current coil

Fig. 5. A 50-ampere 3-wire current coil

electromagnet itself and achieve the necessary results.

The 2 transverse arms (4 and 5) in figure 2 are the usual shunt paths for some of the potential coil flux. Compensation for the inherent small error in the meter caused by changes in voltage of the circuit to be measured is accomplished in the usual manner by having some portion of this path approach saturation as the voltage varies upward. The total amount of flux passing through the potential coil determines the number of turns necessary in this coil for a given impressed voltage. Also, the magnetizing current I_m in the coil varies inversely as the number of turns used. This total flux, and hence both turns and I_m, can be conveniently adjusted by fixing the relative permeance of these shunt paths. The width of the shunt paths, together with the length of the shunt-arm gaps, has, therefore, been chosen to give the desired number of potential-coil turns and desired magnitude of I_m in the finished meter. For the 115 volt coil, this is 0.06 ampere, a value somewhat below the average and one which has demonstrated itself to be acceptable to users of meters over a long period of time. For the 230 volt coil, I_m is, of course, one half this.

Temperature Errors

There are 2 classes of error caused by temperature variation. Class I temperature errors are those resulting from changes in magnitude of the various fluxes operating on the meter disk, and occur at all power factors of the measured circuit. Class II errors are caused by changes in the phase angle relationship of the various alternating fluxes as the temperature changes. Such errors are important only at low power factors in the measured circuit.

Kinnard and Faus[2] have fully analyzed the sources of error resulting from temperature changes and first developed a method of compensating for the so-called class I group by the use of a nickel-alloy shunt having a negative temperature coefficient of permeability on the permanent magnet damping unit. This has since become universal on all American makes of meters. In the present design, this shunt takes the form of a small clip having an approximate nickel content of 30 per cent. It can be seen plainly on the upper tips of the permanent magnets in figures 13 and 14. D. T. Canfield[3] has also presented a complete analysis of temperature errors for both class I and class II. The class II errors are existent only at power factors less than unity. The low resistance potential coil already described in the new design minimizes the class II errors at their source instead of resorting to compensation. This method of minimizing class II errors was also advocated by Kinnard and Faus. They are still further reduced by the use of a phasing and light load adjustment plate having a low temperature coefficient of resistance. Resultant performance of the meter with respect to temperature under unity power factor as well as inductive loads is given in the appendix.

Main Electromagnet Gap

The smaller the gap for the disk, the more effective the electromagnet. Both manufacturing and operating experience must be combined to determine a desirable minimum value. For a number of years gaps as low as 0.1 inch have been used by one company, but this value requires excessive manufacturing care. A value of 0.125 inch was chosen in the present design. This provides ample disk clearance, but more than this would be of doubtful value in that it is necessary in any event to use a gap of less than this amount for the damping magnets which are to fit on the same disk.

With all the main details of the electromagnet thus determined, it is merely routine to draw in the main outlines and yoke, bearing in mind that economy of material and a minimum number of rivets are desirable. The potential electromagnet yoke has been proportioned to give a flux path of uniform cross section. Ineffective excess iron has been eliminated. The resultant shape makes it easy to thread the laminations through the coil in the assembly operation.

Three main spacer and support rivets are used and go through the laminations in such positions that they are out of line with the main flux paths. These are almost equidistant and widely separated so that the 3 ends of the spacers determine precisely a plane which is parallel to the lamination plane.

Torque Requirements

The electromagnet is a motor and it is necessary to decide arbitrarily how much torque it is desired to produce from it. The lower the torque, the less copper and iron are needed in the complete meter. For many years the ratio of torque produced by the moving element in millimeter-grams to the weight of this element in grams has been regarded by manufacturers and users alike as a rough measure of the merit of a meter element. Current opinion for the past few years has sanctioned a ratio of 3 to 1 as good practice. This opinion is supported by service records over a period of years of many meters which have had a ratio of even less than this value. The ratio, however, has always been computed for a *nominal* full load condition and fails to take into consideration that modern meters are capable of 3 times this torque. Giving the overload torque full credit, a 9 to 1 ratio would exist.

Actually, users are not anxious to accept a reduction in this ratio because in the 5-ampere, 115-volt

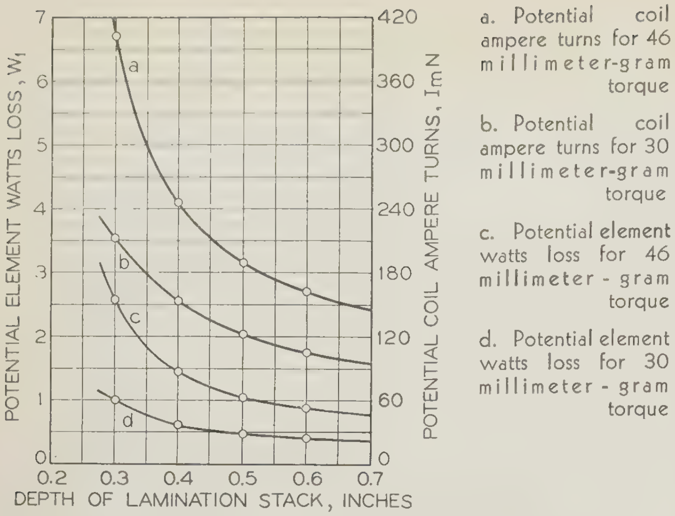


Fig. 6. Relation of electromagnet stack thickness to potential coil ampere turns and losses

meter it would mean less torque at exceedingly light loads. As an example, although a 5 ampere modern meter with a 3 to 1 ratio will measure accurately a 15 ampere load, it will start on only $^1/_3$ the watts of the old form of 15 ampere meter also having a 3 to 1 ratio but without overload capacity, or it will start on the same watts required by the old form of 5 ampere meter. Stated differently, the accuracy on overload was not obtained at the expense of accuracy at light loads.

Low starting watts and extreme light-load accuracy are wanted for 5 ampere meters on small installations so as to record energy flow because of clocks or bell-ringing transformers at times when no other loads are existent. On installations requiring 10 or 15 ampere meters, however, it becomes impractical to detect such a small power flow and, indeed, very unimportant from the standpoint of percentage of total energy measured. With the growth of loads in the future to take full advantage of the capacity of the 5 ampere meter, the problem of light-load accuracy will probably not loom so large. For the near future, however, it was important enough to warrant a maintenance of the 3 to 1 ratio.

Much work has been done in the past to make the weight of the moving element low because it must rest on the jewel bearing. It was possible to carry it over without change from the previous design. It consists of a light aluminum disk mounted on a short aluminum-alloy shaft. It is mounted on the extreme lower end of this shaft so as to give the complete element a low center of gravity and minimize the support, and thus the friction, contributed by the upper bearing. The moving element weighs 15 grams, and a value of 46 millimeter-grams was arbitrarily chosen as the nominal full load torque of the element.

Electromagnet Thickness

Cross section of iron was not considered as an important factor in shaping the lamination because within reasonable limits this can be made anything desired by changing the thickness of the lamination stack. The volume of iron in the completed electromagnet in order to produce the required torque is determined by considering performance with different depths of stack.

Figure 6 shows the relation of potential electromagnet loss, W_1, and potential-coil magnetizing ampere turns, I_mN, as the stack thickness is varied in a completed electromagnet. One set of the curves is drawn on the assumption that the desired 46 millimeter-grams torque is delivered at nominal

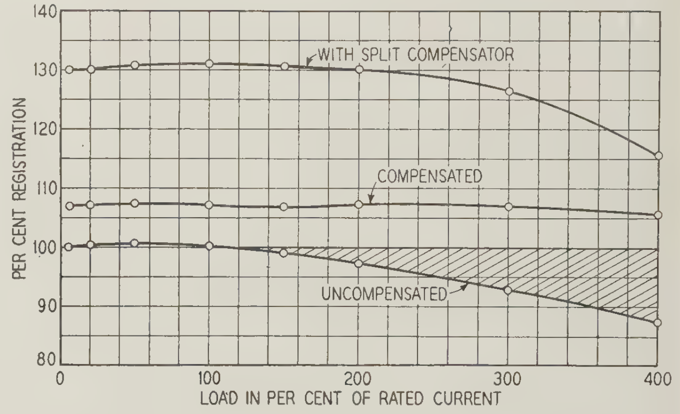


Fig. 7. Accuracy of element on heavy loads before and after compensation

current and voltage at all times regardless of stack thickness. The curves assume proper windings on the core and are easily secured by testing a few electromagnets with different amounts of laminations in each. The stack thickness chosen has been 0.6 inch, giving a watt loss of 0.9. Under these conditions, 162 magnetizing ampere turns are required in the potential coil, and with 2,700 turns, which are used on the 115 volt electromagnet, I_m becomes 0.06 ampere.

It is interesting to note that a substantial saving in stack thickness could be effected if the industry would be willing to accept a smaller torque, a higher loss, or a combination of the 2. To illustrate this a second set of curves has been drawn on the basis of a required torque of 30 millimeter-grams, from which it can be seen that a stack depth of only 0.3 inch is required on the basis of a 1.0 watt loss. Similarly, a torque of 46 millimeter-grams could be had by raising the loss to 1.6 watts with a stack of only 0.375 inch.

Speed of Moving Element

Another arbitrary decision must be made, and this concerns the series of disk constants K_h which are to be used. This, also, is tied in closely with past experience and present standard practice. Three manufacturers have consistently used a constant of $^1/_3$ watt-hour per revolution in the 5-ampere 115-volt meter for a period of years. This corresponds to a nominal speed of 25 revolutions per minute at 500 watts load. There is little incentive to depart from this speed, and considerable resistance would

be encountered from users if it were tried. A lower speed would call for a higher torque, if the same power output of the moving element were maintained. This in turn would eventually require more material in both the electromagnet and damping magnets and hence would be a disadvantage in cost and weight. It would appear that some advantage would accrue in less bearing wear from the smaller number of revolutions because of a higher constant accompanying a lower nominal speed, but little actual evidence has accumulated in service to support this. At best it could only be an improvement proportional to the reduction in the number of revolutions for a given kilowatt-hour consumption, while the demand for bearing improvement far exceeds the modest gain that could be obtained in this way. The subject of bearings merits a study in itself; and, although the jewel and pivot bearing is very old, being inherited from the clock art, there is a fair amount of evidence that improvements will be forthcoming which will be sufficient to fulfill operating requirements at established speeds. Such improvements could include better design, as well as the utilization of new materials which would either not require lubrication or which could be lubricated with greater permanency than present forms.

A further disadvantage of changing to a lower speed is that the meter element must be calibrated at light loads by counting revolutions against a standard. Since a minimum of one or two revolutions must be taken, this increases the time for testing. If stroboscopic testing is used, the difficulties are also increased as, even with the present speeds, this form of testing is difficult at 10 per cent load.

A main disadvantage of the high speed is that it causes the meter to have a markedly drooping characteristic in its registration curve at heavy loads. Until the advent of satisfactory means of overload compensation, this difficulty might have been entirely sufficient to require a lower speed, but, with the form of compensation to be described, it has been completely removed. The decision was therefore made to continue the established nominal speed of 25 revolutions per minute at 500 watts or multiples thereof.

Overload Compensation

The uncompensated element does not have a good registration curve. The lower curve of figure 7 shows this from 5 to 400 per cent nominal current load at unity power factor. The shaded area shows the amount of compensation necessary, which at 400 per cent load amounts to over 12 per cent. With a compensator, the curve can be straightened almost perfectly. The compensator consists of 3 laminations of steel (figure 9) held in the position shown in figure 8 on a brass slide which is driven into 2 notches in the inner faces of the current poles.

Imagine that the compensator is cut into halves by removing the magnetic material between the dotted lines in figure 9. The areas A and B of the compensator, being directly under the potential pole, will then act as extensions of the current pole faces because they are virtually connected with the respective poles through the gaps between the compensator and the pole pieces which extend along the perimeter of these poles.

It was formerly stated that the separation of the current pole tips to eliminate overhang reduced the torque available from them. The compensator in the bisected form completely restores that torque so that a registration curve taken on the meter, without recalibrating it, is as shown in the upper curve of figure 7. It is approximately parallel to the former curve and 30 per cent above it.

If the iron between the dotted lines in figure 9 is now restored, it is evident that areas A and B will tend to assume the same magnetic potential and can only differ in an amount equal to the magnetomotive force drop across the iron in the shunting portion of the compensator, which is roughly represented by the iron between the dotted lines.

At moderate loads the flux leaking on to areas A and B on each side of the compensator is so effectively shunted as to have little effect. On heavy loads, however, the iron between the dotted lines approaches saturation, and the areas A and B come into play as virtual extensions of the current pole pieces and hence increase the torque and thus pull up the drooping registration curve.

The middle curve, which is practically flat at 107 per cent registration, shows the actual effect of the addition of the complete unbisected compensator, assuming that the adjustments of the meter had remained unchanged. Actually, of course, calibration to 100 per cent registration is made with the compensator in place, which gives a flat curve at 100 per cent. In addition to its corrective effect, therefore, the compensator adds 7 per cent to the torque of the meter at all loads.

Industry requirements for the final registration curve are very exact, and the saturation curve of a plain single iron path of uniform cross section and

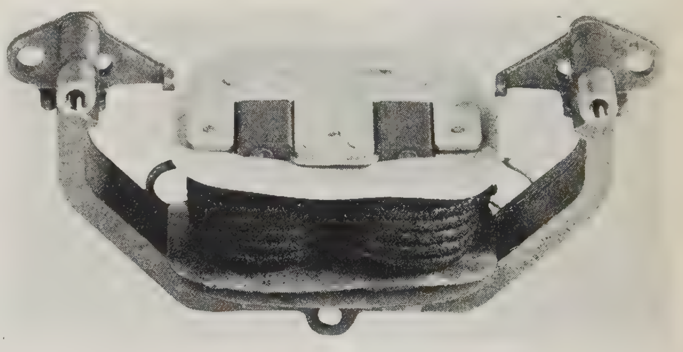


Fig. 8. Current core with overload compensator in place

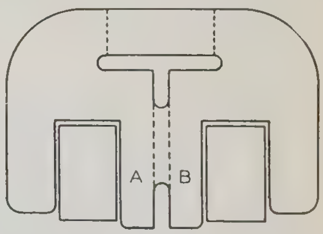


Fig. 9. Plan view of compensator

length is too sharp to match the required amount of compensation as indicated by the shaded area in figure 7. For this reason, the shape of plate shown has been developed, in which there are 2 shunting paths of different length, the longer one passing on the outer side of the T-shaped slot. The shorter path tends to saturate first, followed progressively by the longer one. Thus the knee of the curve is rounded off to match the compensation needed. Correction can be controlled with exactness by the shaping of the laminations and the slots therein. Production tests on many meters indicate that uniform compensation can be obtained by this method. One contributing factor is the extended gap between the compensator and the pole edges, which keeps a relatively constant reluctance despite any small displacements or changes in dimension of the parts in production.

The laminated structure of the compensator cuts down eddy currents in it which would tend to cause slight and nonuniform phase displacement of the current flux by reason of the fact that the current flux passing through the compensator is nonlinear with respect to the magnetomotive force on the current poles. For this reason, the compensator is as accurate on inductive loads as it is on unity power factor circuits. Table II gives the typical compensated registration curve data as determined accurately from a large number of meters of various voltage and current ratings. Note that the accuracy on inductive loads is almost identical with unity power factor accuracy, which means that throughout the wide range of from 3 to 400 per cent load, accuracy is independent of changes in power factor.

Performance Characteristics

The new meter element was arranged primarily to function with a powerful and accurate heavy load compensator such as that described. By taking advantage of what the compensator could do many indirect advantages were obtained. Without the compensator, it would not have been possible to use such a high value for the current pole ampere turns and still keep the potential coil ampere turns at a low value. Hence the compensator is responsible indirectly for the simple series of current pole turn values as well as the low losses in the potential electromagnet. In addition to straightening out the load curve it also contributes substantially to the torque of the element, even at moderate loads. All of the performance characteristics of a meter are interlocked in a substantial amount and there is scarcely one of them which is not affected favorably, at least in some degree, by the presence of the compensator.

A very few comparisons with the superseded design will serve to bring out some of the gains in the motor element. The complete weight of the old type electromagnet was 2.34 pounds and in the new it is 1.76, a reduction of 25 per cent. Even with the decreased weight the new element has 10 per cent higher torque. Potential-coil magnetizing currents are the same for both old and new elements but the new has 10 per cent lower watts loss in the potential coil. Current coil carrying capacities in the old meter were ample for 300 per cent load, and in the new meter these were increased slightly and the current coil insulation given a greater factor of safety.

Problems relating to the performance of meters under variations of power factors, voltage, frequency, and wave form are fully described in texts on meters such as that by Jansky[4] or Knowlton.[5] For a period of years this performance has been so excellent as to leave little to be desired. For reference, a brief tabulation of various standard performance data including such items is given in appendix I.

Table II—Compensated Registration Curve Data

Per Cent of Rated Full-Load Current	Per Cent Registration: At Unity Power Factor	Per Cent Registration: At 0.5 Lagging Power Factor
3	100.5	101.1
5	100.1	100.7
10	100.0	100.3
20	100.1	100.1
50	100.3	100.4
100	100.0	100.0
200	100.0	100.3
300	100.0	100.7
400	98.5	99.7

Above data based on the assumption of correct calibration at 10 per cent and 100 per cent load, and are average or typical performance, from which individual meters should not vary by more than 0.5 per cent

Design of Frame

The electromagnet is supported on a cast frame at only 3 points by means of the spacer rivets passing through it. Immediately below each spacer rivet is a support lug having a hole in it through which a screw may be passed to fasten the electromagnet to any desired base. These 3 support lugs practically coincide in position with the 3 spacer rivets. Thus the 3 point support scheme is carried out in such a manner that warping or displacement of the base cannot possibly communicate any distortion to the meter itself.

Figure 10 shows a complete meter element minus the register so that a better view of the frame can be

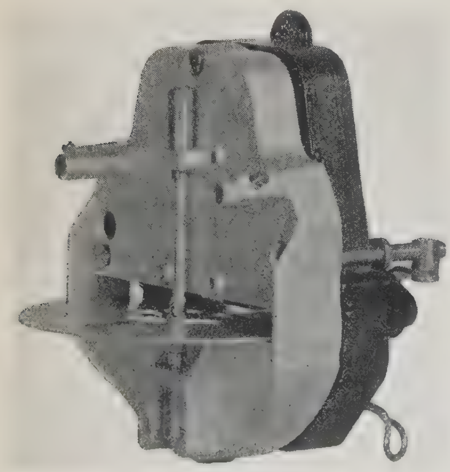


Fig. 10. Meter element without the register, showing the frame

obtained. With only 3 support screws through the lugs on the rear of the electromagnet, the element can be fastened to any type of base without affecting its adjustment or operation. These screws are accessible from the front without difficulty with the meter completely assembled. Similarly, the 3 screws holding the frame to the electromagnet are also easily accessible with the meter completely assembled.

The current coil ends on all ratings of electromagnets are terminated in such a position that the binding screw heads passing through them can be conveniently operated from the front of the frame, if desired, without removing it from the electromagnet. Thus the frame and the parts which it supports, when screwed onto the electromagnet, comprise a complete measuring element, irrespective of any support or positioning contributed by the base and, indeed, can be mounted in any type of base with equal convenience. This is important as trends in the last few years have indicated that the forms of enclosure for meters, including the bases, may be subject to considerable modifications in design.

The new construction is such that a meter, even in its installed condition, can have its entire element removed from its base, working from the front without disturbing any other part. Similarly, the frame alone and all the parts which are carried by the frame can be removed without disturbing the electromagnet in the base. In addition, any part carried by the frame can be removed and put back without disturbing either the frame or the electromagnet. Any of these things can be done with a screw-driver, operated from the front, and without changing the calibration of the meter.

From the standpoint of electrical characteristics, such matters of arrangement of parts may seem trivial, but they loom large to the operator who must take care of large numbers of different makes and types of meters. To the casual observer they may seem obvious and easy to accomplish, and only by comparison with the many models and types which have gone before can the advance in the form of simplicity be recognized. It is in these problems affecting simplicity and space engineering that suggestions and requirements from operating engineers from many sections of the country have counted heavily in influencing the final design.

The frame carries the moving element with its upper and lower bearings together with the damping magnet unit. These are readily visible in figure 10. Much care was used to secure proper strength in the frame because these elements must be supported in fixed and unchanging relation.

Construction of Damping Unit

All of the American meter producers now use a 2 magnet unit similar in general construction to that shown in figure 11. Such units have given successful performance in a great many meters over a long period of years so that there are a number of their design features which need not be discussed, as they are matters of routine. Among these are kind of steel, heat treatment, and forging.

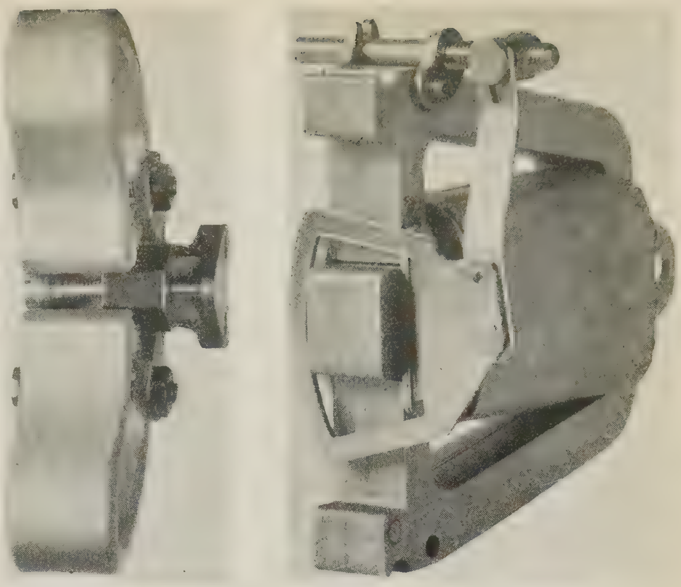


Fig. 11 (left). Damping unit with full load adjustment

Fig. 12 (right). Potential magnet with light load adjustment

The damping strength required is definitely fixed from the torque and the speed, which have already been discussed. The design problem consists in getting this from the unit with maximum economy and with an adequate factor of safety.

The first consideration is the length of gap, and here again the result of much operating experience comes into play. The smaller the gap, the more effective the magnet for a given weight of steel; but small gaps carry with them the disadvantage of very small tolerances and the very important operating factor that any magnetic dirt or filings in the meter will get stuck in a narrow gap and very probably scrape on the disk. Past successful operation has resulted in many meters with gaps as low as 0.078 inch, while some have operated with gaps as high as 0.125 inch. For the present design, a minimum of 0.090 inch was chosen.

The bearings and shaft are designed to bring the disk accurately in the center of this permanent magnet gap without necessity for vertical adjustment. This is checked by gauges at the proper stages of manufacture; and, when accomplished, the position of the disk in the electromagnet gap will also be correct with some margin to spare because the electromagnet gap is 0.125 inch in width.

The shape of the magnet is chosen to give it an adequate length for permanence and also to avoid sharp bends which are difficult to make and which tend to start cracks which are ruinous to a good magnet. The shape must also be worked out to provide as much room as possible above the magnets for the register mechanism, which must also include demand registers for some meters, and in demand registers space is always at a premium. In the present unit, the length of each magnet is approximately 65 times the minimum length of gap, and in past practice 50 times has been considered adequate. A thick magnet section has been avoided as the steel

near the surface responds most readily to heat treatment and is most efficient.

The 2 magnets are permanently fixed in a brass clamp which holds them at the most efficient damping radius from the center of the disk, and adjustment is secured by a soft iron bar below the disk which slides in and out to shunt a variable portion of their flux. This bar is chosen to shunt no more of the flux than is absolutely necessary to secure the proper range of adjustment so that all other available flux is actually effective for damping purposes, with the exception of that passing through a small nickel steel clip on the top poles of the magnets. In figure 11 the clip is not shown on the pole tips so that the view of the full load adjustment bar is not obstructed. This clip has a negative temperature coefficient of permeability and reduces the so-called class I temperature errors of the meter to a negligible value.

The merits of the design can be measured by fully magnetizing a unit and applying it to a meter. With the new design, under this condition, the damping effect is approximately twice that necessary, being sufficient to cause the meter to run at 50 per cent registration instead of the desired 100 per cent. The unit is, therefore, brought down to the desired strength by demagnetization before it is used. An alternating magnetomotive force is applied to each magnet for this purpose. The flux remaining in the magnet will subsequently be unaffected by any other accidental demagnetizing force having a lower value. In the new unit, the demagnetizing force for each magnet exceeds 250 ampere turns.

Mechanical and thermal abuse on meters in service is rarely serious. A far more likely cause of magnet weakening is electrical disturbance such as severe short circuits, or effects of lightning, which result in demagnetizing stray fields. The coercive force or the amount of demagnetizing that a magnet unit is able to stand is, therefore, a most important indication of its stability and permanence. Because permanency is the very heart of value in a meter, the magnet unit should be proportioned not only efficiently but generously. Numerous laboratory and service tests were, therefore, run with respect to the new unit. Its resistivity to abnormal conditions was found to be closely proportional to the a-c demagnetization required. This factor was found to range from 50 per cent to 90 per cent higher than for similar successful designs which had preceded it, when measured under the same laboratory conditions.

Adjustments

A new feature of the full load adjustment on the damping unit is that the soft iron bar is pushed in and out on a screw arranged to be free from back lash so that small movements of the screw in either direction immediately produce corresponding changes in calibration. The screw requires no clamping to make the adjustment permanent so that the effect has been described as a "micro-set" adjustment. A departure from all previous designs is that the adjustment screw is operated by a screwdriver from the front of the meter. The entire magnet unit can, of course, be removed from the frame without affecting the adjustment or the resultant calibration.

The light load adjustment consists of the usual closed turn in the shape of a metal plate surrounding the lower end of the potential pole. It is shown in detail in figure 12. By means of a split nut and lever, it is made to operate in micro-set fashion from the front with a screw-driver. It is important to note that, since the light load adjustment operates by reason of its relative position with respect to the potential pole, the proper place for it to be mounted is actually on the potential electromagnet rather

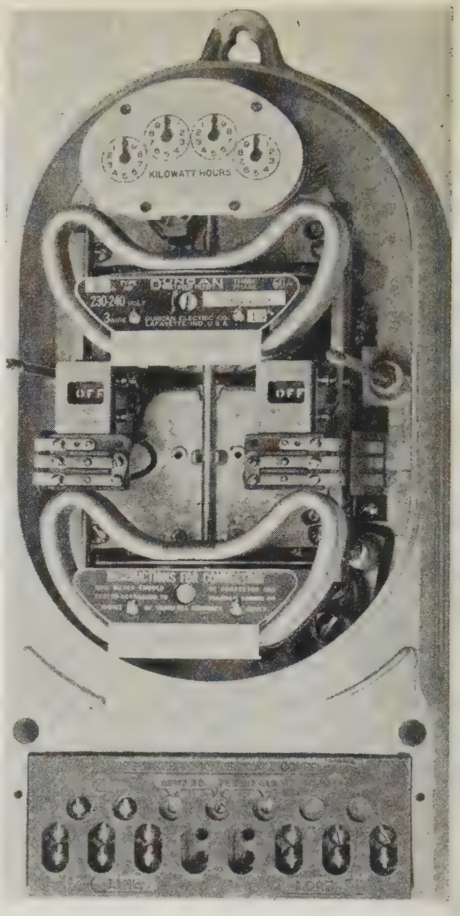
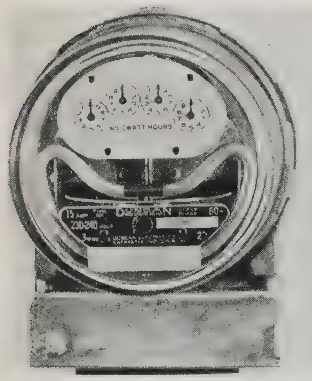


Fig. 15. Two-element meter without cover

Note similarity to single phase construction, and open-potential-coil indicators



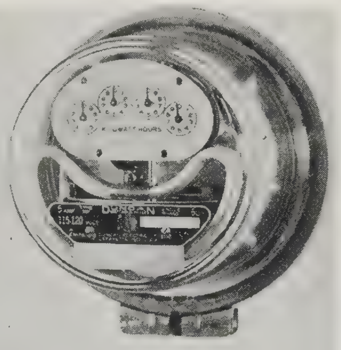


Fig. 13 (left). Bottom-connected single-phase meter

Fig. 14 (right). Socket-type single-phase meter

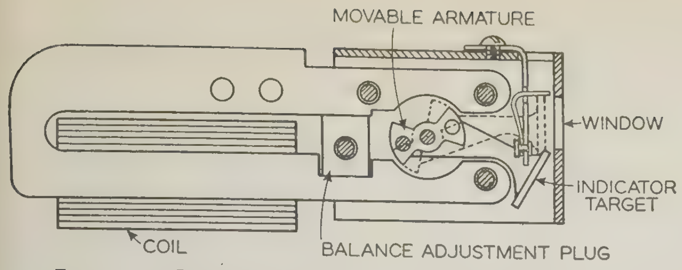


Fig. 16. Open-potential-coil indicator construction

than on some extraneous part, such as frame or base. This detail makes light load adjustment independent of accidental displacement of any other parts with respect to the potential pole.

Very little phasing is required to bring the potential flux of the pole in exact quadrature with the applied voltage. The light load plate more than supplies this, making it possible to accomplish the phasing adjustment differentially by a small coil of a few turns on each current pole. This coil is closed through an adjustable resistance. When adjusted to the correct value, it is soldered and the operation ordinarily is permanent and does not have to be repeated.

Enclosures

Figure 13 shows a completed single phase element in a front connected case, and figure 14 shows the same meter in a base for detachable mounting in a wiring socket. Discussion of various cases and their modifications comes outside the scope of this paper. Their importance with respect to the meter design results from the fact that the meter element must be easily adaptable to them. In this the new design fulfills requirements not only as to present cases, but will allow much flexibility in future requirements.

Polyphase Design

Once complete, it should be possible to apply the single-phase element interchangeably to multiple element designs. The same accessibility of parts from the front of the meter can also be obtained. In general, the multiple element unit should have the same performance as the single element. Only one detail need be added to the single units to make them adaptable, and this is some form of torque adjusting device which may be used to vary the torque output of the electromagnet uniformly at all loads so that balance of torques between elements can be secured when several electromagnets all operate on one moving element.

This should be some device which can easily be applied to any existing electromagnet without changing its form, because interchangeability between elements, as far as stocking for repair or manufacture is concerned, would be disturbed even by a modification as slight as the use of special length leads for the potential coil. In the new design, this balancing has been accomplished by inserting a small adjustable reactance in the potential circuit of each electromagnet. Since this is an entirely separate part, it fulfills the foregoing requirements.

There is an additional advantage in this method because another and separate function can be obtained from this device, since it can be used as an open-potential-coil indicator. Figure 15 shows a complete 2 element watt-hour meter with cover removed and the indicators in place; figure 16 shows the indicator construction.

An indicator of some form is necessary on polyphase meters because open potential coils are difficult to detect as the entire meter does not stop if only one element is affected. In the electromechanical indicator shown in figure 16, the magnetizing current I_m of the associated potential element passes through its coil. The impedance drop through this coil is only 8 per cent of the total applied potential, and hence there is little torque sacrificed. When the indicator is energized, the target is pulled up into a conspicuous position to show that the element is "ON". Similarly, the target will drop down to show "OFF" conspicuously if I_m is in any way interrupted. The indicator coil is as well insulated and of the same size wire as the potential coil which it protects and cannot burn out for any cause less severe than that which would cause failure of the main coil. A high degree of reliance can, therefore, be placed on the indication.

The permeance of the magnetic circuit through the indicator coil can be varied by changing the position of an iron balance plug in this circuit which changes the impedance drop through the coil to give the adjustment required. This adjustment also has been made micro-set by means of a screw passing through the plug. Operating the balance adjustment changes the torque of the meter element a uniform percentage under all conditions of load, and does not affect the phasing adjustment in an appreciable degree.

Appendix

The accompanying tables give miscellaneous performance characteristics of the new design for 60 cycle meters, both single phase and polyphase, in all voltage ratings and in all current capacities, including the 50 ampere rating. Average values are given for the typical meter. Small manufacturing variations and local conditions of test and installation may cause the individual meter to vary slightly from the average performance, but never by a substantial amount. The effect of voltage variation on meter registration is shown in table III. The data in the table are based on the assumption of correct calibration at a reference voltage within the rated voltage limits as given on the nameplate of the meter, and were taken at full load. However, nearly identical performance with respect to variation in voltage is obtained at all other loads between

Table III

	Per Cent Registration at Full Load	
Per Cent of Rated Voltage	Unity Power Factor	0.5 Lagging Power Factor
85	100.1	100.1
100	100.0	100.0
115	99.7	99.6

10 per cent and 300 per cent. Note that the readings with inductive load practically coincide with the readings at unity power factor loads.

The effect of frequency variation on registration is shown in table IV. For present day electric distribution circuits, these errors are negligible.

Variation in registration caused by temperature is shown in table V, for which correct calibration at 10 per cent load and 100 per cent load and correct power factor adjustment at 20 degrees centigrade are assumed. The variations shown are typical and average for all loads between 10 per cent and 300 per cent.

Table IV

Cycles	Per Cent Registration at Full Load: Unity Power Factor	Per Cent Registration at Full Load: 0.5 Lagging Power Factor
57	100.4	99.8
60	100.0	100.0
63	99.7	100.3

Table V

Temperature, Degrees Fahrenheit	Temperature, Degrees Centigrade	Per Cent Registration At Unity Power Factor	At 0.75 Power Factor	At 0.5 Power Factor
−4	−20	99.9	100.15	102.3
14	−10	100.1	100.1	101.8
32	0	100.1	100.05	101.2
50	10	100.0	100.0	100.7
68	20	100.0	100.0	100.0
86	30	100.0	99.95	99.5
104	40	100.1	99.90	99.0
122	50	100.3	99.85	98.4

Temperature coefficient per degree centigrade at unity power factor = 0
Temperature coefficient per degree centigrade at 0.5 lagging power factor = 0.055 per cent
At approximately 0.75 lagging power factor, all temperature errors are substantially zero

Miscellaneous Data
(For 5 amperes, 115/120 volt, single phase meters)

Potential element exciting current at 115 volts	0.06 ampere
Potential element coil resistance	72 ohms
Potential element loss at 115 volts	0.9 watt
Power factor of potential coil	0.13
Electromagnet disk air gap	0.125 inch
Diameter of disk	3 1/2 inches
Thickness of disk	0.036 inch
Weight of moving element	15 grams
Full load torque (at 575 watts)	46 millimeter-grams
Ratio-torque to weight	3
Speed of disk at 500 watts	25 r.p.m.
Watt-hour constant	1/3
Resistance of current coil circuit	0.010 ohm
Current coil loss at 5 amperes	0.25 watt
Range of adjustment at light load	80 to 120 per cent
Range of adjustment at full load	85 to 120 per cent
Weight of service type	7.2 lbs.
Weight of plug-in type (without socket)	6.6 lbs.
Weight of socket for plug-in type	2.1 lbs.

References

1. Handbook for Electrical Metermen, 1923 edition, chapter III.

2. Temperature Errors in Induction Watthour Meters, I. F. Kinnard and H. T. Faus. A.I.E.E. Trans., v. 44, 1925, p. 275.

3. Theory of Action of the Induction Watthour Meter and Analysis of Its Temperature Errors, D. T. Canfield. A.I.E.E. Trans., v. 46, 1927, p. 411.

4. Electrical Meters (a book), C. M. Jansky, McGraw-Hill Book Company, N. Y., 1917, chapter XIII.

5. Electric Power Metering (a book), A. E. Knowlton, McGraw-Hill Book Company, N. Y., 1934, chapters VIII and IX.

Control of Potential Over Insulator Surfaces

To reduce radio interference caused by corona from pin insulators, adherent conducting coatings or films usually are applied to the central portions of the heads of the insulators, and metal thimbles or conducting coatings in the pin holes. By extending the coating to cover the entire head and by using a coating of the proper resistivity, the voltage at which corona occurs can be raised considerably without materially lowering the flashover voltage. This paper calls attention to the principle of controlling the potential distribution over surfaces of insulators by utilizing the resistance drop in potential resulting from the flow of charging current in high resistance films.

By
EDWARD BENNETT
FELLOW A.I.E.E.

GORDON FREDENDALL
ASSOCIATE A.I.E.E.

Both of University of Wisconsin, Madison

BRUSH DISCHARGES in the film of air that separates the porcelain surfaces of pin type insulators from the adjacent surfaces of the line and tie wires have been a troublesome source of radio interference on moderate-voltage power-transmission lines. Laboratory tests indicate that any commercial pin insulators, regardless of the manufacturer's voltage rating, may be expected to give rise to radio interference if operated at effective line voltages exceeding 8 kv to ground, unless the insulators be provided with caps or conducting coatings which eliminate the discharges from the line and tie wires to the insulator surfaces.

To eliminate these brush discharges it is the usual practice to apply an adherent conducting coating or film to the central portion of the head of the insulator extending about one inch out beyond the tie wire groove and to employ a metal thimble or a conducting coating in the pin hole. The prevailing practice is to have the resistance of these coatings so low that the entire coating assumes the potential of the line wire or of the pin with which it is in contact. This

A paper recommended for publication by the A.I.E.E. committee on power transmission and distribution, and scheduled for discussion at the Great Lakes District meeting, Lafayette, Ind., Oct. 24–25, 1935. Manuscript submitted July 1, 1935; released for publication August 9, 1935.

means that there is a tendency for a brush or corona discharge to occur at the edge of the coating, although the voltage between line and ground necessary to cause this corona usually is higher than the voltage necessary to cause corona on the uncoated insulator.

This paper points out that by extending the coating to cover the entire head, the voltage at which corona occurs can be raised greatly without materially lowering the flashover voltage of the insulator provided the following principle is utilized: The resistivity of the coating must be increased to the range of values at which the *IR* drop in potential from the line wire to the peripheral edge of the coating is as great as possible consistent with no material decrease in the flashover voltage of the insulator. The current that causes the drop in potential toward the periphery is the radially directed charging current that flows in the coating.

Briefly, the paper calls attention to the principle of controlling the potential distribution over surfaces of insulators by utilizing the resistance drop in potential resulting from the flow of charging currents in the high resistance films with which the surfaces are coated. It presents the current-voltage equations for several simple geometrical forms, and the results of experimental confirmations of the principle.

Potential Equations for High Resistance Films

Disk. If the head of a pin insulator be coated with a conducting film of known resistivity, the differential equations of the film relating current, potential, and distance cannot be solved because of the impossibility of expressing the capacitances of the successive circular zones of the film. A simple geometrical form, susceptible of solution, which to some extent resembles a coated insulator head, is illustrated in figure 1. It consists of an insulating disk of glass or porcelain, A, resting on a metal plate, B. On the upper surface of the disk is a thin circular conducting sheet or coating, C, of high (and uniform) resistivity. Concentric with this coating and resting on it is a circular metal electrode, D.

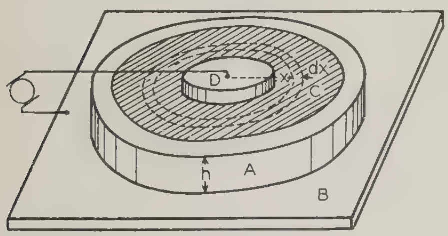


Fig. 1. Disk insulator model somewhat similar to a coated insulator head

If a sinusoidal electromotive force be applied between the circular electrode D and the metal plate B, a conduction current from the electrode flows radially in the high resistance coating to charge the successive circular zones of the coating to the appropriate potentials. If for figure 1:

x represents the radial distance from the center of the coating to any circular zone of width dx

a and b represent, respectively, the radii of the metal electrode D and the coating C

i represents the instantaneous value of the current in the outward direction in the zone of radius x

e represents the potential of the zone of radius x relative to the metal plate A

p represents the permittivity of the disk in coulomb-volt-centimeter units

r represents the resistance between opposite edges of a centimeter square of the high resistance coating

then the resistance of the zone of radius x is $rdx/2\pi x$, and its capacitance to the metal plate, a, is $2\pi xpdx/h$.

h = 1.3 centimeters
b = 15.0 centimeters
a = 5.0 centimeters
$p = 6 \times 8.85 \times 10^{-14}$ in coulomb-volt-centimeter units
$\omega = 2\pi 60$

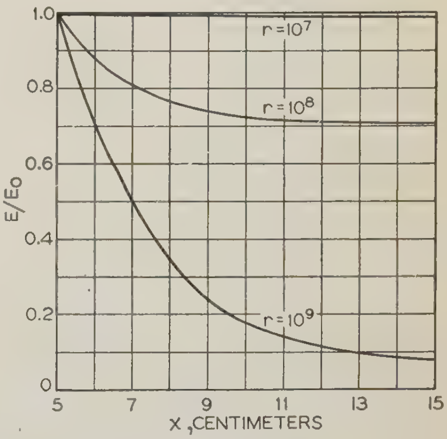


Fig. 2. Distribution of potential over high resistance coating of disk insulator shown in figure 1 for 3 coating resistivities (in ohms per centimeter square)

The capacitance from the upper surface of the zone to ground may be ignored, or it may be allowed for approximately, depending on the configuration of the ground surfaces.

Accordingly, the differential equations for the film are

$$\frac{\partial i}{\partial x} = -\frac{2\pi p x}{h}\frac{\partial e}{\partial t} \tag{1}$$

$$\frac{\partial e}{\partial x} = -\frac{ri}{2\pi x} \tag{2}$$

Writing the expressions for the sinusoidal current and potential difference in the exponential forms,

$$\left.\begin{aligned} i &= I \exp j\omega t \\ e &= E \exp j\omega t \end{aligned}\right\} \tag{3}$$

in which ω represents $2\pi f$ and the amplitudes I and E are functions of x. Substituting these expressions in equations 1 and 2 the following equations result:

$$\frac{\partial I}{\partial x} = -j\frac{2\pi p x \omega E}{h} \tag{4}$$

$$\frac{\partial E}{\partial x} = -\frac{rI}{2\pi x} \tag{5}$$

Eliminating I between these equations,

$$\frac{\partial^2 E}{\partial x^2} + \frac{1}{x}\frac{\partial E}{\partial x} - \frac{jr\omega p}{h}E = 0 \tag{6}$$

This is Fourier's equation; its solution written in terms of the "ber" and "ker" functions is[1]

$$E = E_1\,(\text{ber}\ mx + j\ \text{bei}\ mx) + E_2\,(\text{ker}\ mx + j\ \text{kei}\ mx) \tag{7}$$

in which

$$m = +\sqrt{\frac{rp\omega}{h}} \tag{8}$$

Evaluating the integration constants E_1 and E_2 to satisfy the boundary conditions, namely, $I = 0$ at $x = b$ and $E = E_0$ (the impressed voltage) at $x = a$, their values are found to be

$$E_1 = -\frac{AE_0}{BC - AD}$$

$$E_2 = \frac{BE_0}{BC - AD} \tag{9}$$

in which

$$\left.\begin{aligned} A &= \text{ker}'\ mb + j\ \text{kei}'\ mb \\ B &= \text{ber}'\ mb + j\ \text{bei}'\ mb \\ C &= \text{ker}\ \ ma + j\ \text{kei}\ \ ma \\ D &= \text{ber}\ \ ma + j\ \text{bei}\ \ ma \end{aligned}\right\} \tag{10}$$

When numerical values are assigned to the constants appearing in the solution, a series of curves may be constructed giving the potential variation over the high resistance film for different values of the surface resistivity r. Such a family of curves appears in figure 2.

Parallel Stream Line Flow of Charging Current in High Resistance Films. Instances arise in practice in which the presence of corona at the peripheries of capacitor plates is undesirable. In many such instances the potential gradients in the vicinity of the edges of a highly conducting plate can be reduced to values lower than the critical value for corona by applying a high resistance border strip as illustrated in figure 3. The charging current which flows through the dielectric between high resistance strips causes a resistance drop over the strips thus leading

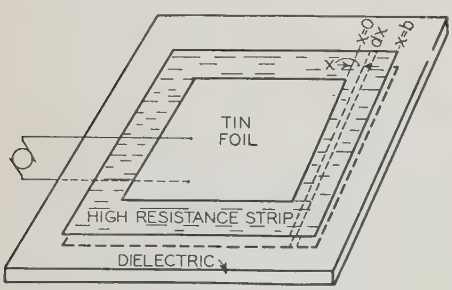


Fig. 3. Parallel plate capacitor

to lower potential gradients around the periphery of the modified capacitor plate than would obtain otherwise without the strips.

In performing puncture tests on metal sheathed cable, some expedient usually is employed to prevent abnormally high potential gradients at the edge of the sheath. The principle of obtaining controlled potential distribution by means of a resistance drop in a high resistance film may be used to considerable advantage in this practical instance. Figure 4 illustrates how the effect may be obtained with high resistance paint or tape applied over the cable dielectric beginning at the edge of the sheath.

The differential equations that approximately govern the flow of current and potential distribution over high resistance films applied to a cable test specimen or to a parallel plate capacitor as shown in figures 3 and 4 are linear and possess a solution expressible in terms of exponential and trigonometric functions. This is true also for numerous other applications that are characterized by parallel stream line flow of current in the high resistance film.

If a sinusoidal voltage be impressed across the parallel plate capacitor or cable specimen of figures 3 and 4, respectively, and if

x represents the distance from the edge of the metal to any zone of width dx

i represents the instantaneous value of the current in the x direction per unit width of the zone dx

e represents the potential of the zone dx relative to the corresponding zone on the opposite side of the dielectric

C represents the capacitance per square centimeter of coating

r represents the resistance measured between opposite sides of a centimeter square of high resistance film

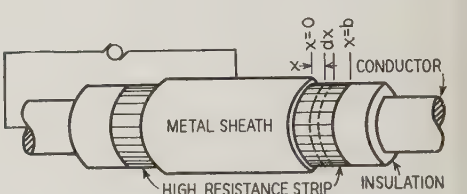
then the differential equations for the film are

$$\frac{\partial i}{\partial x} = -C\frac{\partial e}{\partial t} \tag{11}$$

$$\frac{\partial e}{\partial x} = -Ri \tag{12}$$

For the parallel plate capacitor of figure 3, having resistance strips on both faces, the resistance R is

Fig. 4. Cable test specimen



equal to $2r$; for the cable illustrated in figure 4, the resistance R is equal to r.

Performing the following substitutions for the sinusoidal current and potential difference in the exponential forms,

$$\begin{aligned} i &= I \exp j\omega t \\ e &= E \exp j\omega t \end{aligned} \tag{13}$$

the following equations result

$$\frac{\partial I}{\partial x} = -jC\omega E \tag{14}$$

$$\frac{\partial E}{\partial x} = -RI \tag{15}$$

Eliminating I between equations 14 and 15 yields

$$\frac{\partial^2 E}{\partial x^2} - jRC\omega E = 0 \tag{16}$$

1. See reference at end of paper.

The solution of this linear differential equation is

$$E = E_1 \epsilon^{-kx} (\cos kx - j \sin kx) + E_2 \epsilon^{kx} (\cos kx + j \sin kx) \quad (17)$$

in which

$$k = + \sqrt{\frac{R C \omega}{2}} \quad (18)$$

Upon evaluating the integration constants E_1 and E_2 to satisfy the boundary conditions, namely, $I = 0$ at $x = b$ (the distance to the edge of the coating) and $E = E_0$ (the impressed voltage) at $x = 0$, it is found that

$$E_1 = \frac{E_0 \epsilon^{kb} (\cos kb + j \sin kb)}{2 (\cosh kb \cos kb + j \sinh kb \sin kb)} \quad (19)$$

and

$$E_2 = \frac{E_0 \epsilon^{-kb} (\cos kb - j \sin kb)}{2 (\cosh kb \cos kb + j \sinh kb \sin kb)} \quad (20)$$

Numerical values pertaining to any example to which the differential equations 11 and 12 apply may be substituted in equation 17 for E, and curves may be drawn for the potential distribution over the film.

Application of a High Resistance Film to the Pin Type of Insulator

The method of controlling the potential distribution over the surface of a dielectric by utilizing the resistance drop resulting from the charging current in a high resistance surface film may constitute a practical solution of the problem of eliminating radio interference from the pin type of insulator. As already pointed out, radio interference is caused by corona at the tie wire and pin hole, or on some special insulators, at the edge of a metallic flux distributor. In some instances, interference at low line voltages may arise also from corona in constricted regions between shells or over cement junctions, but it has been demonstrated by several manufacturers that these sources of radio interference can be eliminated by one piece construction and attention to proper geometrical configuration if the operating voltage does not exceed the rated voltage appreciably.

In order to obtain experimental confirmation of the effectiveness of the principle when applied to a pin insulator, a 50-kv one-piece commercial insulator without metal cap or thimble was selected as a model to receive coatings of different resistivity and diameter on the head and complete coating in the pin hole. The procedure followed in determining the lowest voltage giving rise to radio interference was to reduce the applied insulator voltage, beginning with a high value, until no interference could be detected by a superheterodyne radio receiver placed 6 feet from the insulator. The results of these tests are shown in figure 5. Radio interference measurements were not extended above a line voltage of 66 kv (16 kv above the manufacturer's rating), at which voltage corona appeared between the shells of the insulator. For all other tests, corona made its first appearance at the periphery of the coating.

It should be noticed especially that the low resistance coatings ($r < 10^2$ ohms per centimeter square) offered relatively little protection against radio interference and that prohibitive reduction in the flashover voltage occurred whenever the diameter of this coating was large enough to afford any appreciable advantage over the simple uncoated insulator. However, figure 5 depicts also the unique property of the high resistance coating of reducing the potential at the coating periphery to the extent that no radio interference existed in the voltage range 0–66 kv with coatings of large diameter. Furthermore, it is important to note that there was no material decrease in flashover voltage of the insulator.

Diameter of tie wire groove, 10 centimeters; diameter of top shell, 28 centimeters

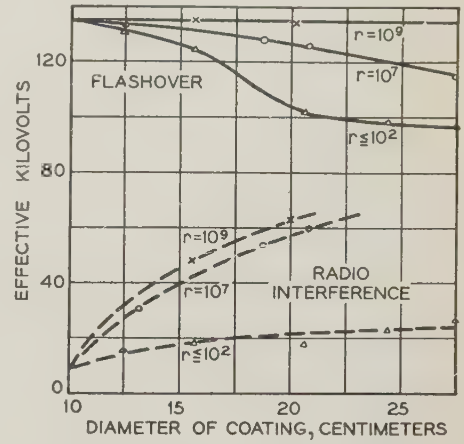


Fig. 5. Dry flashover voltage and lowest voltage causing radio interference from a 50 kv pin insulator versus diameter of conducting coating on the head for 3 coating resistivities (in ohms per centimeter square)

It may be observed also that a latitude of at least 100 fold in the value of coating resistance on the head and in the pin hole is permissible without leading to radio interference. The upper limit for resistance will be attained when the potential gradient near the tie wire becomes large enough to cause corona and a lower limit will be set when the reduction of potential at the periphery of the coating is not sufficient to prevent corona.

A suitable manufacturing process for imparting the proper resistance to the head and pin hole of the pin insulator has not been devised. Temporary films were employed successfully for experimental purposes in the laboratory, but it is evident that a coating intended for service in the field must be permanent under the influence of a variety of destructive agents. The best method of imparting the resistance to the proper surfaces of the porcelain insulator would appear to be the use of a surface glaze possessing the desired resistivity. If this plan be followed, the resistance of the glaze in the line and tie wire grooves should be much less than that required on the head in order to avoid poor tie wire contacts and high potential drops in the region surrounding these points of contact.

Reference

1. Tables of Integrals and Other Mathematical Data (a book), Dwight. The Macmillan Co., N. Y., 1934.

Pyrochemical Behavior of Cellulose Insulation

Tests on unbleached unsized linen paper of high purity indicate that the physical defects implied by the various pyroelectric theories of insulation failure appear secondary to, and manifested as a result of, the chemical changes involved. These chemical changes result in both increased power factor and decreased dielectric strength of the insulation. The investigation shows that chemical results of overheating of cellulose insulation in service may be disastrous even within a range of temperatures not generally associated with dangerous physical effects.

By
F. M. CLARK
ASSOCIATE A.I.E.E.

General Electric Co.,
Pittsfield, Mass.

HEAT DISSIPATION has long been recognized to be one of the most important factors upon which the successful operation of commercial dielectrics depends. Cooling surfaces must be so arranged that excessive heating is avoided. The general acceptance of the various pyroelectric theories of insulation breakdown[1,5] has reflected the universal importance associated with the thermal characteristic. The physical effects of cumulative heating have occupied the attention to such an extent, however, that in many instances the chemical changes produced have been overlooked. Unlike the physical results, the chemical effects of cumulative heating rarely, if ever, are recognized as being directly connected with the dielectric failure of insulation. The chemical results of overheating may be disastrous even within a range of temperatures not generally associated with dangerous physical effects. The insidious chemical action is of slow growth and may be traced only with difficulty. In its final stages, dielectric deterioration from chemical causes becomes physically manifest in accordance with the usual pyroelectric phenomena. The purpose of this paper is to trace the chemical change in cellulose insulation directly produced from thermal causes, the occurrence of which seriously may affect the successful use of the insulation as part of electrical design.

A paper recommended for publication by the A.I.E.E. committee on research, and scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted June 6, 1935; released for publication Aug. 20, 1935.

1. For all numbered references, see list at end of paper.

As illustrative of cellulose insulation, unbleached unsized linen paper of high purity was used in this investigation. It is recognized that chemical reactions are affected greatly by contaminating materials. As such, ligneous matter in other types of paper as well as sizing or other fillers might well be expected to produce variations in chemical stability. Unbleached linen paper was selected as being representative of a commercial type of cellulose insulation substantially free from such secondary effects. The paper used was 0.0005 inch thick.

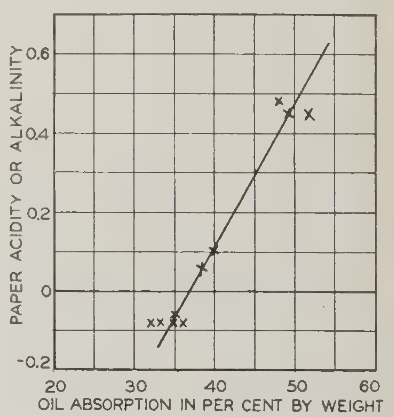
From the results obtained it is concluded that for the successful application of cellulose insulation as an integral part of electrical machine design, proper consideration of thermal phenomena is fundamentally essential. Important though they may be, the physical effects implied by the various pyroelectric theories of insulation failure appear secondary to, and manifested as a result of, the chemical changes involved. Cellulose insulation, as represented by unbleached unsized linen paper, deteriorates mechanically and electrically as a result of thermally produced chemical changes clearly evident at temperatures higher than 100 degrees centigrade even when of relatively short duration. These chemical changes not only produce increased power factor but are accompanied by gas formation with subsequent decrease in dielectric strength. Even for lower temperatures not normally associated with short time chemical change, cellulose insulation may deteriorate with consequent decrease in electrical characteristics if high potential stress be applied concurrently.

Removal of Moisture From Cellulose

Mechanical as well as electrical properties of cellulose insulation are associated with its water content, which is present both as an absorbed sur-

Fig. 1. Relation between oil absorption by paper and thermally produced chemical change gauged by means of the acidity value

This figure is based upon data of tables I and II. For acidity test, see footnote (*) under table I



face film and as an integral part of the chemical structure. The drying of the material in preparation for its use as a dielectric involves the removal of the absorbed water film. Like the removal of all films, the separation of the absorbed moisture presents increasing difficulty as the drying progresses. Elimination of the final traces merges into the removal of the molecularly associated water. The exposure of cellulose to progressively higher and

Table I—Chemical Change in Unbleached Linen Paper Produced by High Temperature in Air, as a Function of the Method of Exposure

	Properties of Paper After Treatment							
	In Loosely Wound Rolls				In Tightly Wound Rolls			
Treatment	Color	Acidity*	Oil Absorption**	Width, Inches	Color	Acidity*	Oil Absorption**	Width, Inches
None	White	−0.08	32	2	White	−0.08	32	8 7/16
110 deg C, 48 hr	White	−0.08	36	1.9	White	−0.08	35	8 1/8
150 deg C, 27 hr	Yellow	+0.10	40	1.8	Brown	+0.44	48	8 1/8
150 deg C, 48 hr	Yellow	+0.52	45	1.8	Deep brown	†	†	8

* Acidity is expressed as positive or negative. A negative value indicates the milligrams of sulphuric acid necessary to neutralize one gram of paper. A positive value indicates the milligrams of sodium hydroxide necessary to neutralize one gram of paper.
** Oil absorption is the amount of oil retained expressed in per cent by weight of the original (dry) paper.
† Deterioration was so great that reliable acidity and oil absorption values could not be obtained.

higher temperatures leads to more and more moisture removal until, at a sufficiently high temperature, chemical degradation clearly is observed. The problem of satisfactory drying of cellulose insulation thus becomes one of distinguishing between those conditions necessary to obtain the best dielectric and mechanical properties and those accompanied by chemical disintegration. The problem becomes increasingly difficult when it is recognized that chemical changes may result from prolonged exposure to almost any temperature. Thus when heated in air, cellulose first will decrease in weight with loss of water to an apparently stable value. Further heating, however, eventually will produce a slowly increasing weight[3] with accompanying changes in chemical acidity and color, and mechanical and electrical deterioration. Factory drying of insulation preparatory for use as an integral part of electrical machines clearly recognizes this behavior. The treatment is carried out under conditions carefully eliminating the second stage. Service use of the insulation, however, knows no such limitations and is favored only by the characteristically lower temperature encountered, with resultant slower chemical change in the cellulose.

The Chemical Change Is Accelerated by the Products Formed

There is one important factor that necessarily accompanies the use of cellulose as a dielectric in electrical design: The products of chemical change are not easily removed. Even if gaseous or liquid, the products of decomposition usually are adsorbed

Table II—Chemical Change in Unbleached Linen Paper When Subjected to High Temperature and Vacuum in the Form of Tightly Wound Rolls

	Properties of Paper After Treatment			
Treatment	Color	Acidity*	Oil Absorption*	Width, Inches
None	White	−0.08	33	8 7/16
110 deg C, 48 hr	White	−0.06	35	8 1/4
150 deg C, 27 hr	Yellow	+0.08	38	8 1/8
150 deg C, 48 hr	Yellow	+0.49	45	8 1/8

* See footnotes under table I.

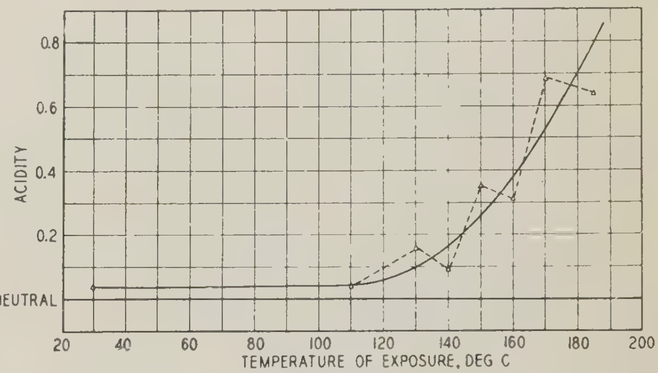


Fig. 2. Acidity change in paper resulting from high temperature treatment

Heating period—48 hours under a pressure of 2 millimeters of mercury
Test samples—Tightly wound rolls as described for table I
For acidity test, see footnote (*) under table I

or at least remain in more or less direct contact with the material from which they are formed. It is not a rarity that chemical reactions of this type are autoaccelerated. That such effects arising from cellulose degradation cannot be ignored is illustrated in the data of table I, which describes the behavior of unbleached linen paper as a function of the method by which it was exposed to high temperature. To obtain the best thermal distribution, the paper was heated with a sheet of aluminum foil between each 4 mil pad. In one group of tests the paper was in the form of loosely wound rolls about 2 inches wide, in order to allow easy volatilization of the products of degradation. In the other group the paper was in the form of tightly wound rolls about 8 inches wide in order to present greater difficulty for the volatilization of the reaction products. In each roll the paper was 50 feet long. The chemical change produced in the paper from such treatment is accelerated in those rolls presenting the greater possibility for the retention of the products resulting from the chemical change in the cellulose.

To illustrate further the retardation produced by treatment that removes the chemically formed products of degradation from contact with the cellulose, rolls similar in construction to the tightly wound samples of table I were heated in a vacuum of approximately 2 millimeters of mercury. The re-

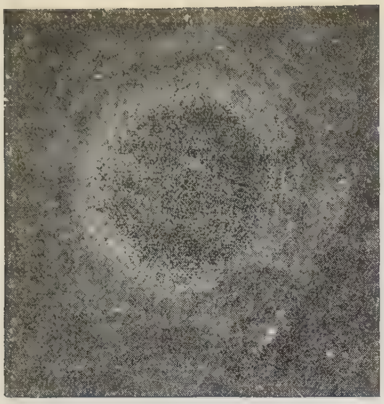


Fig. 3. Mechanical deterioration in paper resulting from ionization effects

This paper was tested at 25 degrees centigrade between sheets of mica in an atmosphere of hydrogen gas. The voltage applied was sufficiently high to cause corona streamers across the mica surface

sults obtained are practically duplicates of the data presented in table I for the loosely wound rolls of similar paper. Typical data are given in table II.

In tables I and II data concerning the oil absorptive properties of the paper as a function of its treatment are tabulated. It is significant that for those papers in which chemical change has been produced as evidenced by the changed acidity, a corresponding change in oil absorption value also is obtained. The relation between the chemical change in the paper (acidity) and the oil absorption is illustrated in figure 1.

Acidity Change as a Function of Temperature

In the data of tables I and II, acidity increase is not pronounced for temperature treatment at 110 degrees centigrade, irrespective of the method of test. At temperatures higher than 110 degrees, even when heated in a vacuum of 2 millimeters of mercury, decomposition of the paper results, as evidenced by increased acidity; figure 2 illustrates this effect. The data of figure 2 are based upon changes that occurred when tightly wound rolls, similar to those already described in connection with table I, were heated for 48 hours under a vacuum of 2 millimeters of mercury. Even for such short thermal exposures under the most favorable conditions for stability, a pronounced deterioration of the paper occurs when temperatures higher than 110 degrees centigrade are used. Longer periods of treatment at lower temperatures must be expected to produce similar chemical changes.

Dielectric Deterioration Accompanying Paper Degradation

Increased electrical losses accompany the chemical changes produced by high temperature exposure of paper. The relation is shown in figure 9 for unbleached oil-treated linen paper. A condition dangerous to the successful commercial operation of cellulose dielectrics arises where chemical change of this type is allowed to continue unchecked. A more immediately dangerous condition, however, would result were noncondensable gases also evolved from the insulation. The increased power factor of the insulation under such conditions would be accompanied by ionization effects under high potential. Figure 3 shows the disintegration of unbleached

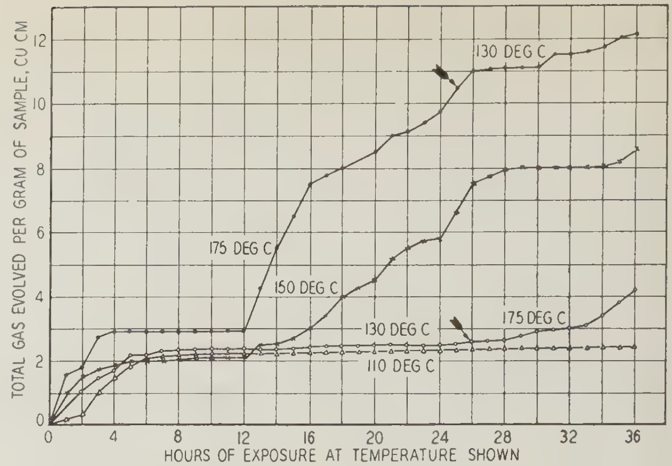


Fig. 4. Gas evolution from paper produced as a result of thermal decomposition under vacuum

Test samples—Tightly wound rolls as described for table I
Test conditions—Paper heated as shown under a pressure of 2 millimeters of mercury
The arrows appearing on the 175 and the 130 degree curves indicate a temperature change from 175 to 130 and from 130 to 175 degrees as illustrated

linen paper produced by high voltage ionization. The paper illustrated was placed between sheets of mica in an atmosphere of pure hydrogen in order to eliminate carbonization and oxidation of the cellulose under corona discharge. The deterioration shown in figure 3 is typical of ionization effects arrested before complete disintegration of the cellulose. If allowed to run its full course, the cellulose sheet is completely disintegrated with the production of a finely divided dust. In figure 3, the disintegration is plainly under way as is witnessed by the screen-like structure which distinguishes the area directly between the electrodes.

That noncondensable gases are evolved from unbleached linen paper when exposed to high temperatures is illustrated in figure 4. These data were obtained with tightly wound rolls similar in con-

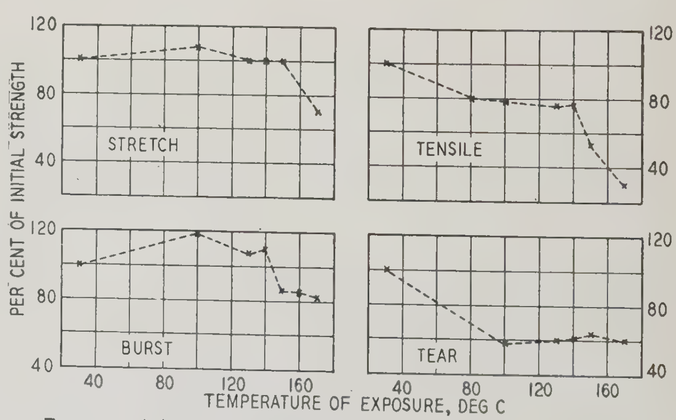


Fig. 5. Mechanical deterioration of paper resulting from high temperature treatment

Mechanical strength was measured after 48 hours' heat treatment at the temperature shown. All data shown here were taken at 20–25 degrees centigrade, during the same day under prevailing atmospheric humidity. (40 per cent relative humidity at 68 degrees Fahrenheit)

struction to those of table I and suspended in a vacuum chamber under a pressure of 2 millimeters of mercury. Means were provided for collecting and measuring the reaction products normally gaseous at 25 degrees centigrade and 760 millimeter pressure. As illustrated in figure 4, irrespective of the testing temperature, the first rush of gas, which was largely air with some water vapor, was succeeded by a quiescent period the duration of which was a definite function of the experimental conditions. With temperatures maintained at 110 or 130 degrees centigrade, no further noncondensable gases were evolved within the limits of the experiment. With higher temperatures, gassing again started after a total heating period of about 12 hours.

The data of figure 4 are important from another viewpoint. In commercial practice, it is often necessary to decide whether insulation once subjected to overheating, is fit for further use. Gauged from the standpoint of gassing tendencies, one would conclude that cellulose insulation once severely overheated is not suited for further successful use. Thus in figure 4, the cellulose when actively "gassing" in the final stage of the 175 degree test showed but little change when the temperature was decreased to the normally safe value of 130 degrees. Contrariwise, the insulation at 130 degrees, as illustrated, showed a prompt response in gassing when the temperature was increased to 175 degrees.

Mechanical Deterioration Accompanying Chemical Change

The mechanical strength of paper, so necessary in some of its dielectric applications, decreases with extended drying. When chemical deterioration of the cellulose is produced, the mechanical strength decreases at an increasing rate. This is illustrated in figure 5. A more definite picture of the decreased mechanical strength accompanying the short-time high-temperature treatment of the cellulose is given in figure 6. Here the mechanical properties of figure 5 are given equal weight and averaged. Taking the normal mechanical strength of the undried paper at 25 degrees centigrade as 100 per cent, it is evident that drying, even within the limits of temperature not conducive to marked acidity or dielectric deterioration, produces decreased mechanical strength. As the temperature rises above 130–140 degrees, the drop in "average" mechanical strength may assume dangerous proportions. It is obvious that longer periods of heating at lower temperatures cannot be ignored in the application of cellulose insulation where the maintenance of good mechanical properties is paramount.

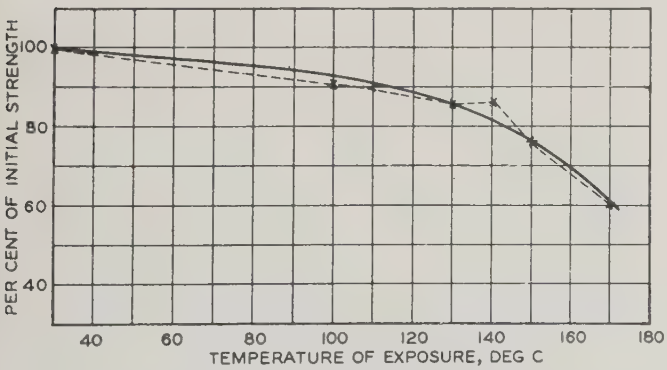


Fig. 6. "Average" change in mechanical properties of paper produced by high temperature treatment

Data of this figure are based upon the experimental results given in figure 5, each point being the average obtained for the deterioration as represented by the tensile, stretch, tear, and bursting strengths, each test being given equal weight in determining the average value

Behavior of Oil Treated Paper

In general the technical procedure used in the investigation of oil treated cellulose insulation duplicated that applied to the study of the insulation not oil treated. It may be observed that the results obtained bear strong resemblance, differing merely in degree. Thus figure 7 shows the changing paper acidity produced by thermal decomposition of oil-treated unbleached linen paper heated under conditions similar to those applying to the untreated paper, the acidity change in which is described in figure 2. The presence of oil merely affects the degree of change. In figure 7 chemical degradation of the cellulose is evident from temperatures higher than 100 degrees centigrade, as is true for the untreated paper of figure 2; but whereas untreated paper shows pronounced acidity change for the higher temperature, figure 7 shows a more slowly increasing acidity. This difference might well be expected, since the cellulose decomposition products are in part at least oil-soluble and are thus difficult to evaluate.

To illustrate the gassing of oil-immersed oil-treated cellulose is difficult because of volatilization of materials derived from the oil and the oil-solubility of the cellulose formed gases. The behavior is illustrated roughly in figure 8, resort being made to a gas bubble counter to express the rate of gas evolution under a pressure of 2 millimeters of mercury. The gases arising in the heated vacuum chamber containing the oil-treated oil-immersed cellulose, were led through cooling towers into the bubble counter. Under a steady vacuum the number of bubbles

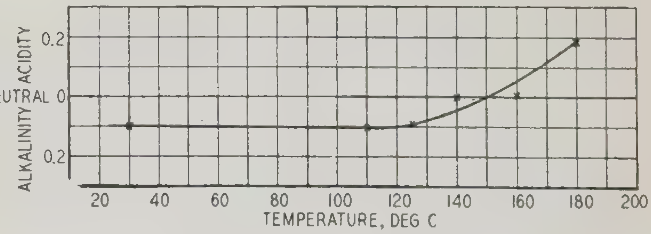


Fig. 7. Acidity change in oil immersed paper resulting from high temperature treatment

Heating period—48 hours under a pressure of 2 millimeters of mercury
Test sample—Tightly wound rolls as described for table I, vacuum oil-impregnated at 50 degrees centigrade
For acidity test, see footnote (*) under table I

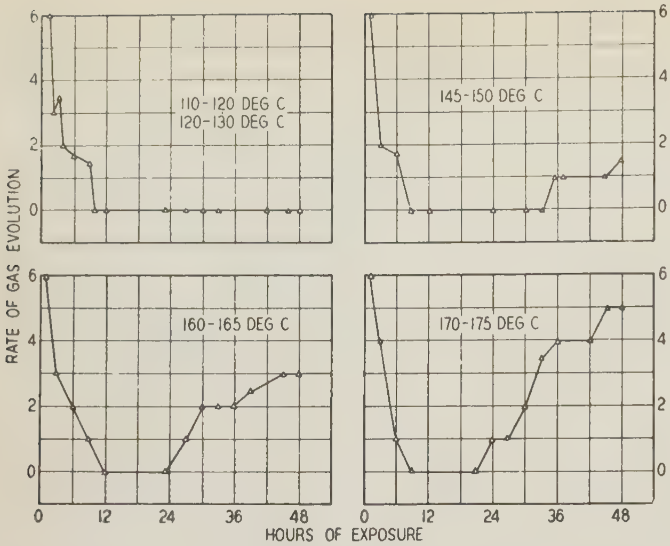


Fig. 8. Gas evolution from oil immersed paper produced as a result of thermal decomposition under vacuum

Test sample—Tightly wound rolls as described for table I
Test conditions—Paper heated as shown under a pressure of 2 millimeters of mercury
Rate of gas evolution expressed as:

0. No gas evolved in bubble counter
1. Occasional bubbles noted in bubble counter
2. Approximately 1 bubble each 3 minutes in bubble counter
3. 3 to 5 bubbles each 3 minutes in bubble counter
4. 5 to 20 bubbles each 3 minutes in bubble counter
5. 20 to 40 bubbles each 3 minutes in bubble counter
6. More than 40 bubbles each 3 minutes in bubble counter

formed per minute from the 2 millimeter gas tube of the bubble counter was determined. A blank correction is applied for the expression of the behavior illustrated in figure 8. For temperatures up to 130 degrees centigrade, no gaseous decomposition of the cellulose was observed within the limits of the experiment. For higher temperatures, a second stage of gassing occurred, indicative of cellulose degradation. The duration of the quiescent period following the first rush of gas (air) is a function of the testing temperature, cellulose decomposition being evidenced by gas formation after an interval that is shorter, the higher the temperature applied. Thus at 145 degrees the second gassing stage began after 33 hours of heating, at 160 degrees after 23 hours, and at 170 degrees after 19 hours of heating. As might be anticipated, the rate of gas formation during the second or cellulose decomposition stage was more pronounced the higher the testing temperature.

The use of cellulose insulation even for short periods of time at temperatures higher than 130 degrees centigrade must be strictly avoided if increased acidity and gas formation are to be prevented.

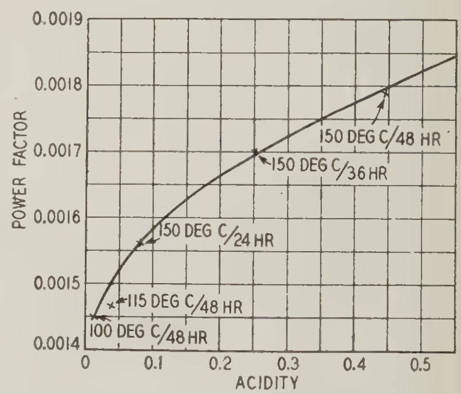
Power Factor of Oil Treated Cellulose

Thermally produced chemical changes resulting in increased acidity is accompanied by pronounced increases in power factor; this is shown in figure 9. Unbleached linen paper was exposed for various periods at selected high temperatures under a pressure of 2 millimeters of mercury and later vacuum-oil-impregnated at 100 degrees centigrade. The effect of the thermal history is clearly evident in the power factor and acidity, which both increase with the severity of the previous exposure.

The service use of oil-treated cellulosic insulation must always be made with clear recognition of the power factor-temperature relation (figure 10). Increased temperature may produce a rapidly rising power factor for temperatures higher than 40 degrees centigrade. Such rapidly rising power factor promotes cumulative heating, which in turn may produce disastrous results because of the resulting thermal degradation of the cellulose itself. Cellulose insulation can be operated at temperatures higher than 50 degrees centigrade under conditions

Fig. 9. Effect of thermally produced chemical change on the power factor of oil treated paper

Power factor measured at 25 degrees centigrade, 1,000 cycles. For acidity test, see footnote (*) under table I



Samples were heated as shown under pressure of 2 millimeters of mercury. Temperature then was lowered to 100 degrees centigrade and the samples oil-treated while still maintained under 2 millimeter pressure. After impregnation, the temperature was lowered to 25 degrees for power factor measurement at atmospheric pressure

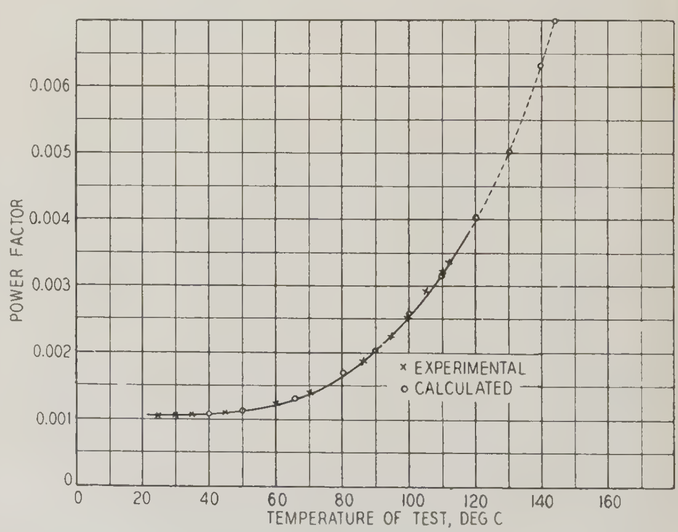


Fig. 10. Relation between power factor and temperature for paper vacuum-dried and oil-treated at 100 degrees centigrade

Calculated values are based upon the logarithmic relation between power factor and temperature of test. Power factor was measured at 1,000 cycles

that reduce to a minimum the possibility of "spot" thermal concentration.

Dielectric Strength of Oil Treated Cellulose

The short-time dielectric strength of oil treated cellulose insulation is recognized to involve a minimum of cumulative heating effects. The fact that increase in testing temperature, entirely aside from chemical degradation, may result in decreased breakdown strength in accordance with figure 11, as well as in increased power factor, serves to limit the maximum permissible temperature of electrical apparatus dependent on impregnated paper or like material for its successful operation.

Chemical change in oil treated cellulose produces decreased dielectric strength as well as disadvantageous effects arising from other mechanical or electrical causes. Figure 12 presents test results obtained on oil-treated unbleached linen paper dried and oil-impregnated under a vacuum of better than 2 millimeters at 100 degrees centigrade for 48 hours, a condition that has been shown to give little, if any, chemical or electrical deterioration. Subsequent to drying and oil-impregnation the dielectric pads were sealed under degassed oil in nonbreathing containers. The assemblies were heated at temperatures from 80 to 150 degrees centigrade for periods up to 170 hours, whereupon the minute-test dielectric strength at 25 degrees centigrade was determined without removal from the sealed container. The dielectric strength for the 80 degree aging test was not affected. For temperatures of 115 degrees or higher, the dielectric strength seriously was affected. At the end of about 100 hours, the breakdown strength of the insulation was only approximately 71 per cent of the initial value for the 115 degree treatment, 68 per cent for the 130 degree treatment, and 57 per cent for the 150 degree treatment.

"Life" Test Characteristics

It is difficult to interpret "life" test information concerning the behavior of insulation in terms of

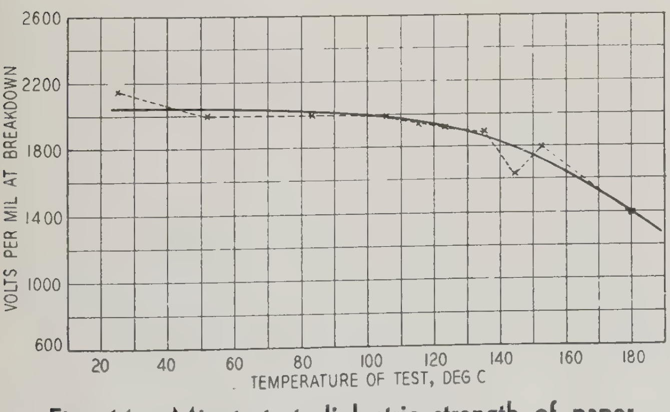


Fig. 11. Minute-test dielectric strength of paper vacuum-dried and oil-treated at 100 degrees centigrade, as a function of the testing temperature

Thickness of dielectric—0.004 inch

commercial practice. Because of time limitations, conditions necessarily must be exaggerated with respect to one or more factors. It is usual in work of this type to exaggerate the temperature of the test, the voltage applied, or both. Because of the evi-

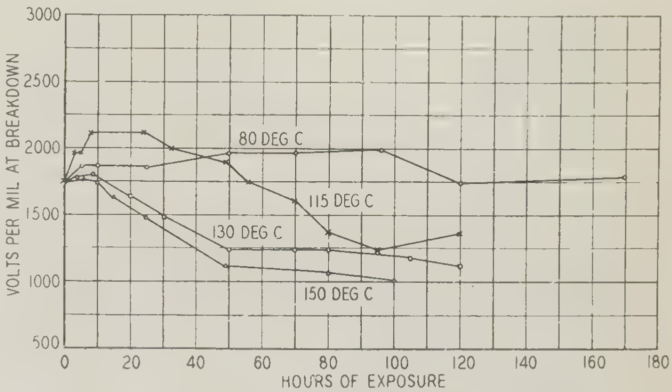


Fig. 12. Decrease in the minute-test dielectric strength at 25 degrees centigrade of vacuum-dried and oil-treated paper resulting from thermally produced chemical change

Thickness of dielectric tested—0.004 inch
Treatment of insulation—Vacuum dried at 100 degrees centigrade for 48 hours and mineral oil impregnated under vacuum; then aged at temperature shown while immersed in degassed oil in sealed containers

dent danger from thermal decomposition of the cellulose at temperatures higher than 115 degrees centigrade, ambient temperatures of 25 and 95 to 100 degrees were chosen in the life test analysis of this investigation. Each life test was carried out in sealed containers to protect from atmospheric oxidation and the loss of thermally formed volatile products. Figures 13 and 14 illustrate typical results using unbleached linen paper of high purity as the cellulose insulation.

In figure 13, 0.0025 inch pads of unbleached oil-treated linen paper were used. Such pads normally have an average minute-test breakdown strength of approximately 2,600 volts per mil. A "life" test potential of 3,300 volts, 60 cycles, giving 1,280 volts per mil, was used to determine the long-time 25-degree dielectric stability of the insulation as a function of its previous thermal history. This electrical stress was chosen since experience had shown that a materially lower potential gradient normally would not produce dielectric failure of the insulation within the period of 100 days assigned for the test interval. At 1,280 volts per mil, previous experience had shown that at least 50 per cent of the samples would fail even when treated in accordance with the best procedure. A comparison of curve *A* with curve *B* of figure 13 shows the effect of the chemical degradation of cellulose insulation in affecting its long time dielectric strength. Curve *A* is based upon insulation heated at 115 degrees centigrade under vacuum for 3 days. Curve *B* is based upon similar insulation heated at 135 degrees under vacuum for 3 days. Subsequently to such treatment, both types of insulation were vacuum oil-treated at 100 degrees.

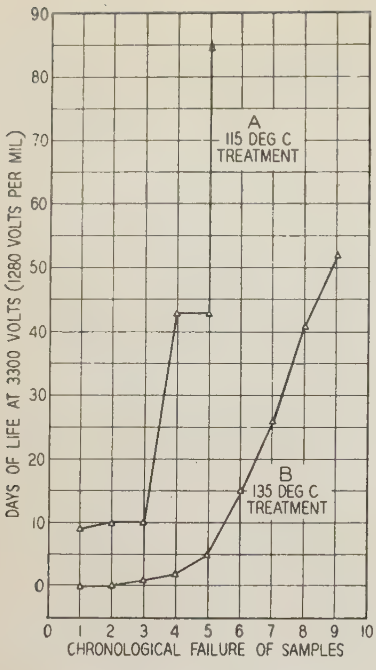


Fig. 13. Effect of thermal treatment on the 60-cycle "life" test dielectric strength of oil-treated paper

Thickness of dielectric pad—0.0025 inch

Life test conditions—Ambient 25 degrees centigrade 1,280 volts per mil

Insulation was dried under vacuum for 72 hours at temperature shown, followed by oil impregnation while still under vacuum at 100 degrees centigrade

The characteristics of the paper after the vacuum temperature treatment were:

	Curve A	Curve B
Paper acidity (See footnote* under table I)	+0.05	+0.19
Power factor (25 degrees centigrade —1,000 cycles)	0.0015	0.0020

The breakdown values of figure 13 are plotted chronologically. The insulation of curve *A* showed only 5 breakdowns out of 9 samples on test. The insulation of curve *B* showed 9 breakdowns out of 9 samples on test. Considering only the first 5 samples of each group, the average breakdown period of Curve *A* is 23 days; that for curve *B* only 1.6 days. In curve *B* a zero value for time to breakdown indicates failure within 1 hour on test. The chemical degradation of the cellulose insulation exhibited in an increasing power factor and acidity is therefore not without marked effect on the long-time dielectric strength of the insulation.

Even without visible chemical or short time electrical affect, the application of high temperature to oil-treated cellulose insulation may cause slow deterioration of a cumulative type capable of producing diastrous results. Thus it has been shown that short time applications of temperature lower than 110 degrees centigrade normally do not produce chemical change as determined by the acidity or electrical change as evidenced by the dielectric strength or initial power factor. Figure 14, however, clearly shows that temperatures as low as 95 to 100 degrees may produce electrical deterioration if coupled with simultaneously applied high potential stress. In figure 14 the insulation consists of unbleached linen paper of high purity similar to that used throughout this investigation. The paper was vacuum-dried for 48 hours at 100 degrees centigrade and oil-impregnated. The arrangement of the insulation was in the form of small, flat pads. Two groups were prepared, one containing 0.008 inch pads of dielectric and the other group 0.004 inch pads. Following the drying and oil impregnation, the dielectric assemblies were placed on test at 95 to 100 degrees centigrade under a potential of 3,250 volts, 60 cycles in sealed containers to protect from atmospheric oxidation and the loss of thermally formed volatile products. The slowly increasing power factor of the 0.008 inch samples operating at 425 volts per mil, indicative of chemical and electrical deterioration, is accelerated by the higher voltage stress (850 volts per mil) of the 0.004 inch insulation. The data of figure 14 represent typical group behavior, each testing group consisting of 6 individual samples. No electrical failures occurred during the duration of tests covered by investigation of this type.

References

1. The Physical Nature of the Electrical Breakdown of Solid Dielectrics, K. W. Wagner. A.I.E.E. Jl., v. 41, April 1922, p. 288–98.

2. The Use of Salts to Produce a Fixed Humidity, Julius Obermiller and M. Goertz. *Z. Physik. Chem.*, v. 109, May 1924, p. 145–64.

3. The Influence of Temperature in the Drying of Wood Pulp, G. K. Davis. *Tech. Proc.*, Paper Makers Assn. of Great Britain and Ireland, v. 4, 1934, p. 1.

4. Solid Insulation Breakdown, W. Rogowski. *Arch. F. Elektrot.*, v. 13, May 1924, p. 153–74.

5. Insulation Failure—A Pyroelectric Effect, J. L. R. Hayden and C. P. Steinmetz. *Elec. Wld.*, v. 80, Oct. 21, 1922, p. 865–8.

6. The Pyrogenic Decomposition of Cellulose, G. Bantlin. *J. Gasbel*, v. 57, p. 32.

7. The Discontinuity of the Drying Process, E. A. Fisher. *Proc.*, Roy. Soc. (London), v. 105, series A, 1924, p. 571–82.

8. The Action of Heat on Cotton Cellulose, Knecht and Miller. *Jl.*, Soc. Dyers and Colourists, v. 41, 1925, p. 43.

9. The Effect of Temperature on Deterioration of Fibrous Insulation, Kujirai and Akahira. *Bul. 21*, Inst. of Phys. and Chem. Research (Tokyo).

10. The Moisture Content of Cereals, O. H. Nelson and G. A. Hulett. *Jl.*, Ind. and Engg. Chem., v. 12, Jan. 1, 1920, p. 40–5.

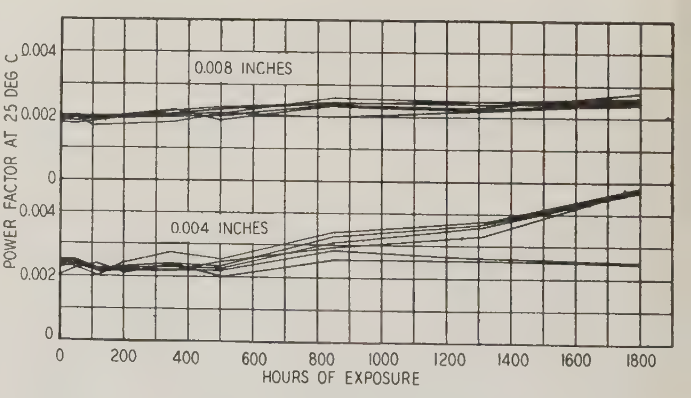


Fig. 14. Dielectric instability produced in oil-treated paper as a result of 60-cycle high-voltage application at a temperature not normally associated with short time chemical deterioration

Life test conditions—95 to 100 degrees centigrade in sealed containers, 3,250 volts applied

Treatment of insulation—Vacuum dried at 100 degrees centigrade for 48 hours, followed by oil impregnation while still under vacuum and temperature

Power factor was measured at 25 degrees centigrade, 1,000 cycles

A Cathode Ray Oscillograph for Observing 2 Waves

A general purpose cathode ray oscillograph is described in this paper which embodies a cathode ray tube of high beam efficiency, a description of which has not hitherto been published. This tube makes it possible to use a high frequency oscillator for supplying both the beam and sweep voltages. An electronic switching circuit is described which greatly extends the usefulness of the cathode ray oscillograph by making it possible to portray simultaneously 2 or more different waves in their proper phase relation, provided both waves have frequencies which are in a simple multiple relationship to each other.

By
R. H. GEORGE
ASSOCIATE A.I.E.E.
Purdue University, West Lafayette, Ind.

H. J. HEIM
ASSOCIATE A.I.E.E.
Purdue University, West Lafayette, nd.

H. F. MAYER*
NONMEMBER A.I.E.E.
Purdue University West Lafayette, Ind.

C. S. ROYS
ASSOCIATE A.I.E.E.
Purdue University, West Lafayette, Ind.

As a result of the development of a glass type cathode ray tube[1] and a system of cathode ray television reception at Purdue University, a number of new tubes and auxiliary circuits were made available which have proved very useful in the development of cathode ray oscillograph equipment. The field of application of the cathode ray oscillograph has also been greatly extended by the development of a stable electronic switching circuit for the simultaneous observation of 2 different wave forms.[2]

Five portable oscillographs have been constructed, 4 of which were developed for special industrial applications, and 1 for lecture room demonstration and general laboratory use at the university. The special oscillographs, however, are not described in the present paper, which is concerned principally with the description of the cathode ray tube, the high voltage power supply, and the electronic switching circuit as embodied in the general purpose oscillograph.

General Description of Oscillograph

The general purpose oscillograph, figure 1, is built as a self-contained unit for operation from a 110-volt 60-cycle power source. The complete schematic wiring diagram is represented by figure 2. On the left of the diagram, the parts, in order from top to bottom are: power and filament transformers, low voltage d-c power supply, synchronizing and sweep circuits, combination amplifier and timing oscillator, and full-wave high-frequency rectifier for modulation measurements. On the right of the diagram in the same order are: the sweep voltage and high voltage d-c power supply, the cathode ray tube, and the electronic switching circuit.

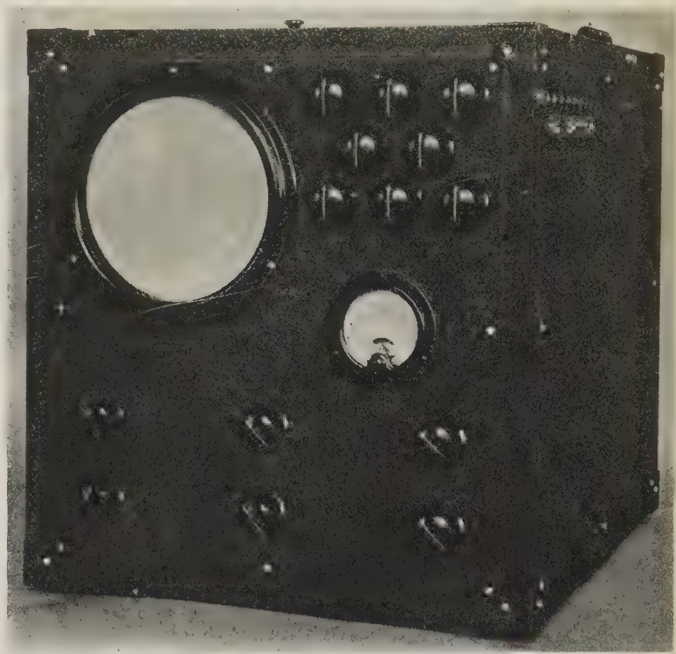


Fig. 1. Front view of the cathode ray oscillograph for observing 2 waves simultaneously

The cabinet consists of a welded "dow-metal" supporting frame with removable sheet metal panels, which permit access to all parts, but provide complete electrostatic shielding. The equipment is mounted in double deck arrangement. In order to minimize 60 cycle disturbances all power supplies are located on the lower deck and provided with special shielding where necessary. The cathode ray tube is mounted in a magnetic shield supported from the top of the frame. The auxiliary circuits are mounted on the upper deck as illustrated in figure 3. The finished cabinet is 17 inches high, 16 inches wide and 20 inches deep and the complete oscillograph has a total weight of 72 pounds.

A paper recommended for publication by the A.I.E.E. committee on instruments and measurements, and scheduled for discussion at the A.I.E.E. Great Lakes District meeting, West Lafayette, Ind., Oct. 24–25, 1935. Manuscript submitted Aug. 8, 1935; released for publication Sept. 9, 1935.

The authors acknowledge the assistance of Dr. C. Francis Harding, and other members of the electrical engineering and experiment station staffs of Purdue University.

* Now with General Electric Company, Schenectady, N. Y.

1. For all numbered references, see list at end of paper.

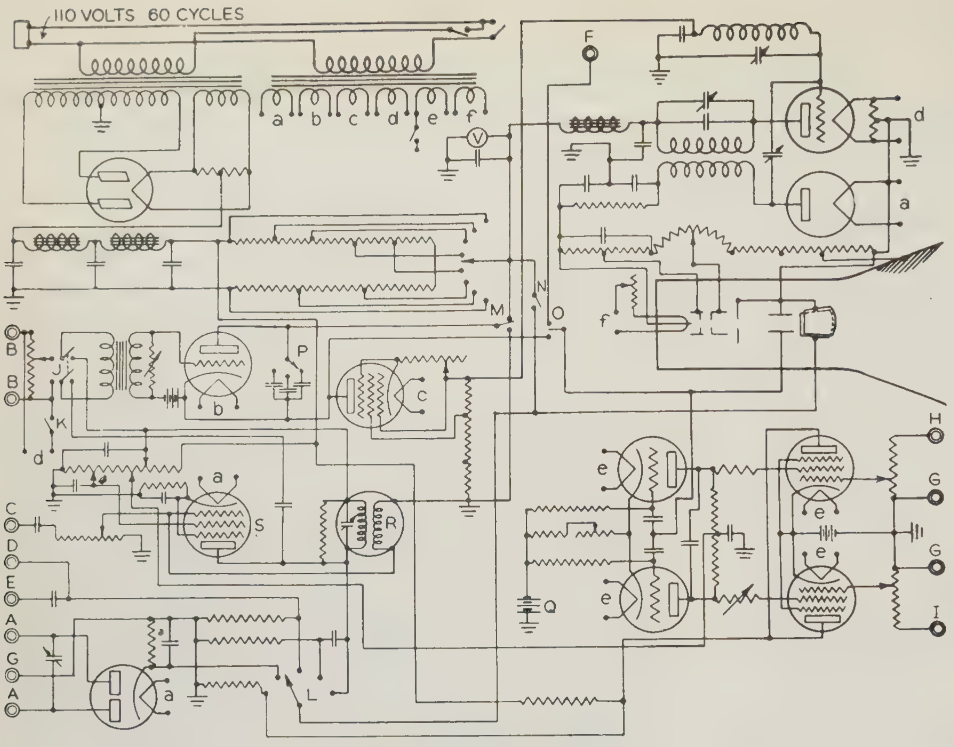


Fig. 2. Schematic wiring diagram of oscillograph

THE CATHODE RAY TUBE

The method of focusing an electron stream developed several years ago[3] has been modified and adapted for use in glass cathode ray tubes developed for television purposes. Because of their ability to produce a sharply defined beam over a wide range of operating voltages, these tubes have proved to be especially well adapted for oscillograph use. The general arrangement of the electron gun is shown schematically in figure 4. Referring to this figure, the electrons are emitted from a small area on the end of the cathode. The shield which surrounds the cathode brings the beam to a short focus near the plane of the aperture and also controls the beam current. Focusing is accomplished by varying the potential on the first anode with respect to the cathode and second anode. The beam reaches its maximum velocity as it emerges from the second anode, which is usually operated at ground potential.

The cathode consists of a piece of No. 22 platinum-nickel alloy wire which has a 0.01 to 0.015 inch hole drilled or pressed in the end, and this hole packed with barium and strontium oxides. The emitting surface is therefore limited to a relatively small area. The tungsten heater wire is wound in the form of a double spiral to make it noninductive and to permit the cathode to be spot welded to the mid-point. Since the cathode is heated by conduction, the heater filament operates at a relatively low temperature. Platinum-nickel cathodes have been used because of the very high temperature required to melt them loose from the heater filament. The filament current can be increased to about double the normal operating value without melting off the cathode. The ends of the double spiral heater filaments are held in place by means of a short section of ceramic tubing. A connection is made directly with the cathode and the lead brought out through the center of the spiral heater and insulating cylinder. The cathode is held in the center of the shield by means of a mica spacer.

The cathode shield serves the double purpose of shielding the cathode and acting as the control electrode for modulating the beam. The stem end of the shield is left open as the heater filament operates at a temperature too low for electron emission.

The first anode is made in the form of a cylinder with a plate across the end facing the cathode. The hole in the plate is approximately 0.04 inch in diameter, which is sufficient to permit the entire beam to pass through it when properly focused. The cylindrical section of the first anode is made sufficiently long to allow the beam to gain the necessary diameter before entering the intense portion of the accelerating and focusing field near the second anode. Usually 90 per cent or more of the total accelerating voltage appears between the first and second anodes.

In the glass type cathode ray tube the second anode is made stationary and in the form of a flat disk. The hole in the second anode should be slightly larger than the diameter of the beam at this

Table I—Average Characteristics of Cathode Ray Tubes

	Minimum	Maximum
Diameter of tube, inches	7	
Length of tube, inches	18 1/2	
Cathode voltage to ground	−250	−5000
Cathode shield voltage with respect to cathode	+ 10	− 60
First anode voltage with respect to cathode	+ 25	+ 500
Second anode voltage with respect to cathode	+250	+5000
Filament current in amperes, average	2.7	
Beam current in microamperes	0 to 250	

point. The beam is focused by the converging electric field between the first and second anodes. As previously mentioned, the beam is brought to a focus at the screen by varying the potential on the first anode with respect to the cathode and second anodes.

When the electron gun is operating properly, from 70 to 90 per cent of the electrons leaving the cathode space charge reach the screen in the form of beam current. Consequently, the energy required to operate the electron gun is so small that it can be supplied from a small high frequency oscillator.

The coating on the inside of the glass envelope provides a return path for the electrons and eliminates instability when the tube is operated as a television receiver. The sharpness of the beam is improved by impressing a negative potential of 20 to 50 volts on the conducting coating.

The fluorescent screen material is sprayed on the inner surface of the tube before the conducting coating is applied. For visual observation the screen material is usually synthetic willemite which produces a yellow-green light, but for photographic reproduction and television reception a screen material which produces a light closely approaching white is used. This screen material was developed for reproducing television pictures in black and white.

Tubes are provided with 2 pairs of deflecting plates, with one plate of each pair connected to the second anode and grounded. In order to eliminate keystone distortion (the distortion of a rectangular figure into the shape of a keystone) due to an uncompensated change in velocity of the beam as it passes through the fringing field from the ends of the deflecting plates and to divergence of this field, the plates are provided with grounded guard plates, as shown in figures 4 and 5. The guard plates prevent the fringing flux from becoming divergent, and also practically eliminate changes in beam velocity by limiting the extent of the fringing flux and providing a compensating component.

In some of the tubes a ground plate is mounted just beyond the deflecting plates for the purpose of minimizing the electron current which may tend to drift back to the deflecting plates.

The electron beam focuses to a sharply defined spot of 0.1 to 1 millimeter diameter over range of voltages from 250 to 3,000 volts, and with only a slight increase in size up to 5,000 volts. Although in the past it has been necessary to use 0.5 to 1.5 microns of argon gas in the tubes to obtain a sharply defined beam over this wide range of beam voltage, recent calculations indicate that a sharp focus may be obtained in a hard vacuum by some changes in the dimensions of the electron gun. Further development is being directed along this line.

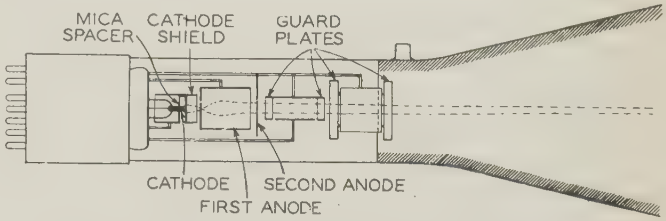


Fig. 4. Schematic diagram of electron gun

Despite the facts that the tubes contain a small amount of argon, and that no effort has been made to prevent positive ion bombardment of the cathode, the life of the tubes usually runs well over 1,000 hours. Moreover, tubes which have lost their emission can be repaired by repacking the oxides in the cathode and re-exhausting. Since the holes in the anodes are larger than the diameter of the cathode the oxides can be replaced without dismounting the gun parts.

The average characteristics of the cathode ray tubes are shown in table I.

The filament current is supplied from a 2.5 volt transformer winding and adjusted by means of a high current rheostat. Although the voltage across the filament at normal current is approximately 0.5 volt the filament cannot be damaged by the full 2.5 volts.

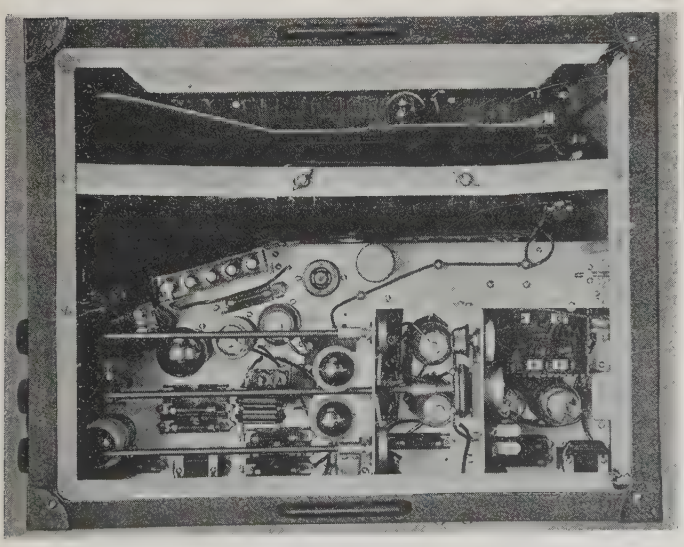


Fig. 3. Top view of oscillograph with cover removed

The High Voltage D-C Power Supply

As a result of the high beam efficiency of the cathode ray tube it has been possible to use a high frequency oscillator and rectifier to supply both the beam and sweep voltage for the cathode ray tube. The circuit developed for this purpose is shown in the upper right hand corner of figure 2. This circuit was originally designed for use in television receivers where safety is an important factor, and where plate voltage for the oscillator is available from the amplifier plate supply.

The high frequency power is supplied from a special tuned-grid tuned-plate oscillator, the plate coil of which forms the primary of a high frequency step-up transformer. Current from the secondary of this transformer is rectified by means of a half-wave high voltage rectifier to supply the beam voltage. In order to obtain maximum high voltage output, the oscillator is tuned to the natural frequency of

the rectifier circuit, which is usually about 100,000 cycles. The tuning of the oscillator circuits is accomplished by means of mica trimmer capacitors which are not disturbed after the correct adjustment is found. At a frequency of 100,000 cycles good filtering is obtained with 2 0.002-microfarad mica capacitors and a 100,000 ohm resistor. The use of such small capacitors in the filter circuit greatly reduces the danger from serious shock by accidental contact with the high potential circuit and eliminates the necessity for a discharging switch.

By the use of high excitation on the grid of the oscillator, sufficient power is obtained by grid rectification to supply the sweep voltage for the linear time axis. The correct ratio between the beam and sweep voltages is obtained by adjusting the capacitance between the grid of the oscillator and the plate of the high voltage rectifier.

With an oscillator of this type operating at a plate voltage of 400 volts and drawing about 35 milliamperes plate current, it is possible to supply a beam voltage of 4,500 volts and a sweep voltage of 500 volts. However in the case of the oscillograph being described, the oscillator is so adjusted that beam voltages of 500 to 3,500 volts can be obtained in 500 volt steps by varying the plate voltage of the oscillator by means of a tap switch.

The deflection sensitivity can be checked readily by closing a push-button switch on the front of the panel, which applies the plate voltage of the oscillator to one pair of deflecting plates, and reading the plate voltage on the panel voltmeter. This is necessary when making accurate voltage measurements with the oscillograph since the regulation of

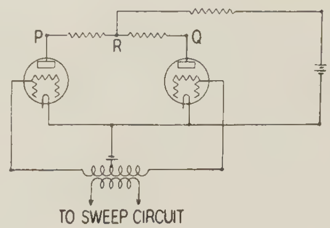
Fig. 5. View of section of cathode ray tube showing the guards on the deflecting plates

the high frequency power supply permits some change in beam voltage with change in beam current.

The high frequency power supply has proved itself to be quite flexible and well adapted to most applications.

An Electronic Switching Circuit for the Simultaneous Observation of 2 Waves

The application of the cathode ray oscillograph has been greatly restricted, in the past, because of its inability to portray more than a single phenomenon at a time. Since it is often desirable, or even essential, to show 2 or more waves simultaneously in their correct relative phase positions, this limitation has proved to be a serious handicap in the general usefulness of this type of oscillograph. One possible



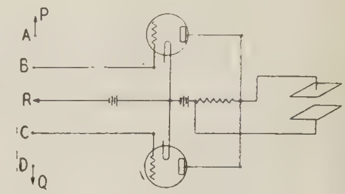
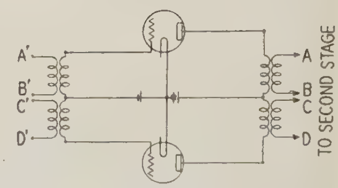


Fig. 6 (left). Schematic diagram of a circuit that might be used as the basis for an oscillograph for simultaneous observation of 2 waves

Fig. 7 (right). Schematic diagram of an auxiliary circuit for use with figure 6

Fig. 8. General arrangement of first stages of amplifiers used with circuit finally adopted



solution is to devise a single tube with a multiplicity of beams, corresponding to the multi-element vibrator type of oscillograph. Since it is recognized that such a plan would greatly complicate the manufacture and functioning of the tube, a much more feasible scheme would be to utilize a single beam. This could be accomplished by impressing the different signal voltages across the deflecting elements of the tube in succession, one during each sweep. In this manner, due to the persistence of vision and provided that the frequency of the cycles of operation were not too low, 2 or more waves whose frequencies are in a simple relation could be caused to appear simultaneously in their correct relative phase positions on the screen.

Such a method has been devised to show 2 waves, the switching being accomplished by means of a rotary switch[4] operated by a synchronous motor that was driven from the same power supply as the observed phenomena. Although this arrangement has proved to be quite practical as far as power applications are concerned, it cannot be synchronized when either the frequency is too high or the power supply too limited to permit the use of a synchronous motor. It is likewise obvious that any system of electromechanical relays would suffer similar limitations. The latter could be removed, however, by means of an electron tube relay system that was actuated by the sweep circuit. It is the purpose of this portion of the article briefly to discuss the development of such a device.

A circuit that might be used as the basis in the development for an oscillograph for the simultaneous observation of 2 waves is shown schemati-

cally in figure 6, the tubes being small grid-controlled rectifiers. Since this is similar in construction and theory of operation to the simple 2 tube inverter,[5,6] its action will not be discussed in detail at this time.

In connection with the actual application of this circuit to the particular project being undertaken,

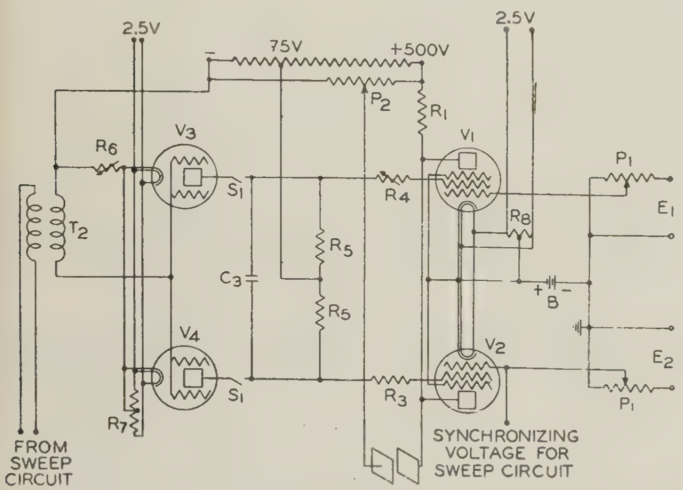


Fig. 9. Diagrams showing use of pentodes in control circuit

it was first proposed that the tubes themselves might be caused to function as switches. Because of the necessarily high impedance of the deflecting plate circuit, however, no simple arrangement of this sort that would be applicable over a wide range of frequencies could be devised. Accordingly, instead of permitting the tubes to function directly as switches, it was then proposed that they might be utilized to control the bias on a pair of amplifier tubes, the grids being connected to the 2 signals and the plates being tied together and feeding into the deflecting circuit of the cathode ray tube. The schematic diagram for this arrangement is shown in figure 7. In this, points P, Q, and R would be connected to the correspondingly lettered points in figure 6, the large differences of potential that occur alternately across the resistors permitting first one amplifier to pass a signal and then the other.

In the actual working out of a circuit to give the desired result when triode amplifiers were used, it was found necessary to introduce 2-stage rather than single-stage amplifiers. This was for the purpose of simplifying the plate supply problem as well as because of the necessity of isolating the switching circuit from the signal circuits; the additional stages being inserted to act as buffers rather than for the purpose of introducing additional amplification. The general arrangement of the first stages is shown in figure 8, the 2 signals now being impressed across $A'B'$ and $C'D'$, respectively.

The first electronic switching circuit built into an oscillograph made use of the 2-stage amplifier. Although it operated in a very satisfactory manner in most cases, the amplifier was the weak point, as it would obviously neither pass the average component of a pulsating wave nor faithfully reproduce a flat-topped alternating wave. One possible solution considered in this regard was to develop an appropriate direct-coupled amplifier. It was instinctively felt, however, that such an amplifier would be prohibitively complicated in that it would require several separate plate voltage supplies. No attempt was accordingly made to carry out such a project but another method was developed.

It has already been mentioned that the 2-stage amplifiers were made necessary because of both the blocking and signal voltages being applied to the same grids. With this inherent disadvantage in mind, it was proposed that single-stage, direct coupled, screen-grid tube amplifiers might be used. In this, the signal voltages would be impressed across the control grids as before but the blocking would be done on the screen grids. In the actual carrying out of this proposal, pentodes were used as shown in figure 9, the power supply, sweep circuit, etc., being essentially the same as used in the earlier set-up.

The performance of the multiwave oscillograph in this final form proved to be quite satisfactory. The stability of the switching circuit has been materially improved in the later model. This was done by introducing the synchronizing impulse into the switching circuit by means of capacitive rather than inductive coupling. It is expected that it might be

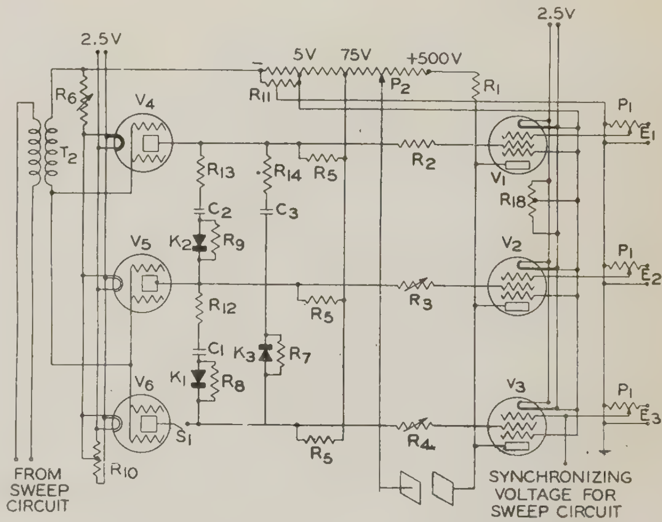


Fig. 10. A possible circuit for an oscillograph for observing 3 waves simultaneously

otherwise improved, or even simplified in certain respects. In this regard, the following topics are in mind for investigation:

1. Substitution in place of 2 tube inverter circuit of:
   (*a*) Single-tube inverter circuit; or
   (*b*) Vacuum tube oscillator of an appropriate type.
2. Modification of switching circuit in such a manner that waves whose frequencies were not necessarily in simple multiple relationship could be stabilized.
3. Construction of a circuit for showing more than 2 waves.

So far as item 3 is concerned, a possible circuit for showing 3 waves is given in figure 10 and the same

idea could be extended to the case of an unlimited number of waves. It should be recognized, however, as soon as the frequency of the complete cycle of operation drops below 20 cycles per second that there would be a noticeable flicker.

The electronic switching circuit as embodied in the general purpose oscilloscope is illustrated in the lower right hand corner of figure 2. It may be noted that the switches in the plate leads of the inverter tubes have been eliminated in this circuit as it was found that the inverter would start of its own accord if the cathode resistor was properly adjusted. The battery Q is used for the purpose of compensating for the drop in potential in the inverter tubes, thus causing the inverter to reduce the potential on the screen grids of the amplifier to zero or even to a negative value. This prevents the blocked tube from passing current when the maximum normal signal is impressed on the control grid.

The use of capacitive coupling between the sweep and inverter circuits has proved very satisfactory for tripping the inverter. With this arrangement the inverter has been made to operate satisfactorily when the sweep circuit was being discharged as many as 14,000 times per second, thus permitting observation of phase relation between 2 waves at frequencies as high as 70,000 cycles per second.

Other Auxiliary Circuits

In the sweep circuit, figure 2, a type 57 tube is used to charge the sweep capacitor at a constant rate, and a special gaseous discharge tube is used to discharge it. The discharge tube has a control factor of 100 or more and is capable of withstanding the entire sweep voltage which may exceed 800 volts. By throwing the switch M to the upper position the sweep voltage can be almost doubled by adding the plate voltage of the oscillator to that obtained by grid rectification. In either case the sweep voltage varies with the beam voltage in such a manner that changes in beam voltage produce very little change in the rate of sweep. By throwing switch J to the left the sweep can be synchronized with an external signal applied across BB; or by throwing it to the right and applying the signal between C and G the signal is amplified first. If switch K is closed, with switch J thrown to the left, the sweep can be synchronized with the 60 cycle power supply.

When the coils R are plugged in, the tube S acts as a fixed frequency oscillator for use in calibrating the time axis, but when the coils are out it acts as a resistance coupled amplifier.

If a suitable loop antenna is connected to the terminals AGA, strong radio frequency signals can be rectified for modulation measurements, and for checking distortion in the modulating waves.

Lissajou's figures can be studied by impressing the waves between CG and FG with switch O in the upper position.

References

1. Discussion, R. H. George. Elec. Engg. (A.I.E.E. Trans.), v. 54, July 1935, p. 750.

2. Discussion, C. S. Roys. Elec. Engg. (A.I.E.E. Trans.), v. 54, July 1935, p. 749–50.

3. A New Type of Hot Cathode Oscillograph and Its Application to the Automatic Recording of Lightning and Switching Surges, R. H. George. A.I.E.E. Trans., v. 48, July 1929, p. 884–90.

4. Simultaneous Traces with Cathode Ray Oscillograph, C. Bradner Brown. *Electronics*, v. 6, June 1933, p. 170.

5. Electronics (a book), Ralph G. Hudson. John Wiley & Sons, Inc., New York, N. Y.

6. Electron Tubes in Industry (a book), Keith Henney. McGraw-Hill Book Company, New York, N. Y., first edition, 1934.

The Production of Impulse Test Voltages

In performing surge tests on electrical power apparatus to determine ability to withstanding lightning or switching surges, the shape of the impulse voltage wave should be approximately the same as 1 of the 3 test wave shapes commonly in use. Methods of producing a wave shape having the prescribed time constants are described in this paper, and oscillograms are presented which show the correctness of calculated design characteristics of the generator for producing these surges.

By
C. S. SPRAGUE
ASSOCIATE A.I.E.E.

Purdue University,
West Lafayette, Ind.

Much of the equipment used on modern electric power systems is tested by being subjected to artificially produced surge voltage waves which simulate the lightning or switching surges which may occur on the system. With the rapid development of surge testing equipment and technique during the last decade, has come an increasing tendency toward standardization of testing methods and procedure. The various factors which affect the wave shape and the accuracy with which the wave may be measured and recorded have been investigated and reported in previous papers.[1–4] Mathematical and experimental determinations of the wave shape with calculated or measured generator con-

A paper recommended for publication by the A.I.E.E. committee on instruments and measurements, and scheduled for discussion at the A.I.E.E. Great Lakes District meeting, West Lafayette, Ind., Oct. 24–25, 1935. Manuscript submitted Apr. 18, 1935; released for publication Sept. 3, 1935.

1. For all numbered references, see list at end of paper.

Table I—Values of "a" and "b" for the Proposed Waves in the Equations RC = a, seconds; LC = b, seconds squared

Wave Shape	a	b
$1^1/_2$ x 40	55.35×10^{-6}	15.54×10^{-12}
1 x 10	12.90×10^{-6}	3.180×10^{-12}
1 x 5	5.595×10^{-6}	1.800×10^{-12}

stants have been made under various conditions of load and generator circuit,[5–7] the characteristics of resistance and capacitance potentiometers have been studied, and material progress has been made toward standard calibrations for sphere gaps for measuring voltages under impulse conditions.[8–13] Naturally, many of the factors which influence the methods and technique of impulse testing depend upon local laboratory conditions and cannot be subject to definite specifications; nevertheless, certain general rules of established procedure have been noted.[8,14]

One of the most important steps of the standardization process was the recommendation of the 3 impulse wave shapes,[14] designated as the $1^1/_2$ x 40, the 1 x 10, and the 1 x 5 microsecond waves; the first figure in each of these 3 designations indicates the time in microseconds for the wave to increase from zero voltage to crest voltage and the second figure indicates the time from zero voltage to one-half crest voltage on the tail of the wave. The time constants of these impulses were the result of a careful study, not only of the impulse voltages produced by lightning, but also of the facility with which they could be reproduced in the various laboratories throughout the country.

Recently the engineering experiment station at Purdue University has devoted a portion of its time to the determination of the constants of the impulse generator for producing impulses of the prescribed time constants. The present paper shows the degree to which the actual waves, as recorded by the cathode ray oscillograph, check the calculated values. Traveling wave effects and reflections which always exist in surge generator circuits are shown in their relation to the several constants of the surge generator.

Conclusions

1. With a suddenly applied electromotive force, a series circuit of resistance, inductance, and capacitance is inherently subject to reflections and traveling wave phenomena, due to the finite lengths of the circuit and the finite velocity of propagation of energy through the circuit. In proportion to the main response characteristic of the circuit the relative importance of these reflections depends upon the lengths of wire in the circuit and upon the circuit constants, including the load, if the latter is appreciable. The effect of the reflections upon the main response characteristic is minimized with short wire lengths and low discharge resistances and high load capacitances.

2. In the surge generator the obvious and direct method of minimizing the effect of reflections is to use a large value of capacitance, which, for a given wave shape, reduces the necessary values of inductance and resistance. Unfortunately for the economic aspects of the case this entails a rather large investment in the condenser units. Theoretically, the connection of an appreciable load capacitance in parallel with the load resistance should diminish the effect of reflections, providing sufficient series resistance is incorporated in the generator gap circuits to prevent oscillation of the generator circuit inductance and the load capacitance. The load capacitance might be provided by a capacitance potentiometer or by the test piece itself. The desirability of using some such arrangement as the above will of course depend upon the conditions and the type of testing in a particular laboratory.

3. The wave shapes calculated from equations for the simple series circuit of capacitance, resistance, and inductance check closely the actual wave shapes, under the following conditions:

(*a*). The load resistance (including sphere gaps and test piece, if any) is essentially noninductive and has negligible capacitance for the wave shape under consideration.

(*b*). Reflections are sufficiently minimized by the use of low values of inductance and resistance.

(*c*). The load resistance is slightly increased to compensate for the decrease in the effective capacitance due to the phenomena of residual charge and the slight loss of energy in the resistors of the charging circuit.

4. From the standpoint of the length of wire and the flashover characteristics under transient conditions, the most efficient inductances are those having an axial length approximately equal to their diameter (single layer, helically wound coils with uniform turn spacing).

Description of Apparatus

The condenser bank of the surge generator consisted of 48 25-kv $^1/_4$-microfarad units, mounted in such a manner that adjacent banks might be readily connected in parallel, providing several values of potential and capacitance.

The series inductance was provided by a coil wound upon an octagonal form 2 or 3 feet in diameter. Turn spacings of from $^1/_2$ inch to 2 inches were available depending upon the number of turns and the surge potential.

The load resistance and the dividing resistance were provided by a water column resistor contained in several parallel lengths of rubber hose. With this arrangement a wide range of resistance values could be obtained by varying the conductivity of the water solution, and, if necessary, the number of lengths of hose in parallel. Taps were provided along the hose for a rough adjustment of the test voltage, with fine control obtained by varying the charging voltage of the surge generator. With the load and divider resistance each having one end grounded, a large volume of the water solution could be readily circulated through the hose, eliminating any appreciable change in resistance due to heating. A resistance-cable divider was used in obtaining all the oscillograms reproduced in this paper. The cable was terminated, at the cathode ray oscillograph, in a resistance equal to its surge impedance.

The cathode ray oscillograph was of the George hot-cathode type, having a beam potential of about 15 kv and employing automatic vacuum tube circuits for initiating and synchronizing the sweep and the cathode beam.

Calculation of Constants

The values of resistance and inductance necessary to produce the desired wave shapes were calculated for the several values of capacitance afforded by the surge generator. These calculations were made from the equations developed for the simple series circuit, assuming lumped constants and negligible load on

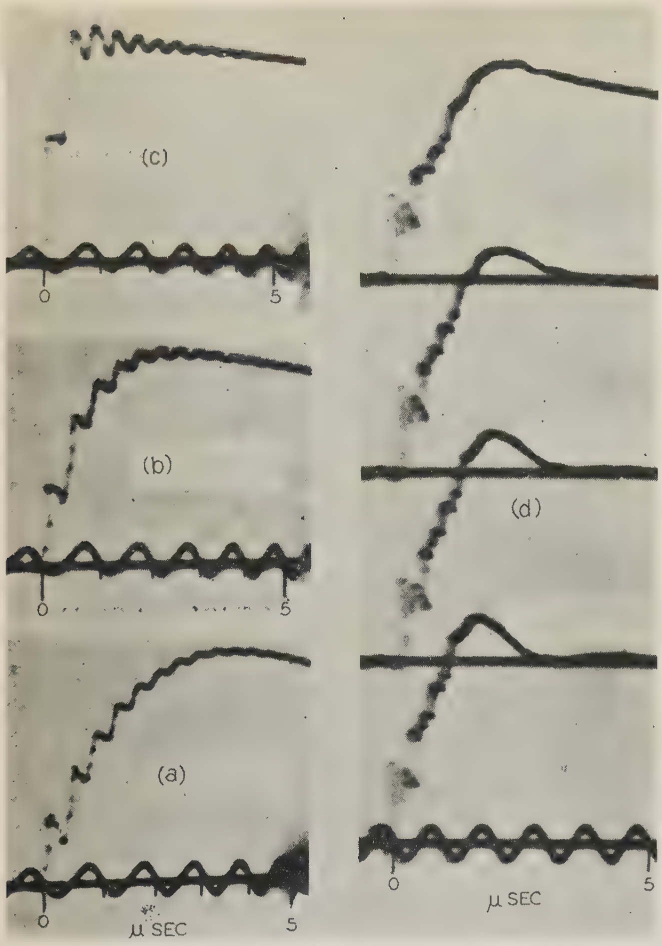


Fig. 1. Showing reflections across load resistance in surge generator circuit. Time in microseconds

(*a*). $C = 0.096 \times 10^{-6}$ farads, $L = 0.18 \times 10^{-3}$ henrys, $R = 175$ ohms
(*b*). $C = 0.096 \times 10^{-6}$ farads, $L = 0.18 \times 10^{-3}$ henrys, $R = 353$ ohms
(*c*). $C = 0.096 \times 10^{-6}$ farads, $L = 0.18 \times 10^{-3}$ henrys, $R = 860$ ohms
(*d*). $C = 0.00614 \times 10^{-6}$ farads, $L = 2.53 \times 10^{-3}$ henrys, $R = 4{,}090$, 5,700, 7,700, and 9,800 ohms, respectively, reading from top down

the surge generator. It was realized, of course, that with certain types of testing the capacitance or reactance of the equipment under test might not be negligible, and that further adjustments would be necessary to provide the prescribed wave shape with the test piece connected. However, one of the objects of this work was to determine to what degree the surge generator approximated the simple series circuit, hence the calculations were made as stated.

For a particular wave shape and capacitance the values of resistance and inductance were calculated with a good degree of accuracy, then for this same wave shape, but for other values of capacitance, the values of resistance and inductance were determined by the relations, $RC = a$ and $LC = b$. (See appendix.) Table I presents the values of the constants a and b for each of the 3 wave shapes, and table II gives the calculated values of inductance and resistance for several available values of capacitance.

Both experience and theoretical considerations lead to the conclusion that traveling waves and reflections must exist in the surge generator circuit. In particular, reflections occur across the load resistance. These reflections may be positive or negative depending upon the value of the resistance and the constants of the inductance coil.

With fair analogy, the inductance coil and the discharge resistance may be considered as similar to a transmission line grounded through a resistance and having potential suddenly applied at the open end. According to the usual relations governing the reflection of traveling waves, the voltage across the resistance at the first reflection will depend upon the value of the resistance in relation to the surge impedance of the line, being greater than the voltage of the approaching wave if the resistance is greater than the surge impedance of the line (positive reflection), and less than the wave voltage when the resistance is less than the surge impedance of the line (negative reflection). With positive reflection, the successive reflections at the resistance produce voltages alternately greater and less than the wave voltage. In actual practice the losses which are always present give the appearance of a damped oscillation riding upon the main wave. This type of wave is sometimes called an "overshooting" wave. With negative reflections the voltage across the resistance increases at each reflection by successive increments and approaches the wave voltage, causing the wave front to have the familiar "stair-step" appearance, characteristic of negative reflections. Reflections of both types have been encountered in surge generator work, even though the circuit lengths are relatively short compared to those of a transmission line.

Figure 1 shows several oscillograms illustrating positive and negative reflections and the effect upon the reflections of varying the load resistance. It is obvious that the surge travels through the inductance and reflects across the load resistance in a manner analogous to surges upon a transmission line. If the load resistance can be made low, and if the inductance coil contains only a short length of wire it is apparent that the wave front will be built up by a series of negative reflections of low amplitude and short period. Under these conditions the characteristic stair steps are usually rounded off quite rapidly and a relatively smooth wave results. On the other hand, with a greater length of wire in the inductance the period of the reflections will be greater and the individual reflections will be more pronounced. Similarly, with a fixed inductance, the period of the reflections is constant while the amplitude varies with the value of load resistance as shown in figure 1, parts *a*, *b*, and *c*. With the higher values of resistance positive reflections occur as shown by figure 1*d*.

It is obvious from the foregoing discussion that the necessary inductance should be obtained with a minimum length of wire. Primarily, the axial length of the coil and the spacing between turns must be sufficient to prevent the impulse voltage from flash-

ing over the inductance. If, say, the spacing between turns is assumed, the axial length and the coil diameter may be varied to determine the proportions which give the required inductance in a minimum length of wire If the resulting coil will not withstand the impulse voltage, a greater spacing between turns must be assumed, and the optimum proportions determined as before.

Using Nagaoka's formula,[15] inductance values were calculated for turn spacings varying from $^1/_2$ inch to 2 inches, and for coil diameters varying from 2 feet to 10 feet. For each given turn spacing the curves of inductance versus number of turns were plotted for the several diameters.

For any of the above spacings, it was possible to determine, for each diameter, the number of turns necessary to produce any assigned inductance within the range covered. From the diameters and the corresponding numbers of turns, the necessary lengths of wire were determined readily. The necessary lengths of wire were plotted against the corresponding coil diameters; the diameter providing the minimum length of wire could then be determined easily. The axial length of the coil was found from the turn spacing originally assumed and the number of turns corresponding to the optimum diameter.

Several curves of this nature were prepared for different values of inductance and for different turn spacings. A study of these curves provided the following general conclusions.

1. For any turn spacing the diameter which provides a given value of inductance with a minimum length of wire is such that the ratio of axial coil length to diameter is approximately 0.4.

2. For any turn spacing and given value of inductance the ratio of axial coil length to diameter may vary from about 0.2 to about 1.1 with an increase in the length of wire of about 10 per cent over the minimum length.

From conclusion 2 it is obvious that the axial length of the coil may be equal to the coil diameter

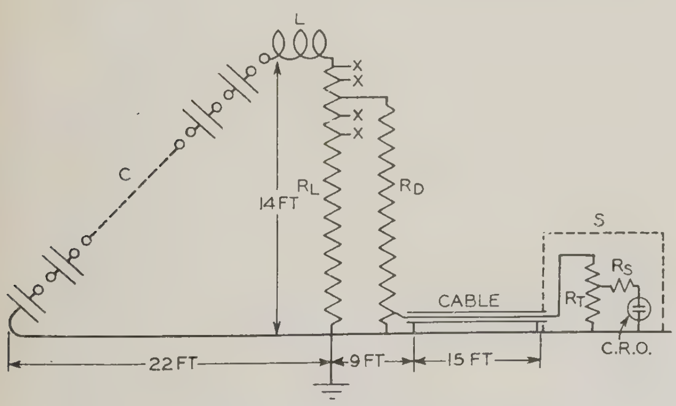


Fig. 2. Surge generator circuit diagram

C = effective capacitance (0.0056 to 0.23 microfarad)
L = added series inductance (0 to 3,000 microhenrys)
R_L = effective load resistance (100 to 10,000 ohms, including R_D)
R_D = divider resistance (1,000 to 15,000 ohms)
R_T = cable terminating resistance (50 ohms)
R_S = series damping resistance (2,000 ohms)
= shielding cage for cathode ray oscillograph
X = taps for coarse adjustment of test voltage

Table II—Calculated Values of Resistance and Inductance to Produce Standard Test Impulses With Different Values of Capacitance*

Wave Shape	Maximum Released Voltage Kv	Capacitance Microfarads	Resistance Ohms	Inductance Millihenrys
$1^1/_2 \times 40$	1,075	0 00613	9030	2.54
$1^1/_2 \times 40$	575	0.0230	2420	0.678
$1^1/_2 \times 40$	375	0.0528	1050	0.295
$1^1/_2 \times 40$	275	0.096	577	0.162
1×10	1,075	0.00613	2105	0.518
1×10	575	0.0230	562	0.139
1×10	375	0.0528	244	0.060
1×10	275	0.096	134	0.0331
1×5	1,075	0.00613	912	0.294
1×5	575	0.0230	244	0.079
1×5	375	0.0528	106	0.0341
1×5	275	0.096	58	0.0188

* Calculated from equations for simple series circuit of resistance, inductance, and capacitance.

without serious increase in the length of wire. It is hardly necessary to remark that the longer coils would be more desirable from the standpoint of preventing flashover.

The inductances were wound with iron wire as the damping effect of the distributed resistance was found to be of some benefit in providing smoother wave fronts.

Oscillograms of Actual Waves

For the several values of surge generator capacitance, the values of inductance and resistance were adjusted as closely as possible to the calculated values for the particular wave shape as given in table II. Figure 2 provides the circuit diagram together with the range of values of the circuit constants and with dimensions to indicate circuit lengths. As stated before, all waves were recorded using a resistance cable divider to provide the proper voltage for the deflecting plates. With the short length of cable (15 feet) and with divider resistances not over 15,000 ohms, there was no perceptible distortion of the wave shapes.

Oscillograms were obtained of the impulse voltages produced with the circuit constants adjusted to the calculated values. In general, the values of time to reach the crest were closely equal to those of the standard impulses but the time at which the voltage decreased to 50 per cent of crest value was less than the prescribed time. Two probable causes of this were apparent. The first was the loss of energy due to the series resistors in the Marx circuit charging arrangement and the second was the phenomena of residual charge or the inability of the condensers to give up all their stored energy during the short interval of discharge. The effect of this latter is to cause an apparent decrease in the capacitance of the surge generator.

Figure 3 shows several of the oscillograms taken after the series resistance had been increased to provide the prescribed attenuation of the wave tail. Each wave was recorded with 2 sweeping rates, one to show the wave to the 50 per cent value on the tail and one to show the wave front alone and pro-

vide a more accurate determination of the time to reach the crest. Oscillogram *a* of figure 3 illustrates a $1^1/_2$ x 40 wave taken under conditions of low generator capacitance and high resistance and inductance. The positive reflections are indicated by the overshooting wave. All the oscillograms were obtained under conditions of minimum load capacitance with no sphere gaps or test piece connected. That the overshooting of the wave at its crest was not due to oscillations of the generator circuit inductance with any load capacitance was indicated by the fact that large amounts of series resistance in the generator gap circuits had no effect upon the overshooting. Likewise, the overshooting was not affected by connecting a 50 centimeter sphere gap across the load resistance, the sphere gap having a capacitance of at least of the same order of magnitude as the effective distributed capacitance of the load resistance.

Oscillograms *b*, *c*, and *d* of figure 3 show the 3 standard wave forms obtained with generator capacitances sufficiently large to allow the use of relatively low values of inductance and resistance. In these latter oscillograms, positive reflections are entirely absent; the negative reflections which must exist have sufficiently small magnitude and low period to be readily attenuated, thus providing a smooth wave front.

Table III provides a comparison of the actual with the calculated circuit constants for each of the oscillograms of figure 3, together with the time constants of the actual impulses.

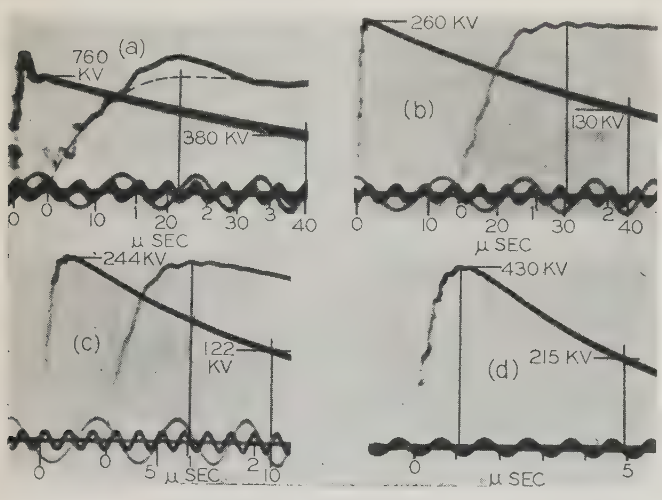


Fig. 3. Oscillograms taken after series resistance had been increased to provide prescribed attenuation of the wave tail. Time in microseconds

(*a*). $1^1/_2$ x 40 wave with positive reflections (low generator capacitance). $C = 0.00614 \times 10^{-6}$ farads, $L = 2.53 \times 10^{-3}$ henrys, $R = 9{,}180$ ohms. Maximum released voltage = 1,075 kv

(*b*). $1^1/_2$ x 40 wave (high generator capacitance). $C = 0.0528 \times 10^{-6}$ farads, $L = 0.270 \times 10^{-3}$ henrys, $R = 1{,}130$ ohms. Maximum released voltage = 375 kv

(*c*). 1 x 10 wave (high generator capacitance). $C = 0.0528 \times 10^{-6}$ farads, $L = 0.060 \times 10^{-3}$ henrys, $R = 259$ ohms. Maximum released voltage = 375 kv

(*d*). 1 x 5 wave (high generator capacitance). $C = 0.023 \times 10^{-6}$ farads, $L = 0.082 \times 10^{-3}$ henrys, $R = 245$ ohms. Maximum released voltage = 575 kv

Table III—Impulse Generator Circuit Constants and Time Constants for Oscillograms of Figure 3

Oscil	Wave	Capacitance µf	Inductance Calc. mh	Inductance Actual mh	Resistance Calc. ohms	Resistance Actual ohms	Time To Crest µSec	Time To 50% µSec
3*a*	$1^1/_2$ x 40	0.00614	2.53	2.53	9,000	9,180	1.6	40
3*b*	$1^1/_2$ x 40	0.0528	0.270	0.270	1,050	1,130	1.4	40
3*c*	1 x 10	0.0528	0.060	0.060	244	259	1.0	10
3*d*	1 x 5	0.023	0.079	0.082	244	245	1.1	4.9

µf = microfarads
mh = millihenrys
µsec = microseconds

Appendix

The calculation of circuit constants to produce a given wave shape is a rather laborious task. In the usual case the capacitance is known and the inductance and resistance are determined by trial and error substitutions.

It is true that the substitution of the time constants of the wave (say $1^1/_2$ x 40 microseconds) and the value of the capacitance into the circuit equations gives rise to 2 equations with 2 unknowns, R and L, but since the equations are of transcendental nature their solution for the values of R and L cannot be obtained in the usual manner. A graphical solution has been presented in a paper by C. M. Foust, H. P. Kuehni, and N. Rohats.[5] If, for any particular wave, the values of resistance and inductance to produce that wave are calculated for several values of capacitance, and then plotted against the capacitance, it will be found that the curves are hyperbolas (or straight lines if plotted on paper having logarithmic scales on both axes). Having once demonstrated the hyperbolic relations, $RC = a$ and $LC = b$, the values of a and b may be quite accurately determined by cut-and-try methods, once and for all. For that particular wave shape, the values of R and L may henceforth be quickly obtained by a simple slide-rule calculation. Since the constants a and b may be determined to any desired degree of accuracy, and since their use is not restricted to any given range of capacitance values, it would seem desirable to determine the values of R and L from these constants rather than from curves.

References

1. Impulse Testing of Commercial Transformers, F. J. Vogel and V. M. Montsinger. A.I.E.E. Trans., v. 52, June 1933, p. 409–10.

2. Impulse Testing of Large Transformers, F. D. Newbury. Elec. Engg., v. 51, April 1932, p. 252–4.

3. Impulse Voltage Testing, C. F. Harding and C. S. Sprague. A.I.E.E. Trans., v. 52, June 1933, p. 428–33.

4. Better Standards Needed for Surge Testing, F. D. Fielder. *Elec. Wld.*, v. 102, Sept. 30, 1933, p. 433–5.

5. Impulse Testing Technique, C. M. Foust, H. P. Kuehni, and N. Rohats. *Gen. Elec. Rev.*, v. 35, July 1932, p. 358–66.

6. Impulse Generator Circuit Formulas, J. L. Thomason. A.I.E.E. Trans., v. 53, Jan. 1934, p. 169–76.

7. Characteristics of Surge Generators for Transformer Testing, P. L. Bellaschi. A.I.E.E. Trans., v. 51, Dec. 1932, p. 936–45.

8. Laboratory Measurement of Impulse Voltages, J. C. Dowell and C. M. Foust. A.I.E.E. Trans., v. 52, June 1933, p. 537–42.

9. Technique of Surge Testing, F. D. Fielder. *Elec. Jl.*, v. 30, Feb. 1933, p. 73–6.

10. Heavy Surge Currents—Generation and Measurement, P. L. Bellaschi. Elec. Engg. (A.I.E.E. Trans.), v. 53, Jan 1934, p. 86–94.

11. The Measurement of High Surge Voltages, P. L. Bellaschi. A.I.E.E. Trans., v. 52, June 1933, p. 544–52.

12. Impulse Calibration of Sphere Gaps, P. L. Bellaschi and P. H. McAuley. *Elec. Jl.*, v. 31, June 1934, p. 228–32.

13. Calibration of the Sphere Gap, J. R. Meador. Elec. Engg. (A.I.E.E. Trans.), v. 53, June 1934, p. 942–8.

14. Recommendations for Impulse Voltage Testing, A.I.E.E. Subcommittee Report. A.I.E.E. Trans., v. 52, June 1933, p. 466–71.

15. Radio Instruments and Measurements. Circular (No. 74) of the U.S. Bureau of Standards, March 23, 1918, p. 252.

Discussions

Of A.I.E.E. Papers—as Recommended for Publication by Technical Committees

ON this and the following 16 pages appear the author's closure of a paper presented at the induction motor session of the 1935 A.I.E.E. winter convention, New York, N. Y., January 22–25, and discussion of papers presented at the electrical machinery, instruments and measurements, power generation, and selected subjects and sessions of the 1935 A.I.E.E. summer convention, Ithaca, N. Y., June 24–28. Author's closures, where they have been submitted, will be found at the end of the discussion on their respective papers.

Members anywhere are encouraged to submit written discussion of any paper published in ELECTRICAL ENGINEERING, which discussion will be reviewed by the proper technical committee and considered for publication in a subsequent issue. Discussions should be (1) concise; (2) restricted to the subject of the paper or papers under consideration; and (3) typewritten and submitted in triplicate to C. S. Rich, secretary, technical program committee, A.I.E.E. headquarters, 33 West 39th Street, New York, N. Y.

Induction Motor Locked Saturation Curves

Author's closing discussion of a paper published in the April 1934 issue, pages 536–41, and presented for oral discussion at the induction motor session of the winter convention, New York, N. Y., January 24, 1935. Other discussion of this paper was published in the September 1934 issue, page 1312, and in the July 1935 issue, page 761.

H. M. Norman: In reply to the point brought up by P. H. Trickey, it seems that there is possibly a misunderstanding regarding the limitation placed on the method outlined in the paper when dealing with totally bridged slots. As explained in the first section of the paper, the reason for the work was to enable the designer to calculate the starting condition more accurately, and therefore it is the upper part of the locked saturation that stress was laid upon and not the very low values of locked current of value approximately equal to full load current. These low values of current would enable the calculation of a more accurate power factor for various loads and such curves as Trickey suggests would help in this direction because for each value of load current there is a different value of rotor slot reactance which causes the locus of the primary current to deviate from a circle.

The paper does cover the case of bridged slots, however, when the upper part of the locked saturation curve is required, by the use of equations 8 and 9. The comparison of test and calculation is given in figure 7 of the paper.

In discussing the curve and formulas given by Trickey I would like to point out that if 2 machines of identical construction but different stacking be compared, then the value of H given in his discussion would be inversely proportional to the stacking. This would result in a different value of slot constant for each machine, while actually they should be the same because there is the same value of current per rotor slot in each machine resulting in the same amount of saturation of the bridge. Perhaps the value of slot constant given on this curve is supposed to include the stacking so that in finding the rotor slot reactance the stacking length is not introduced, but if so then the only curve that would check would be a hyperbola, and this curve deviates somewhat from a hyperbola. I cannot agree with the quantity H being called flux density. For a given magnetomotive force per slot, the flux density in the bridge is fixed regardless of the stacking, and also regardless of the thickness of bridge for all similarly shaped punchings.

Notwithstanding our different view points on this matter, I think that a set of curves on this order would materially help to obtain the load power factors more accurately but would have to be used with discretion for starting conditions on account of the assumption of a constant zigzag reactance.

The points brought up regarding punching strains, machining tolerances, and varying permeability of the iron are true; but it is fortunate that these various disturbing factors seldom pull all in the one direction. Indeed, if this were not so, then the method outlined in the paper would itself not be reliable, because the same disturbing factors apply to the saturated zigzag flux path.

S. F. Henderson has brought up a case of an extreme machine which had only 46 per cent zigzag leakage at starting compared with load conditions. This is a lower percentage than any motor I have checked so far, and as stated in the paper 50 per cent was the lowest percentage checked.

I might comment on the fact that the maximum torque of this particular machine has been calculated with and without correction for saturation of the leakage paths. It brings out the point that the method used in the paper can be applied all along the speed torque curve, as has been done successfully on other machines, including double cage motors where the calculated speed torque curve did not check the test, but came reasonably close to it when the calculated curve was adjusted for saturation.

Regarding the reactance formulas of the machines which Henderson requests, those used for the various slot shapes in figure 6 are given here. They are given in the same sequence, and the same nomenclature applies, as in figure 6, and are for the tooth tip part only.

$\frac{a_1}{t}$ for the first 2 slot shapes

$$\frac{a_1}{t} + \frac{3a_2}{2t + t_1}$$

$$\frac{a_1}{t} + \frac{4a_2}{3t + t_1}$$

$$\frac{a_1}{0.02} + \frac{3.3a_2}{t_1}$$

$$\frac{a_1}{0.02} + \frac{2a_2}{t_1}$$

The last 2 of these formulas, being for bridged slots, are correct for only one value of current and might well be replaced by a system of curves as suggested by Trickey.

The zigzag formula used was

$$W^2{}_t f\,10^{-8}\,\frac{\text{total area of gap}\left(\dfrac{K^2{}_c + K_s}{2}\right)1.14}{S_1 S_2 g_e}$$

where g_e is the equivalent air gap; for the other terms see the nomenclature used in the paper. The end leakage formulas were developed to suit the construction of the line of motors from which the various tests were taken and as they could not be generally applied they are not included here.

Rehabilitation of the Connors Creek Plant

Author's closing discussion of a paper published in the June 1935 issue, pages 610–17, and presented for oral discussion at the power generation session of the summer convention, Ithaca, N. Y., June 25, 1935. Other discussion of this paper was published in the September 1935 issue, pages 1001–2.

R. E. Greene: C. M. Gilt correctly states that each problem presented in rehabilitating and increasing capacity of old power plants must be settled on its own merits. He further points out that frequently an old plant can be advantageously used for stand-by and peak load service. Athough such a considerable portion of system standby concentrated at one location would have been in some degree awkward in view of The Detroit Edison Company's practice of allocating load areas to each plant with loose-linking ties between, this difficulty would not have been insuperable. Serious

consideration was in fact given such an alternative. Apart from some relatively minor objections, the controlling reason for rehabilitating the old plant, rather than holding it for peak or emergency reserve, was the doubt in the minds of the company's engineers of the real value of reserve capacity of this nature if the considerable expense of keeping the plant manned and banked is to be avoided.

Gilt also states that Table II of the paper shows that for new money plus value of equipment from other locations amounting to a total of $20,826,100, an increase of 150,000 kw in capacity is obtained at $139 per kilowatt. This is not a proper interpretation of this table as the sum mentioned not only covers the cost of this additional capacity but also includes money required to modernize the old plant. He refers also to the ultimate book value of $30,367,200, showing that the final book value per kilowatt will be approximately $92. This is true but in considering these figures it should be remembered that the final book value includes nearly $10,000,000 in land, buildings, and equipment retained from the old plant.

C. A. Powell questions the statement "if high pressure units had been superimposed on the old plant, more maintenance and lower reliability, inherent with old equipment, would be retained." This statement applied to the old low pressure units and is borne out by the fact that the turbines were due for rebucketing in a very short while if they were to be retained. Additional factors which led to the decision not to superimpose were: (1) the low pressure turbines were nonbleeding and superimposition would have involved an undesirable system for feed water heating; (2) 2 30,000 kw generators were available for the construction of the new units and many parts of the old units could be used; (3) the use of existing machine foundations, which were adequate for the new and larger high pressure machines, would provide a very acceptable station arrangement; and (4) as the new machines were to be single shaft and operated on a simple regenerative cycle, they would provide a very rugged and easily operated group of units. Obviously, many of these factors could not be evaluated in terms of dollars and a great deal of personal judgment, based on local conditions, entered into the decision.

D. F. Pennell evidently feels that with the load area system of operation it is necessary to predict locations of load increases very closely or else the necessary added capacity will not be able to serve them. No difficulty from this occurs in Detroit as the system ties are capable of transmitting considerable blocks of power between areas. He also believes that this method of operation does not permit the best use of machines in obtaining system efficiency. Again this is a matter of size of area ties, and as evidence that these are ample in Detroit, the net average system heat rate for the company's entire system in 1934 was 14,630 Btu per kilowatt-hour. During that year the new and more efficient machines at Connors Creek were, as yet, supplying practically no energy. With 2 of the new 30,000 kw units now in service, the average system heat rate of the company's territory is approaching 14,000 Btu per kilowatt-hour and will presently better that figure. Additional improvement may be expected with further development of the Connors Creek rebuilding program.

Capacitive Excitation for Induction Generators

Discussion of a paper by E. D. Bassett and F. M. Potter published in the May 1935 issue, pages 540–5, and presented for oral discussion at the electrical machinery session of the summer convention, Ithaca, N. Y., June 26, 1935.

R. E. Hellmund (Westinghouse Elec. and Mfg. Co., E. Pittsburgh, Pa.): As an old induction motor designer, I always have taken some interest in the possible application of induction machines as generators. In the early days when it generally was considered that induction machines were cheaper and simpler than synchronous machines and when certain difficulties were experienced from hunting with reciprocating prime movers, there seemed to be a possibility of applying induction generators. More recently, however, the economic relation between synchronous and induction machines for larger capacities has become such that the synchronous machine is replacing the induction machine in a great many applications.

For very small capacities it appears that the recent development of a great many different permanent magnet steels will tend toward the design of small synchronous generators with permanent magnets. There is, however, an intermediate range, such as mentioned by the authors, where there still seems to be a possibility of induction generators finding occasional application; particularly in the case of isolated engines and machines for some special purposes where higher frequencies are used, capacitive excitation may work out economically.

In the paper, an arrangement for obtaining self-compounding of these generators is discussed. It may be quite worth while, however, to consider voltage regulators for adjusting the capacity, or, in some cases where there is no need for maintaining constant frequency, the regulation might be accomplished through speed regulation of the prime mover.

Charles Kingsley, Jr. (Massachusettes Institute of Technology, Cambridge): This paper presents interesting test data concerning certain characteristics of induction machines which, for the most part, have been well known for a number of years. I should like to mention that the determination of the operating characteristics of a 3-phase induction generator with shunt capacitive excitation has been used for several years as the basis of a dynamo laboratory experiment regularly assigned to senior students in electrical engineering at M. I. T. (See "Electrical Engineering Laboratory Experiments," C. W. Ricker and C. E. Tucker, McGraw-Hill Book Co., second edition, third impression, 1930, p. 297.) The obvious disadvantage of this method of excitation is the resulting poor voltage regulation, as shown in figure 2 of the paper. The method of "compounding" the induction generator by the use of the appropriate value of series capacitance is interesting and, I believe, original.

In discussing in the paper the effects of short circuits suddenly applied to the induction generator with shunt capacitive excitation, the statement is made that the machine current "never can be greater than that flowing in the windings at the time of application of the fault." This statement is incorrect, as is clearly pointed out in a paper entitled "Short-Circuit Current of Induction Motors and Generators" by R. E. Doherty and E. T. Williamson (A.I.E.E. Trans., v. 40, 1921, p. 509–39). The transient current can be calculated readily by the methods presented by S. J. Levine ("An Analysis of the Induction Motor," Elec. Engg., v. 54, May 1935, p. 526–9) or by the methods given in the references at the end of Levine's paper.

Although the final steady-state short-circuit current is zero for 3-phase short circuit of an induction generator with shunt capacitive excitation, the first peak of the transient current may well be several times rated full load current. In fact, since the transient reactance of an induction machine is usually smaller than that of a synchronous machine of the same rating, the transient short-circuit current of an induction generator is usually larger than that of a synchronous generator of the same rating.

The experimental work of this paper was done on small machines which usually have relatively high ratios of resistance to reactance, and consequently the transient currents die out rapidly. It is probably for this reason that the authors did not observe the large transient short-circuit currents which can be expected when larger machines are short-circuited.

C. G. Veinott (Westinghouse Elec. and Mfg. Co., Springfield, Mass.): This paper is an unusually fine exposition on the subject treated, and furthers existing fundamental knowledge of induction machines. I was particularly interested in this paper because this phenomenon is likely to occur in a high

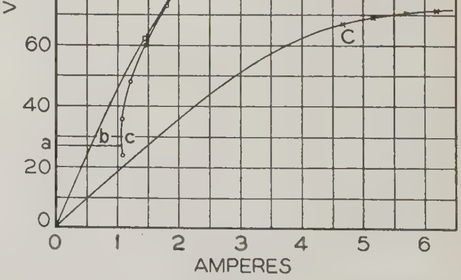


Fig. 1. Test curves taken on a capacitor-start motor
See text for explanation

torque capacitor motor which coasts for a long time after the power is removed.

To check the theory given by the authors, a $^1/_4$ horsepower capacitor-start motor was selected and a no load saturation curve taken in the manner shown by the authors in their figure 1. For this test, different values of capacitance were connected across the main winding and the motor was driven by a direct connected duplicate motor. The points indicated by squares on curve A in figure 1 of this discussion were obtained.

For comparative purposes, the results of a conventional "running saturation"—such as is normally taken to separate core loss from friction and windage—were plotted as curve B in figure 1 of this discussion. In the middle range of voltages, the agreement is practically perfect; at the lower voltages, the current ac is greater than the current ab because of the power component necessary to drive the motor, whereas in ab this power component is absent because the machine is externally driven; the discrepancy between the curves at the higher voltages is explained almost entirely by the drop in speed of the driving motor at the high loads. (At 5 amperes, the speed was only about 1,750 revolutions per minute.) These results thus substantiate the theory of the authors.

In an actual motor of the capacitor-start type, the capacitor is not connected directly across the main winding but may be considered as being shunted across the 2 windings in series, as shown in figure 2 of this discussion. It was assumed that these 2 windings, for the purposes of our discussion, could be thought of as a single winding having a number of turns equal to $\sqrt{T_m^2 + T_a^2}$, where T_m and T_a are the turns on the main and auxiliary windings, respectively.

To check this assumption, the starting switch was short-circuited and a saturation curve was obtained by connecting different values of capacitance in place of the capacitor shown in figure 2. For each value used the machine built up to voltage E with an exciting current I being supplied by the capacitor. To compare the points obtained in this manner with those previously taken, I was multiplied by a constant C and the corresponding E was divided by C where $C = \sqrt{\frac{T_m^2 + T_a^2}{T_m}}$. The points thus obtained were plotted as crosses in figure 1 of this discussion.

It will be noted that these points agree remarkably well with curve A, thus substantiating the authors' theories, as well as justifying the assumption that the 2 windings may be considered as a single winding. (The voltages across the main and auxiliary windings were measured separately, as well as the voltage across the condenser, and it was found that the voltages across the main and auxiliary windings were not in exact quadrature but were slightly more than 90 degrees apart, although this angle seemed to remain nearly constant for several different values of capacitance.)

It was particularly interesting to note that values of less than half the normal capacitance used with this motor would cause the motor to build up to such high voltages that the power losses were 2 to 3 times the motor rating. If the starting switch of this motor had been short-circuited, so that the motor's own capacitor had been connected across the windings, and the motor driven at synchronous speed, the motor would have burned up in a very short period, and it is quite likely that this is typical of any high torque capacitor motor.

However, this situation is not nearly so serious as it seems and very rarely causes trouble. The starting switch normally does not reclose until the motor has slowed down to approximately half speed where the conditions are very much different. Curve C in figure 1, shows a saturation curve taken with the motor driven at approximately 850 rpm. It may be noted that at any given current, the voltage is approximately half of that obtained at full speed. This is as might be expected. However, the number of microfarads required for the machine to build itself up to any given current at half speed is 4 times as great as the number of microfarads to build it up to the same current at full speed because not only is the voltage halved by the reduction in speed, but so is the frequency. Below a certain critical value of capacitance the machine will not build up as a generator, as pointed out by the authors. This critical value is inversely proportional to the square of the speed. Thus it happens that, at half-speed, difficulty from the motor building up as a capacitor-excited generator is seldom encountered.

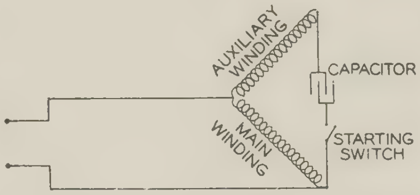


Fig. 2. Connections of a capacitor-start motor

The methods of the authors may be extended as indicated in this discussion to study just such phenomena.

Time-Temperature Tests to Determine Machine Losses

Discussion and author's closure of a paper by M. D. Ross published in the May 1935 issue, pages 513–5, and presented for oral discussion at the electrical machinery session of the winter convention, Ithaca, N. Y., June 26, 1935.

R. E. Hellmund (Westinghouse Elec. and Mfg. Co., E. Pittsburgh, Pa.): In view of the fact that time-temperature curves given by the author have met the rather exacting requirements of loss determination in a very satisfactory manner, it is surprising that no greater use is made of such curves in connection with the rating of electrical machinery where intermittent duty cycles have to be precalculated. During the International Electrotechnical Commission meeting at Prague last fall, serious consideration was given to standardizing on a multiplicity of intermittent duty ratings for electrical machinery. It may readily be seen what an enormous amount of testing would be required with the great number of machinery ratings to establish for each of them a number of intermittent duty ratings. It has been repeatedly shown in literature that intermittent duty cycles can be readily calculated, with an accuracy sufficient for such purposes, from the continuous rating and certain data relating to the rate of temperature rise as can be determined from the tangent of the time-temperature curve. Therefore, it would seem that when ratings of machinery are to be standardized to assist in the application of machinery to intermittent duty cycles, it should be along such lines.

As pointed out in the paper, it is not convenient to obtain time-temperature curves for the rotating parts, and at times it may be difficult to obtain such curves even for the stationary parts. It is, however, quite possible in such cases to obtain short time ratings, that is, 5, 10, or 15 minute ratings, which give a rather close indication of the time-temperature characteristics of the machine. The American Committee on Railway Machinery, which deals with one of the applications where intermittent duty cycles are always encountered, is seriously considering the proposal of short time ratings to take the place of the former one hour rating. This, I believe, is a step in the right direction and one worth-while following in the standardization of ratings of other electrical machinery.

L. A. Kilgore (Westinghouse Elec. and Mfg. Co., E. Pittsburgh, Pa.): This paper presents a method of measuring the loss density in the various parts of electrical machines which has proved very useful in determining the actual distribution of losses throughout the machine.

Where the loss is uniform throughout a body which is fairly well insulated thermally from other bodies the simple relations given in the paper are sufficiently accurate for practical measurements. However, in some cases where the thermal resistance between 2 bodies having different loss densities is relatively low, the initial slope measured in the 5 minutes required by the ordinary methods of test deviates considerably from the initial value which is a true measure of the loss density. Methods of empirically correcting for these effects have been given in the paper.

It is interesting to consider the theoretical analysis of the thermal transient of 2 masses with some thermal conductivity between them. This approximates the condition of the stator copper and stator iron and also the conditions in the ring sample used for calibration.

Let q_1 and q_2 be the rates of heat generation of the 2 bodies in watts per second; C_1 and C_2 the thermal capacities in watts second per degree centigrade; K_1 the thermal conductivity from the first body to the air, and K_2 the thermal conductivity between the 2 bodies. Then

$$C_1 \frac{d\theta}{dt} + K_1\theta_1 - (\theta_2 - \theta_1)K_2 = q_1$$

and

$$C_2 \frac{d\theta_2}{at} + K_2(\theta_2 - \theta_1) = q_2$$

Writing P for $\frac{d}{dt}$ and reducing to one equation for the general solution with q_1 and $q_2 = 0$,

$$[C_1C_2P^2 + (K_2C_1 + K_1C_2 + K_2C_2)P + K_2K_1]\theta_1 = 0$$

Let $-\alpha$ and $-\beta$ be the roots of this equation, then

$$\theta_1 = A\epsilon^{-\alpha t} + B\epsilon^{-\beta t}$$

$$\theta_2 = A\left[\frac{K_1 + K_2}{K_2} - \frac{\alpha C_1}{K_2}\right]\epsilon^{-\alpha t} + B\left[\frac{K_1 + K_2}{K_2} - \frac{\beta C_1}{K_2}\right]\epsilon^{-\beta t}$$

where A and B are constants to be determined from the initial conditions.

It will be noticed that each temperature now contains 2 exponential terms. It can be shown that the initial value of $d\theta/dt$ is a measure of the loss density. However, if numerical examples are taken, it will be found that the slope over a 5 minute interval deviates appreciably from the initial values.

Calibration tests were made on a ring sample with an exciting winding wound over a thin layer of insulation. The specific heat determined from the known loss and the slope of the time-temperature curve based on the readings taken for 5 minutes deviated widely from the theoretical value. However, when the above analysis was used and the complete theoretical curves drawn, it became apparent that the experimentally determined slope was being influenced by the surrounding copper and the copper loss. At low values of flux the exciting current and the copper loss were very low so that the thermal capacity of copper caused the apparent specific heat of the iron to be too high. At high values of flux the exciting current was such as to cause a much higher loss density in the copper than in the iron, causing the apparent specific heat to be too low. At the point where the loss densities were equal the test checked the theoretical specific heat quite accurately.

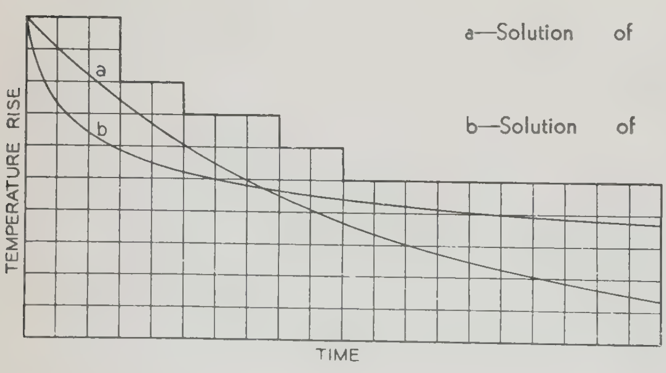
P. M. Lincoln (Cornell University, Ithaca, N. Y.): I note that certain deductions as to losses are made by determining the slope of the cooling curve during the first few minutes after shutting off the load. It is obvious that under the test conditions described, cooling takes place almost entirely by forced convection. I would like to ask whether or not any tests have been made to determine the law which connects cooling rate with temperature elevation under forced convection. It is well known that convection in still air does *not* follow a first power law. Is the same true with forced convection? This is important since any deductions drawn from the slope of the cooling curve during the first few minutes of cooling are affected by this cooling law. A slope taken on a given machine at a given temperature elevation does not hold for slopes taken at other temperature elevations unless the first power cooling law holds. Has this first power law for forced convection been confirmed?

It is obvious that if the first power law for cooling holds for forced convection, the differential equation for cooling takes the form $\frac{d\theta}{dt} = -\theta$ where θ is the temperature elevation and t is the time. The solution of this equation is the well known exponential. If, however, the cooling law for forced convection is the same as for convection in still air, it is well known that the differential equation takes the form $\frac{d\theta}{dt} = -\theta^{1.25}$. The solution of this equation is *not* an exponential, although it has some of the characteristics of an exponential. The solution of these 2 differential equations is plotted in figure 1 of this discussion. It is obvious from these curves that any deductions drawn from the rate of cooling during the first few minutes after shutting off the load are vitally affected by the cooling law.

May I suggest that the author would do the electrical industry a great favor if he will demonstrate by experiment whether or not the first power law holds for forced convection?

There is no intention of disputing the soundness of the method of loss determination suggested in the paper. The slope of the time-temperature curve at zero time is a true measure of losses. The curves in figure 1 of this discussion are deceiving in that they indicate a marked difference in the slopes at zero time. If we were at liberty to change the law of cooling at will, the temperature at zero time would be different with different cooling laws, but slope at zero time would always be the same. If the first power law holds, the resulting exponential time-temperature curve has the property that the intercept between a tangent and a vertical at the point of tangency on the $\theta = 0$ line is the same at any point on the curve. If the first power law does not hold, the resulting time-temperature curve does not possess this property. Consequently, any deductions drawn from the slope at zero time will depend on the temperature elevation at which the slope is taken.

If this discussion has emphasized the need for more accurate experimental data on the laws which govern forced convection, it will have accomplished its object.



a—Solution of differential equation $\frac{d\theta}{dt} = -\theta$

b—Solution of differential equation $\frac{d\theta}{dt} = -\theta^{1.25}$

Fig. 1. Comparison of slopes of cooling curves

V. M. Montsinger: (General Electric Company, Pittsfield, Mass.): While not tied up directly with cooling of revolving apparatus windings after shutdown, I believe it may be of interest to point out some of my experience in connection with the cooling of transformer windings after shutdown. Since it is not possible to measure the resistance of transformer windings at the instant of shutdown, it is necessary to have a simple method of correcting for the temperature drop between the instant of shutdown and the time of resistance measurement, which in some cases may amount to several degrees. A cooling curve is, of course, the most accurate method but for economic reasons is not always practical.

A few years ago I undertook the problem of working up a simple rule for the cooling of transformer windings after shutdown. The general formula for the cooling of a body is of the form

$$T = T_0(1 - \epsilon^{-\beta t}) \qquad (1)$$

where

T = the temperature rise at any instant over its ambient temperature
T = the temperature rise at shutdown
β = time constant
t = time

As shown in my paper on "Cooling of Oil-Immersed Transformer Windings After Shutdown" (A.I.E.E. TRANS., v. 36, 1917, p. 711–30) the value of β may be written

$$\beta = \frac{0.676\,\dfrac{aW_c}{A + a}}{\theta_0}$$

where

a = the cross-sectional area of the copper, and
A = the cross-sectional area of the copper plus the insulation

Equation 1 is not rigorously correct since it is based on the assumption that the loss of heat is proportional to the temperature rise, which is seldom true. The loss by convection (for either gas or liquid) is generally proportional to the temperature rise raised to the 5–4 power.

I soon discovered that this formula could not be used for transformer windings, apparently for the reason that the cooling medium or ambient was the oil (flowing through the ducts) which changed in temperature along and with the winding temperature during the cooling period. Furthermore, the temperature of the oil in the ducts could not be determined very easily.

According to equation 1, for the same initial temperature rise the winding with considerable insulation on it should cool off at a slower rate than a winding with a small amount of insulation on it. Tests made on a variety of transformer windings showed that for the first 3 or 4 minutes the cooling

appeared to be approximately independent of the amount of insulation. At first this was difficult to understand. It soon became evident that some other factor was present which offset the increase in thermal capacity and expected slower cooling.

A further study showed that this other factor was the higher initial temperature rise due to the insulation. This is well illustrated by 2 cooling curves taken on 2 duplicate coils, one of which had no tape on it, and the other of which had 18 layers of 0.012 inch varnished cambric wound over both sides. These curves are given in figures 4 and 5 of the above mentioned paper.

These curves show that while the taped coil started cooling at a slower rate than the untaped coil, yet due to its greater initial rise the 2 curves crossed when considered only from the standpoint of cooling in degrees centigrade. For practical purposes the cooling of transformer windings can, therefore, be based on the copper loss only for the first 3 or 4 minutes. For periods longer than about 4 minutes, the cooling of the taped coil became greater than that of the untaped coil.

It should be stated, however, that this condition does not hold where the increase in insulation increases the surface dissipating the loss as, for instance, where heavy turn insulation is used in line end or buffer coils, and where the extra insulation does not materially increase the initial rise. In most transformer windings the influence of the heavier turn insulation in the buffer coils is negligible and, hence, the cooling after shutdown can be made a function of the copper loss only. The present rule in A.I.E.E. Standards No. 13 for the cooling of transformer windings after shutdown is based only on the copper loss.

M. D. Ross: The discussions by V. M. Montsinger and R. E. Hellmund bring out the importance of time-temperature measurements to the designer of electrical machinery, especially in regard to short time ratings. In predetermining the performance of machines on intermittent loading, it has been customary to calculate the losses in the machine parts and from these losses and the heat storage and cooling properties of the machine, determine the shape of the time-temperature curve corresponding to the estimated values of losses. The procedure described in the present paper is merely a reversal of this process by which we are able to obtain the actual value of losses which could not otherwise be experimentally obtained.

P. M. Lincoln in his discussion asks if the rate of change of heat flow in machines such as were tested is a linear function of the temperature difference between the machine surfaces and the cooling air. The following is quoted from page 642 of an article by G. E. Luke ("The Cooling of Electric Machines," A.I.E.E. Trans., v. 42, 1923, p. 636–51) in regard to heat transfer rates with forced convection as in turbine generators.

"The unit heat transfer increases slightly with the temperature of the surface and air, but tests by the writer show that, for all practical purposes, the unit heat transfer is independent of the temperature over a range of from 20 degrees centigrade to 100 degrees centigrade."

Even where natural ventilation is used, the test method is theoretically correct. If the rate of heat flow is assumed to be proportional to $(\theta - \theta_c)^{1.25}$, equation 1 of the paper becomes

$$q_1 = k(\theta - \theta_c)^{1.25} + C\frac{d\theta}{dt}$$

Under steady conditions,

$$q_i = q_c = k(\theta_0 - \theta_c)^{1.25} \text{ or } k = \frac{q_i}{(\theta_0 - \theta_c)^{1.25}}$$

At time $t = 0$,

$$C\frac{d\theta}{dt} = -k(\theta_0 - \theta_c)^{1.25}$$

$$= \frac{-q_i}{(\theta_0 - \theta_c)^{1.25}}(\theta_0 - \theta_c)^{1.2}$$

or

$$\frac{d\theta}{dt} = \frac{-q_i}{C}$$

In other words, the initial slope of the time-temperature curve is the same for natural or forced ventilation. As the slope is measured by taking readings over about a 5 minute period, there might be some slight difference in the results depending upon the type of ventilation.

L. A. Kilgore's discussion is very important as it gives a method of determining the possible error due to rapid heat flow from one part of the machine to another which might affect the accuracy of the loss measurements. A rough calculation, using the equations given in his discussion, will quickly establish the possibility of errors in the values determined by test.

Sparking Under Brushes of Commutator Machines

Discussion and authors' closure of a paper by R. E. Hellmund and L. R. Ludwig published in the March 1935 issue, pages 315–21, and presented for oral discussion at the electric machinery session of the summer convention, Ithaca, N. Y., June 26, 1935.

J. C. Aydelott (General Electric Co., Erie, Pa.): Those who have examined the contact surfaces of brushes are familiar with the characteristic marking consisting of discolored areas in the otherwise polished surface. These areas may be of greater or less width extending back from the trailing edge of the brush and ending at a very distinct line.

Possibly this brush face marking may be related to the phenomenon which is the subject of this paper. In figure 9 of the paper there is shown a ripple in the current flowing from a commutator segment at the instant that the preceding segment breaks contact with the brush. If commutation is bad enough these ripples may be quite pronounced. The rapid increase in current density, according to the paper, will cause sparking under the brush resulting in the discolored brush surface just mentioned. If the phenomenon is caused by the preceding segment under the same brush, the discoloration will extend exactly one segment width under the brush face. In some cases a similar result is caused by flux linkages in common between upper and lower conductors in the slot when a segment breaks contact with a brush of opposite polarity. In such a case, the width of the discoloration on the brush face may be some fraction of the width of a commutator segment.

In some instances there are 2 or more grades of discoloration, but in each there is a distinct line of demarcation between the various grades of discoloration or between the discolored areas and the polished surface. It is the sharpness of this line which confirms the authors' observation that sparking is caused by the rate of rise of current density. The location of the line as explained above is related to segment spacing and brush spacing.

V. P. Hessler (Iowa State College, Ames): I would like to add a few thoughts to the authors' statements concerning the theory of the sliding contact. The various theories of the sliding contact as presented in the literature fall into 3 classes: the point conductance theory, the thermal emission theories, and the oxide film theory. The point conductance theory alone fails to explain most of the phenomena of the sliding contact but is useful as a part of the oxide film theory.

The various thermal emission theories have been most widely advocated and seemed to explain many of the voltage phenomena of the sliding contact. The fact that visible sparking does occur on occasion between the brush and the ring indicates that thermal emission does take place under certain conditions, and that it must be considered in the complete theory of the sliding contact. The decided effect of ring temperature upon contact drop was the first phenomenon to come to the writer's attention which seemed inexplicable on the basis of thermal emission. It was found by E. Arnold (*E.T.Z.*, v. 28, 1907, p. 263–6) that increasing the ring temperature from 20 to 90 degrees centigrade sometimes produced almost a 10 to 1 decrease in contact drop. These results seemed so paradoxical to the writer that they were repeated and similar results were obtained. The magnitude of the effect varies quite widely with the various grades of brushes.

A. Schliephake (*Elektrotechnische Zeitschrift*, v. 55, 1934, p. 814–15) and R. M. Baker (*Elec. Jl.*, v. 31, 1934, p. 359–60 and 448–50) have found that an oxide-free ring acts as a constant resistance as the current is varied. Also, the contact drop of the oxide-free contact is much less than the ordinary contact. Thus the oxide film seems to be rather largely responsible for the voltage phenomena of the sliding contact.

If the temperature effects mentioned above and the drooping characteristic of the voltage current curves are to be explained upon the basis of the oxide film it would seem that cupric oxide should have a negative temperature coefficient of resistance. This is found to be the case as shown by F. Horton ("The Electrical Conductivity of Metallic Oxides," *Phil. Mag.*, v. 11, 1906, p. 521). The conductivity of fused cupric oxide increases many times as the temperature is increased from 12 to 150 degrees centigrade. Thus it is apparent that certain sliding contact phenomena are ex-

plained upon the negative temperature coefficient of resistance of cupric oxide.

On the basis of the above theory the transient voltage drop characteristic of a sliding contact would be expected to lie above the static characteristic as found by the authors. In some similar experiments made by the writer on stationary contacts the resistance continued to decrease for 5 seconds after the current was suddenly increased.

W. A. Boyer (Anaconda Copper Mining Co., Butte, Mont.): In connection with the operation of d-c machines, the problem of commutation is an ever interesting one. The general opinion is that the various coils of an armature as they come into the commutating zone are all treated alike; in other words, that a voltmeter reading from a point on the surface of the commutator under the leading and trailing edges of a brush to the brush arm indicates a commutating condition which all the coils on the armature are subjected to when they come within that zone. Also, it is more or less taken for granted that this condition remains the same with respect to time unless some change in the mechanical adjustment of the machine has been made.

This general opinion does not give a true story of what actually is taking place. In figure 1 of this discussion is shown a portion of an oscillogram taken on a 1,650 kw d-c generator shortly after it was stoned. This particular machine is used in connection with a Ward-Leonard controlled electric mine hoist, so that its working conditions are continually changing.

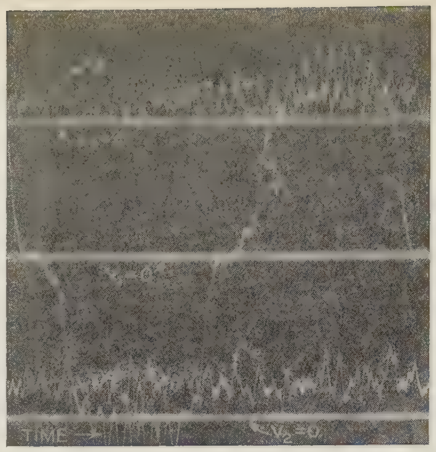
Figure 2 shows a portion of an oscillogram of the same machine taken after 14 days, and figure 3 shows another taken just before the commutator was stoned, or 3 months after that in figure 1 was taken.

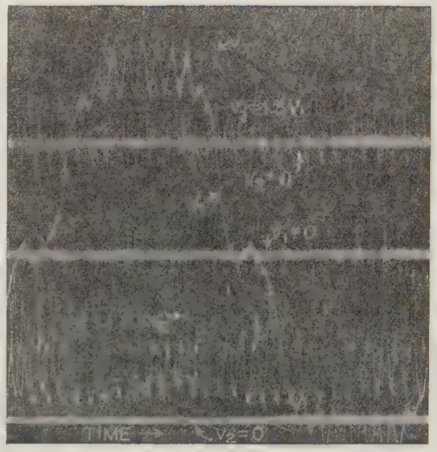
When these various figures are compared the contact drop curves are seen to change considerably. The change in figure 2 from figure 1 has been the result of growth of commutator film. When figure 2 was taken commutation was very good, no visible sparking taking place even on peak loads of 250 per cent of normal load.

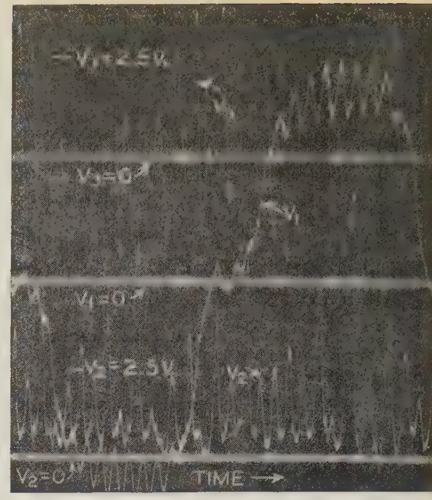
In figure 3 commutator film and mechanical roughness of bars causes the extreme fluctuations. The edges of some of the bars were burned in this instance, and commutation had become very poor.

After a commutator has been newly stoned, the film does not enter into the commutation problem. The differences in contact drop as the various bars are passing under the brush are due to the variation in the voltages that the various coils are subjected to as they pass through the commutating zone. Their voltages of self and mutual induction may vary due to the construction of the coil and their placement in the slot. The commutating flux that the coils pass through may vary due to either a magnetic oscillation caused by varying reluctance of the flux path as the armature rotates or a magnetic oscillation caused by a variation of the magnetomotive force causing the flux to pass through the commutating zone. All of these influences may be seen clearly in figures 1 and 2.

With this variation in the resultant voltages affecting the coils as they come into the commutating zone, the short circuit currents vary. Thus the commutator bars







Figs. 1–3. Oscillograms of commutation of a 1,650 kw d-c generator operating at full load of 750 volts and 2,200 amperes

Figure 1 (upper left) taken shortly after stoning commutator; figure 2 (left) taken 14 days after stoning; figure 3 (upper right) taken 3 months after stoning

V_1 = voltage between 2 adjacent bars

V_2 = voltage between a point on the commutator under the leading edge of the brush and the brush arm

V_3 = voltage between a point on the commutator under the leading edge of the brush and a point under the trailing edge

will have formed on them a commutator film of different physical and electrical characteristics, as may be observed on practically any machine by the slight variation in color of the commutator bars. In all instances it will be found that the order of coloring or burning of bars repeats itself more or less closely, every single or double pole pitch.

The contact voltage variations continue to increase with the growth of the film. If the film reaches a stable condition and the contact drop voltages are within the sparking limit of the brush, commutation may hold for an indefinite period and the brush application is said to be all right. When the film does not reach a stable condition before the voltage variation reaches the sparking limit of the brush, sparking results with its burning of bars and poor commutation. The length of time necessary for the film to reach a stable condition depends upon a great number of factors, such as the design of the machine, nature of the brush, nature of the load, speed of the machine, spring tension of the brush, temperature, character of the surrounding atmosphere, etc. The length of time to reach the sparking limit of the brush may be from a few days to several years.

My observations check fairly well with those given in the paper, in that transient voltages under the brush may reach over 3 volts with no visible sparking. From a large number of observations I am led to believe that the transient characteristic curve for any grade of brush will vary when taken on copper rings upon which a film has been formed previously using various current densities. That is, the transient characteristic curve taken while using copper rings upon which a film has been formed previously with the brushes having a current density of 60 amperes per square inch will be different from a curve taken the same way but with the film on the rings formed while the brushes had a current density of 120 amperes per square inch.

The static characteristic curve may vary somewhat, also, when using these different rings but I expect the greatest difference in the transient characteristic curve. I am led to believe, also, that the static and transient characteristic curves may not differ so much using various grades of brushes on rings newly polished, the difference appearing when rings are used upon which the films have been formed with the various current densities.

R. E. Hellmund and L. R. Ludwig: Since our paper made but very brief reference to the various theories previously advanced for contact phenomena, the discussion contributed by V. P. Hessler, giving a brief review of such theories and also adding information regarding the temperature coefficient of cupric oxide and its bearing on the phenomena, is very appropriate. The paper mentions that we did not wish to discount the importance of the oxide film theory. There is a great deal of evidence supporting this theory. For example, Doctor Holm in Germany has made striking tests by developing oxide on a slip ring and subsequently removing it over half the circumference. Voltage drop tests with such

a ring very clearly demonstrated the difference caused by the oxide film. The work done by R. M. Baker also gives quite conclusive evidence along this line. The entire matter may be further complicated, however, because of other variables in addition to the temperature coefficient of the oxide.

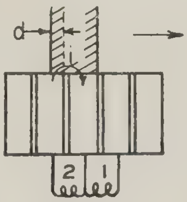
There have been described in some publication certain tests made with constant current on knife switches closed over a considerable period, which showed periodic variation in temperature. The explanation given was that a certain oxide film was formed with a low temperature, such film having a rather high resistance. Because of this high resistance, the temperature of the switch would further rise, resulting in the formation of a different oxide having a lower resistance. This in turn reduced the losses and the temperature to the point where the first oxide began to form, leading to a repetition of the cycle described. It is quite conceivable that similar conditions may prevail with a sliding contact and account for some of the phenomena encountered in practice. Particularly on account of the relatively low average temperature of the commutator and the very high local temperatures at certain points, especially where sparking occurs, the formation of different oxides at different portions of the surface and for different load conditions is quite likely. It has further been definitely established in Westinghouse laboratories that the humidity of the air affects the contact resistance over certain ranges, so that we may actually expect changes in commutation with changes in the weather.

Most of the theories advanced by various research workers for the sliding contact phenomena relate chiefly to normal load conditions. While the consideration of these conditions is the first step necessary in such investigations, the designer is really more interested in what occurs under abnormal conditions, that is, with very high current densities, etc. The reason for this is that under normal conditions he does not encounter any unusual problems and it is only in overcoming the limitations imposed by abnormal conditions that his best efforts are put forth. It is quite conceivable that a certain set of phenomena are involved when a glossy oxide surface is formed, while different or additional phenomena have to be considered under conditions causing a smudgy contact surface.

The great variety of conditions and variables is well demonstrated by the discussion given by W. A. Boyer. The data presented by him are particularly valuable because they have been collected under operating conditions such as found in actual practice. This is information of a kind that usually cannot be obtained by the designer and research worker and it would, therefore, be highly desirable for him to publish a rather full account of his work, giving the results obtained together with certain design data relating to the machines tested, such as the number of slots, the number of coils per slot, the armature chording, and so on.

We were interested particularly in his statement to the effect that different characteristics will be obtained if the oxide film has been formed with a current density of 60 amperes, for instance, than if it has been formed with a density of 120 amperes per square inch. This is quite in line with other experience to the effect that the conditions obtained during a certain load and operating condition are much influenced by the character of the preceding loads and the previous history of the commutator surface of certain parts thereof.

Figure 2 given by Boyer particularly seems to indicate that there can be a difference in different parts of the commutator not only because the electrical conditions vary periodically with the number of slots and the inductive relations of the armature coils and the slots to each other, but also because different portions of the commutator have had a different history. It will be noted for instance that the voltage V_2 varies periodically with the pole pitch. This is a condition frequently observed and which also agrees with his statements that certain coloring and burning of bars repeat themselves more or less closely every single or double pole pitch. The explanation for this seems to be that whenever the machine is standing still for some time after an operating period, the cooling conditions of certain segments near the brushes are different from the cooling conditions of the segments between brushes. Such a difference in cooling conditions seems to leave the various portions of the commutator surfaces in a different condition with regard to the oxide film and other characteristics. If subsequently the machine begins to rotate again, this difference seems to persist, as indicated in figure 2 of Boyer's discussion, even though various portions of the commutator carry the same load during the operating period.



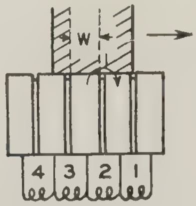
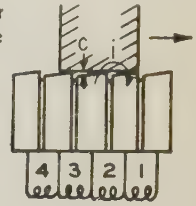


Fig. 4 (left). Transfer of short-circuit current concentrated in portion of brush

Fig. 5 (right). Transfer of short-circuit current with relatively low density in brush

The discussion contributed by J. C. Aydelott brings up a very interesting question. With conditions as shown in figure 4 of this discussion, where the short-circuit current of coil *1* may be suddenly transferred to coil *2*, which is assumed to be located in the same slot, there will occur in the brush portion d not only a sudden change in current but also a high density because d is very small. However, if in figure 5 of this discussion the short-circuit current of coil *1* is suddenly transferred to coil *2*, there will be a sudden change in current but the density will not be very high because the entire width w of the segment carries such current. As pointed out in the paper, it requires a combination of sudden current change and high density to cause sparking. However, previous considerations were under the assumption that there was contact between the entire width of the segment and the brush. With the many irregularities occurring in the contact phenomena, it is quite conceivable that one edge of the commutator segment, for instance the leading edge, will heat to a greater extent than the trailing edge, causing greater expansion of the leading edge and resulting in conditions indicated in a very exaggerated manner in figure 6 of this discussion. In actual practice, the difference in expansion may be very small and the clearance c may be in the order of only 0.0001 or 0.0002 inch. Nevertheless this may be sufficient to cause most of the current i to flow through the leading edge and thus bring about a combination of sudden change and high current density. As a matter of fact, it frequently can be noted that the appearance of the 2 edges of the segments of commutators is entirely different, indicating that their participation in the carrying of the current and in the commutating phenomena is different.

Fig. 6. Effect of greater heating of leading edge of commutator segment



In order to avoid misunderstandings, it may be well to point out that the references made in our paper to phenomena at or near the edges of the brush relate to what happens near the edges but under the brush. In a previous paper, entitled "Commutation Considered as a Switching Phenomenon," (R. E. Hellmund and L. R. Ludwig, A.I.E.E. Trans., v. 51, June 1932, p. 465–8) consideration was given to what occurs at the very edge which forms the last or first point of contact in the opening or closing of the circuit of an armature coil. In this earlier paper it was pointed out that certain damping effects are beneficial to commutation at the very edge of the brush. In the present paper it is shown that sudden changes in current are undesirable, which in turn means that any damping effects that can be introduced to avoid such sudden changes would be beneficial in preventing sparking under the brush as well. Similarly, such features as split-throw coils, which were considered beneficial in the previous paper, may prove of advantage with reference to the phenomena discussed in the present paper.

Saturated Synchronous Reactance

Discussion and author's closure of a paper by Charles Kingsley, Jr., published in the March 1935 issue, pages 300–5, and presented for oral discussion at the electrical machinery session of the summer convention, Ithaca, N. Y., June 27, 1935.

S. B. Crary (General Electric Co., Schenectady, N. Y.): This paper is of particular interest as it presents a comparison between a calculated and a test power angle characteristic which is further evidence as to the correctness of the well-known cylindrical rotor vector diagram which was used.

There is a point in the paper, however, which probably could be clarified.

In the second paragraph of his paper the author states that "the synchronizing power of the equivalent machine," as given in reference 3, "will not in general be the correct value for the actual saturated machine" and "This may introduce considerable error as shown in table II." It should be noted that the error he speaks of is in the synchronizing power coefficient $dP/d\delta$ and not in the calculated power limit, i. e., $dP/d\delta_{eq}$ of reference 3 for the cylindrical rotor case does not necessarily equal $dP/d\delta$ (actual). This was discussed in the closing discussion of reference 3 (A.I.E.E. TRANS. [ELEC. ENGG.], v. 53, April 1934, p. 605–7) in which a method was given for obtaining $dP/d\delta$ and in which it was shown that for a particular case, $dP/d\delta_{eq}$ approaches $dP/d\delta$ at pull-out. It would be of interest if Kingsley would use the method of reference 3 and compare it with the results given in his table I.

An equivalent unsaturated machine for the cylindrical rotor case can be obtained, in a similar manner as was obtained for the salient pole case, which will have all the characteristics of the actual machine for small changes. This more accurate equivalent would require 2 equivalent reactances instead of one so that the additional condition of $dP/d\delta_{eq} = dP/d\delta$ can be satisfied. However, as in the salient-pole case, this additional refinement is not usually necessary for practical calculations as both salient pole and cylindrical rotor machines are quite accurately represented at pull-out by one equivalent reactance.

H. B. Dwight (Massachusetts Institute of Technology, Cambridge): This paper is characterized by a feature which is of great advantage in this type of work, namely, the author of the paper has made tests which show the accuracy of his method.

If a stability calculation whose purpose is to show the load angle for a certain case or the maximum load for certain conditions, transient or otherwise, is put forward, then the person making the calculation should, if at all possible, make tests on practical machines to show the correctness of the calculation. Such tests should involve as complete a set-up of apparatus as possible and a variety of machines, and should include actual measurements of load angle and of maximum power, since these are the results for which the calculation is made. Greatly improved instruments for measuring load angle are now available, among the new types being those developed by Prof. H. E. Edgerton. It is to be hoped that more of such tests will be published.

L. A. Kilgore (Westinghouse Elec. and Mfg. Co., E. Pittsburgh, Pa.): This paper presents in a very clear manner one method of dealing with saturation in calculating power angle curves and maximum power for a synchronous machine. Any method of accurately calculating the excitation of a machine for different loads is suitable for calculating maximum power in a step-by-step process. The A.I.E.E. method of calculating excitation at different power factors from the no load and zero power factor curves gives good results, although it is largely empirical.

The assumption is made that the saturation is the same under load as at no load for the same air gap voltage. This assumption is not so incorrect for turbine generators but for salient pole machines it is seriously in error. If the author had used Potier's reactance x_P throughout instead of stator leakage, the accuracy would be greatly improved for Potier's reactance empirically makes an allowance for the field leakage and although it does vary somewhat, it gives quite good results. Using Potier's reactance equation 2 becomes: $x_s = x_P + \dfrac{x_d - x_P}{k}$ and none of the other equations are affected.

It is interesting to consider the difference between the "saturated synchronous reactance" as described in this paper and the "equivalent reactance" described in reference 3. The saturated reactance represents at least approximately the correct relations between the fluxes in the machine and gives the true angle. It is necessary to correct the internal voltage by a factor k to get a measure of the field current and since k and the reactance are variable, the synchronizing power or rate of change of power with respect to angle can only be calculated by a step-by-step process.

The equivalent reactance as described in reference 3 was intended as a value which would give the correct change in terminal conditions for a small change away from a given operating point with field excitation. The value of the equivalent reactance is generally much lower than the saturated reactance, and has less physical meaning since it does not represent the flux in the machine and it does not give the true internal angle. The equivalent reactance is easier to use once it has been determined, since it requires no factor k for the internal voltage. The problem of calculating or testing this quantity is difficult for an accurate analysis shows that the value is not only different for each operating condition, but varies appreciably for different directions of change away from each point.

The conclusion is that the equivalent reactance is a quantity easy to use but difficult to determine whereas the saturated reactance as defined here is more readily determined and has a direct physical meaning. Both of these quantities have their use in special problems but it is doubtful if either will supplant the short circuit ratio as a measure of a machine's stability under steady state conditions.

Sterling Beckwith (The Metropolitan Water District of Southern California, Los Angeles): The author has made a valuable contribution to the knowledge of saturated synchronous reactance. He has, by the artifice of representing a machine by a "saturated" reactance that must be used with a changing internal voltage, obtained a much simpler expression for saturated synchronous reactance than is otherwise possible, so that in many cases his method should be superior to others.

A question I wish to raise, however, is whether as a general method of attack in a complicated problem the method of using varying internal voltages is as powerful or simple a method as the method (references 1, 2, 3, 4, and 5 of this discussion) which uses either a fixed internal voltage, or an internal voltage which is constant during the incremental interval in which stability is being checked.

In Kingsley's method there is a simple expression for saturated reactance; a simple expression for the voltage behind it; a reasonably simple method of calculating power angle characteristics; a fairly simple method of calculating maximum power; and the limitation that the external system must be reduced to an equivalent series impedance.

In the other methods, there is a complicated expression for saturated or equivalent or adjusted reactance (but a simple approximation) a fairly simple expression for internal voltage (it must be merely the internal voltage that gives proper terminal conditions) a simple method of checking stability at the point of operation (any of the conventional methods may be used) a difficult step-by-step method of obtaining a power angle characteristic (because of the changing saturation) and a simple means of allowing for complicated external systems by means of transfer and driving point impedances. There is also a further advantage in that 2 machines of equal reactance at the pull-out point will have equal stability limits, which is not true of the reactance Kingsley defines as saturated synchronous reactance.

In the problems with which I have dealt in the past, the second method has appeared to be better than Kingsley's method but different problems and more experience with his method are necessary before drawing a final conclusion.

1. EQUIVALENT REACTANCE OF SYNCHRONOUS MACHINES, S. B. Crary, L. A. March, and L. P. Schildneck. ELEC. ENGG. (A.I.E.E. TRANS.), v. 53, Jan. 1934, p. 124–32.

2. ADJUSTED SYNCHRONOUS REACTANCE, AND ITS RELATION TO STABILITY, H. B. Dwight. *Gen. Elec. Rev.*, v. 35, Dec. 1932, p. 609.

3. Discussion by G. C. Dahl of ref. 1. ELEC. ENGG. (A.I.E.E. TRANS.), v. 53, April 1934, p. 604–5.

4. Discussion by J. W. Butler of ref. 1. ELEC. ENGG. (A.I.E.E. TRANS.), v. 53, March 1934, p. 484.

5. Discussion by Sterling Beckwith of ref. 1. ELEC. ENGG. (A.I.E.E. TRANS.), v. 53, March 1934, p. 486–7.

Charles Kingsley, Jr.: S. B. Crary believes that the statement in the second paragraph of the paper in regard to the synchronizing power of the equivalent machine may be misleading. I believe that the statement in the paper is correct, though perhaps it should be amplified; i. e., "the synchronizing power of the equivalent machine will not *in general* be the correct value for the actual saturated machine." However, as Crary has pointed out, it is expected that this error usually will be small at pull-out.

I did not use the method of reference 3 of the paper to calculate the maximum power, as Crary has suggested, since I do not believe that this method is readily adapted to this type of calculation. As stated in the fourth paragraph of the paper, the method of reference 3 is, practically speaking, restricted to cases where the terminal operating conditions are explicitly

known. In calculating maximum power at given constant values of field excitation and receiver bus voltage, the terminal load conditions at pull-out are not explicitly known. Hence, in order to use the method of reference 3, it would be necessary to resort to a rather laborious cut and try process.

I agree with H. B. Dwight that the publication of further test data showing power angle characteristics for a variety of machines under both steady-state and transient conditions would be of great interest.

L. A. Kilgore's discussion raises several interesting points. He mentions the use of the A.I.E.E. method for calculating maximum power. Methods *D* and *E* of table I of the paper are adaptations of the A.I.E.E. method, and the table shows the accuracy of these methods for several different cases. Although the A.I.E.E. method and its various modifications give reasonably close results in calculating excitation at lagging power factors, larger errors may be introduced in calculating maximum power, since the A.I.E.E. method does not give the correct value for the load angle of the machine.

Kilgore suggests that the use of Potier's reactance instead of armature leakage reactance may give increased accuracy, since Potier's reactance empirically makes an allowance for the change in field leakage under load conditions. However, as Kilgore points out, the effects of field leakage are usually not very great in turbine generators, and as shown by March and Crary (ELEC. ENGG., v. 54, April 1935, p. 378) Potier's reactance is usually approximately equal to armature leakage reactance for cylindrical rotor machines (though I have found a case in which Potier's reactance was about twice the calculated value of armature leakage reactance). Since this paper is confined to the cylindrical rotor case, probably there is in most cases little difference between Potier's reactance and armature leakage reactance. Also I should like to point out that the results obtained by the methods of my paper are only slightly affected by the value of leakage reactance used in the equations. For example, I have calculated case I, method *B* of table I of the paper using a value of $x_a = 0.052$ (half the value used in calculating table I). The result so obtained checks the value given in table I to within a fraction of one per cent. However, I am attempting to extend the methods of this paper to the salient-pole machine, and in this case his suggestion may prove valuable.

Kilgore makes an interesting comparison between the "saturated synchronous reactance" as described in my paper and the "equivalent reactance" of reference 3. I should like to add that an analytical solution for the steady-state synchronizing power can be obtained from saturated synchronous reactance for the simple system of figure 1 as follows:

Differentiating equation 6 of the paper, remembering that (kZ) and α are variables:

$$\frac{dP_s}{d\delta} = \frac{E_f V}{kZ}\left(1 - \frac{d\alpha}{d\delta}\right)\cos(\delta - \alpha) - \frac{1}{kZ}\left[\frac{E_f V}{kZ}\sin(\delta - \alpha) + \frac{2E_f^2 r}{(kZ)^2}\right]\frac{d(kZ)}{d\delta} \qquad (1)$$

It is necessary to evaluate $d\alpha/d\delta$ and $d(kZ)/d\delta$. It can be shown that these derivatives can be expressed as follows:

$$\frac{d\alpha}{d\delta} = \frac{x_m}{kZ}\sin\alpha\,\frac{1}{k}\frac{dk}{d\delta} \qquad (2)$$

$$\frac{d(kZ)}{d\delta} = (kZ - x_m\cos\alpha)\,\frac{1}{k}\frac{dk}{d\delta} \qquad (3)$$

The derivative $dk/d\delta$ can be evaluated in terms of the slope of the saturation curve by differentiating equation 8 of the paper and making use of equation A-6 derived in Appendix A of reference 3. The result is:

$$\frac{1}{k}\frac{dk}{d\delta} = -\frac{sE_f V Z_c x_m \sin(\delta - \alpha_c)}{(kZE_a)^2\left(1 - \frac{sx_m}{kZ}\cos\alpha\right)} \qquad (4)$$

where

s = slope saturation factor = a/E_a
a = intercept shown in figure 2 of reference 3.

For given terminal load conditions, all quantities on the right hand side of equation 4 can be calculated. Hence $\frac{1}{k}\frac{dk}{d\delta}$ can be calculated from equation 4 and the result substituted in equations 2 and 3. Substituting the resulting values of $d\alpha/d\delta$ and $d(kZ)/d\delta$ in equation 1, the corresponding value of the synchronizing power can be obtained. If the field excitation E_f and receiver bus voltage V are given, but the terminal load conditions are not explicitly known, the saturated values of $(kZE_a)^2$, (kZ), α and s to be used in the equations can be obtained as functions of δ by using equation 8 of the paper and auxiliary curves similar to those of figure 5 of the paper, adding to figure 5 a curve of s as a function of $(kZE_a)^2$.

Sterling Beckwith has pointed out a limitation to the method described in the paper. Equations 2 and 3 of the paper give the saturated synchronous reactance and excitation voltage of a cylindrical rotor machine regardless of the external system, and can be used in the analysis of a complicated system by employing a cut and try process to determine the air gap voltage at which the saturation factor k of each machine must be taken. I believe that something of this nature also may be necessary in using the methods of references 3, 4, and 5 of Beckwith's discussion. However, in order to use equation 8 and the auxiliary curves of figure 5 of the paper, the external system must be reduced to an equivalent impedance in series with a constant voltage. I believe that this same limitation is imposed on the method of reference 3 of the paper (reference 1 of Beckwith's discussion), and that any approximations which can be made in reducing the external system for use with the method of reference 3 can also be used in connection with the method of my paper. Since the publication of my paper, I have developed a way of applying my method to more complicated systems by using it to calculate machine performance charts somewhat similar to those described in Beckwith's paper (ELEC. ENGG., v. 54, July 1935, p. 728–34).

I believe that in any comparison between the methods of this paper and other methods of calculation, consideration should be given to the fact that the methods of this paper are based on a more nearly correct theoretical foundation than the others (with the possible exception of reference 3 of my paper), and that the accuracy is accordingly greater. The accuracy with which the system to be analyzed can be reduced to a simple equivalent system and the accuracy desired in the solution, as well as the relative simplicities of the various methods of calculation, will be important factors in deciding which method should be used.

Effects of Saturation on Machine Reactances

Discussion and author's closure of a paper by L. A. Kilgore published in the May 1935 issue, pages 545–50, and presented for oral discussion at the electrical machinery session of the summer convention, Ithaca, N. Y., June 27, 1935.

Charles Kingsley, Jr. (Massachusetts Institute of Technology, Cambridge): Satisfactory theoretical methods are known for taking into account the effects of saturation on the synchronous reactances under balanced steady-state operating conditions. Under unbalanced or transient conditions, the effects of saturation on the various reactances become extremely complicated, and at present no satisfactory theoretical solution for these effects is known. This paper presents empirical methods based on the average of the results obtained from a large number of tests on representative machines of various types. The paper summarizes the best information which is at present available. The proper use of these empirical methods should, in most cases, increase the accuracy of calculations involving the unbalanced or transient operation of machines which do not differ greatly from average designs. However, it is to be hoped that methods eventually will be developed which take into account more accurately the various factors which affect the results.

Kilgore has raised a question in regard to the specification of the degree of saturation to be understood in giving standard definitions of the various reactance coefficients. This question is one which should be definitely settled. He recommends that some of the reactances be defined as the unsaturated values, while others be defined as the saturated values applying for sudden short circuit at rated voltage with no load.

I should like to suggest that all of the reactance coefficients and time constants be defined as the unsaturated values, and that this be the value to be understood unless the saturation is otherwise specified. My reasons for making this recommendation are:

1. The synchronous machine theories from which these reactance coefficients are derived are based upon the assumption that the magnetic circuit is mathematically linear. This assumption is usually most nearly correct at low flux densities. Therefore, if the reactances are all defined as the unsaturated values, the definitions will have a reasonably correct theoretical basis.

2. The definitions should be consistent. If some of the reactances are defined as the unsaturated values, while others are defined as the saturated values, it is necessary to remember in which manner each of a rather large number of reactances and time constants is defined with respect to saturation.

3. It is easier to measure the unsaturated values. Kilgore suggests that the unsaturated values can be measured and the saturated values obtained from them by the use of empirical saturation factors. This reduces the accuracy of the saturated values so obtained to about the accuracy of these saturation factors. This seems scarcely accurate enough to be standardized in a definition. It is desirable that the value of each reactance given in the standard definitions should be directly and easily measurable.

Kilgore raises the practical point that the reactances and time constants which he suggests be defined as the saturated values are most commonly used in calculating short-circuit currents at full voltage, and that considerable error may be introduced in these calculations if the unsaturated values are used. This practical objection to the use of the unsaturated values in standard definitions can readily be overcome by supplying those who make short-circuit calculations with the saturated values, definitely stating that the values given are the saturated values. The use of the unsaturated values in standard definitions does not mean that these values should be used in full voltage calculations. Perhaps a brief statement to this effect should be included in the definitions, together with a convenient table of approximate saturation factors, such as table III of the paper.

Whenever there is any ambiguity as to the exact meaning of the "unsaturated" value, I recommend that it be understood as the value obtained at rated armature current.

S. B. Crary (General Electric Co., Schenectady, N. Y.): This paper is a valuable contribution to the literature of saturation and its effect on synchronous machine reactances in that it summarizes extensive test data and at the same time presents an approximate method of calculating the saturation factor for transient reactance.

In calculating the effect of saturation in a salient-pole machine, the author has obtained his result by calculating 2 saturation factors k_{sm} and k_{sl} which are determined independently, the net factor is then k_{sm} times k_{sl}. Further, he has assumed that k_s which is used in the calculation of k_{sm} does not change during the transient. Actually k_s changes as the distribution of flux changes. These assumptions are not necessary in order to obtain a comparatively simple expression for x_d' (saturated), as may be shown as follows. Consider the case of a salient-pole synchronous machine with laminated magnetic paths and which for the sake of simplicity in this discussion has no stator saturation. Then the saturation characteristics are defined by the 2 sets of curves in figure 1 of this discussion. The following nomenclature is used:

$k_2 = 1 + \dfrac{\text{rotor saturation mmf}}{\text{air gap mmf}}$ = air gap flux rotor saturation factor

k_3 = field leakage flux saturation factor

i_d = direct axis armature current

i_{fd} = field current

ψ_{ad} = direct axis air gap flux or flux mutual with field and armature

ψ_{fld} = field leakage flux

$\psi_{ffd} = \psi_{ad} + \psi_{fld}$ = total field flux

x_l = armature leakage reactance

x_{afd} = mutual reactance between field and armature

x_{fld} = field leakage reactance

Then

$$\psi_{ffd} = \left[\frac{x_{afd}}{k_2} + \frac{x_{fld}}{k_3}\right] i_{fd} - \left[\frac{x_{afd}}{k_2}\right] i_d \quad (1)$$

$$\psi_{ad} = \frac{x_{afd}}{k_2} i_{fd} - \left[\frac{x_{afd}}{k_2} + x_l\right] i_d \quad (2)$$

Initially, since $i_d = 0$,

$$\psi_{ffdo} = \left[\frac{x_{afd}}{k_{20}} + \frac{x_{fld}}{k_{30}}\right] i_{fdo} \quad (3)$$

$$\psi_{ado} = \frac{x_{afd}}{k_{20}} i_{fdo} \quad (4)$$

At the end of the first half cycle since

$$-\psi_{ado} = \psi_{ad} \quad (5)$$

and

$$\psi_{ffdo} = \psi_{ffd} \quad (6)$$

$$\left[\frac{x_{afd}}{k_{20}} + \frac{x_{fld}}{k_{30}}\right] i_{fdo} = \left[\frac{x_{afd}}{k_2} + \frac{x_{fld}}{k_3}\right] i_{fd} - \frac{x_{afd}}{k_2} i_d - \frac{x_{afd}}{k_{20}} i_{fdo} = \quad (7)$$

$$\frac{x_{afd}}{k_2} i_{fd} - \left[\frac{x_{afd}}{k_2} + x_l\right] i_d \quad (8)$$

Solving for i_d,

$$i_d = \frac{\dfrac{x_{afd}}{k_{20}}\left[\dfrac{x_{afd}}{k_2} + \dfrac{x_{fld}}{k_3}\right] + \dfrac{x_{afd}}{k_2}\left[\dfrac{x_{afd}}{k_{20}} + \dfrac{x_{fld}}{k_{30}}\right]}{\left(\dfrac{x_{afd}}{k_2} + x_l\right)\left(\dfrac{x_{afd}}{k_2} + \dfrac{x_{fld}}{k_3}\right) - \dfrac{x_{afd}^2}{k_2^2}} i_{fdo} \quad (9)$$

Similarly i_f may be calculated. Therefore, from equation 9,

$$x_{d\,(saturated)}' = 2 \; \frac{\left(\dfrac{x_{afd}}{k_2} + x_l\right)\left(\dfrac{x_{afd}}{k_2} + \dfrac{x_{fld}}{k_3}\right) - \dfrac{x_{afd}^2}{k_2^2}}{\dfrac{1}{k_{20}}\left[\dfrac{x_{afd}}{k_2} + \dfrac{x_{fld}}{k_3}\right] + \dfrac{1}{k_2}\left[\dfrac{x_{afd}}{k_{20}} + \dfrac{x_{fld}}{k_{30}}\right]} \quad (10)$$

When no saturation exists, $k_2 = k_3 = 1.0$, and equation 10 reduces to the well-known relation for transient reactance

$$x_d' = x_d - \frac{x_{afd}^2}{x_{ad} + x_{fld}}$$

The initial saturation coefficients k_{20} and k_{30} are known and the coefficients k_2 and k_3 can be obtained from figure 1 and equations 1 and 2 for estimated values of i_{fd} and i_d. The above expression is more general than that obtained by Kilgore in that it takes into account the change in the flux distribution from the initial condition to that at the end of the first half cycle.

In determining the saturation factors for the case of transient stability we would be interested in knowing in more detail what assumption Kilgore has made. We have made an analysis of the effect of saturation on the performance of synchron-

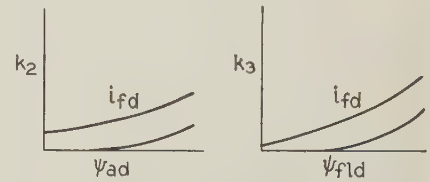


Fig. 1. Saturation characteristics assumed for salient pole machine

See text for explanation

ous machines for small oscillations. In this analysis the rate of change of saturation with respect to the flux, $\partial k/\partial \psi$, constitutes an important factor which cannot be neglected. It is apparent, therefore, that for any study which involves any oscillation such as that obtained in stability that the correction for the reactance cannot be arrived at as simply as in the case of calculating the peak of the first half cycle of armature short-circuit current. The equivalent transient reactance obtained from this analysis of small angular oscillations for the case of a machine having laminated magnetic paths, no stator saturation, and no saturation affected by quadrature axis flux, is

$$x'_{d(eq)} = x_{d(eq)} - \frac{x_{afd(eq)}\, x_{fad(eq)}}{X_{ffd(eq)}} \quad (11)$$

where

$$X_{ffd(eq)} = x_{fad(eq)} + x_{fld(eq)} \quad (12)$$

$$x_{d(eq)} = x_l + \frac{x_{afd}}{k_2\left[1 + \dfrac{\psi_{ad}}{k_2}\dfrac{\partial k_2}{\partial \psi_{ad}}\right]} \quad (13)$$

$$x_{afd(eq)} = \frac{x_{afd}}{k_2\left[1 + \dfrac{\psi_{ad}}{k_2}\dfrac{\partial k_2}{\partial \psi_{ad}}\right]} \quad (14)$$

$$x_{fad(eq)} = \frac{\left[x_{afd} - \psi_{ad}\dfrac{\partial k_2}{\partial i_{fd}}\right]}{k_2\left[1 + \dfrac{\psi_{ad}}{k_2}\dfrac{\partial k_2}{\partial \psi_{ad}}\right]} \quad (15)$$

$$x_{fld(eq)} = \frac{\left[x_{fld} - \psi_{fd}\dfrac{\partial k_3}{\partial i_{fd}}\right]}{k_3\left[1 + \dfrac{\psi_{fld}}{k_3}\dfrac{\partial k_3}{\partial \psi_{fld}}\right]} \quad (16)$$

Equation 12 corresponds to equation 9 of reference 2 for the same conditions, i. e., no stator saturation or rotor saturation affected by quadrature axis flux.

S. H. Mortensen (Allis-Chalmers Mfg. Co., Milwaukee, Wis.): This very able paper shows that a number of the reactances and time constants used in the method of

symmetrical components and analysis of transients are not constants, but due to saturations in various leakage paths, change with the current values involved. This means that each current value has a corresponding reactance, and to prevent complications the author suggests that only one value of the respective constants be standardized which, unless otherwise specified, he proposes should be the saturated value. This suggestion would have merit if the current values involved always would correspond to one operating condition, such as a short circuit at machine terminals, etc. As this is not the case where line constants enter into the picture, it would seem preferable to both operators and designers to base guaranteed reactance constants upon a current value that can readily be obtained and checked by test, such as would be the case if reactances corresponding to rated current were standardized upon. Corrections corresponding to other current values could then be made with sufficient accuracy with the aid of characteristic saturation factor curves similar to those shown in this paper. For that reason it would seem justifiable to recommend that the one value of reactance and time constant corresponding to the rated machine current be standardized.

E. H. Freiburghouse and **C. E. Kilbourne** (both of General Electric Co., Schenectady, N. Y.): The author has presented a very interesting paper summarizing tests on the effects of saturation on synchronous machine reactances and also has given a useful approximate method of obtaining a saturation factor for the transient reactance. As a result of this study a suggestion is advanced that one value of certain so-called constants is sufficient to define the machine behavior and that that value is the saturated value, this meaning the value obtained from a sudden short-circuit from full voltage. This is an excellent and simple treatment of the problem and will undoubtedly give the industry more accurate values of these constants than an indiscriminate use of the unsaturated values would give. However, these are instances when the industry could be served to further advantage with some additional information and other cases where the limiting values of these constants are not the minimums as given by the saturated values but the maximums as given by the unsaturated values.

A survey of approximately 20 constants, including reactances and time constants which can be affected by saturation, indicates that for salient pole machines only 2 frequently used constants, the transient reactance x_d' and its associated quantity T_d' show significant changes with saturation in the operating ranges where they are used in calculations. On wound rotor machines saturation has a much more important effect. Here the transient reactance x_d', the subtransients x_d'' and x_q'', the negative sequence x_2, the zero sequence x_0, and all their associated time constants change with saturation. On double winding generators the through reactance is also affected, and on a few machines the stator leakage reactance x_l has appreciable variations. However, there are reactances x_d, x_q, etc., which are essentially unsaturated.

In each case where saturation plays an important part it is dependent upon the value of armature current existing under the conditions being calculated. Consequently, any one value of a constant that saturates is truly accurate only under the particular conditions existing when it was obtained. This can easily be seen by an inspection of the curves presented in the paper. Therefore, the giving of a single saturated value for these quantities will have their use subject to interpretation for any except the one condition.

Also, the defining of some constants as saturated for all synchronous machines, both salient pole and round rotor, and others as not saturated may produce confusion. Particularly is this true when some of those defined as saturated for one type of machine are unsaturated for the other. Instead of simplifying the existing method as applying to the entire synchronous machine field and producing more accurate results of calculations with less experience required to handle the quantities correctly, this procedure might have the opposite result.

The advantages, which Kilgore has pointed out, in giving saturated values of these constants can be combined with the present general practice so as to further the needs of the industry without omitting some information which can be made available. This can be done if the basis of giving this information remains in nearly its present form, that is, that all constants be given as "normal values," that value being defined as the value existing when normal 3-phase armature current flows under the conditions dominated by the quantity. In addition to these normal values, a "saturation factor" could be given for the particular constants on any machine which needs these values to permit their accurate use.

In order to standardize on practice, the saturation factor generally given can be defined as the ratio existing between the value of the particular constant obtained from a sudden short circuit from full voltage and the normal value. As an equation for the transient reactance this is

$$f = \frac{x_d' \text{ saturated}}{x_d' \text{ unsaturated}}$$

where

f is the "normal saturation factor"

x_d' saturated is the transient reactance obtained from a short circuit from full voltage

x_d' unsaturated is the transient reactance obtained from a short circuit where normal armature current existed

Further, certain constants can be specified as having saturation factors appreciably different than unity. These are so shown in table III of the paper. In addition, average values of these normal saturation factors can be published and if felt desirable can be standardized. Also the methods of obtaining such factors can be standardized. Because of the fact that full voltage short-circuit tests are very expensive to make on test set-ups for large machines it is desirable that these factors be obtained as outlined by Kilgore.

If this should be done the basic method now in use in specifying these constants would not be greatly changed and hence little confusion would result. Also the magnitude of these "saturation factors" will definitely point out which constants on a particular machine are subject to saturation without affecting the same constants on other machines and, further, the use of "ratios" to point out these affects indicates the degree of saturation which a single value cannot. This is quite desirable information when interpretations from one condition to another are to be made.

L. A. Kilgore: The chief purpose of this paper was the presentation of data on the effects of saturation on the various machine reactances. The choice of which value should be used as a reference is somewhat arbitrary, but it is important that some agreement be reached on the value to be understood for each constant when the degree of saturation is not specified.

In their discussions, Mortenson, Kingsley, Kilbourne and Freiburghouse state their preference for rated current values rather than saturated values as recommended in the paper. There are good reasons for both methods of dealing with the problem. I do not believe that Kilbourne and Freiburghouse meant to imply, as it would seem from the fifth paragraph of their discussion, that the present practice is to give all reactance values on a rated current basis. It has been our practice to give saturated values, except for synchronous reactance, unless the unsaturated value was specified. Previous to the time when locked tests were first seriously considered for measuring reactances (about 5 years ago), reactances were generally measured by short circuits and the values used generally were saturated values. The new standard decrement curves were worked up on the basis of typical relations between saturated values.

The chief objection to having the rated current values as the reference is that if no correction is used, serious errors may result in certain cases. The worst case is for a 2 pole turbogenerator where the use of rated current values of subtransient reactance will give a short-circuit current 28 per cent low for a short circuit at the terminals. Kingsley suggests that this objection can be readily overcome by giving saturated as well as rated current values where there is appreciable difference. Kilbourne and Freiburghouse suggest a saturation factor which would be the ratio of saturated to the rated current value. Both of these suggestions are good in that they would give more information; nevertheless, they will undoubtedly lead to some complication and confusion. However, it would seem that some such method would be most satisfactory to the majority of those who have so far expressed an opinion.

S. B. Crary asks concerning the assumptions made in calculating the saturation of the transient reactance for use in stability calculations. The methods given in the appendix and the curves of figure 7 are sufficiently general to calculate the saturated value for each point in the transient swing. The typical curves of figure 2

were intended to be used with the maximum value of current which occurs as pull-out is approached.

Crary's discussion gives another method of calculating a saturated transient reactance for each instant. He also presents a new concept of an "equivalent transient reactance." It would be interesting if he can publish some quantitative data on such a constant and compare it with the "saturated value" as used in the paper. It would not seem reasonable to expect so large a difference between "saturated" and "equivalent transient reactances" as between "saturated" and "equivalent" synchronous reactances described in references 1 and 2 of the paper.

Armature Leakage Reactance of Synchronous Machines

Discussion and authors' closure of a paper by L. A. March and S. B. Crary published in the April 1935 issue, pages 378–81, and presented for oral discussion at the electrical machinery session of the summer convention, Ithaca, N. Y., June 27, 1935.

P. L. Alger (General Electric Co., Schenectady, N. Y.): It is desirable to keep clearly in mind the object of continuing this rather involved discussion of various kinds of reactance.

Originally, there was thought to be just one important reactance of a synchronous machine, the reactance which, in conjunction with the armature reaction, determined the voltage regulation. Potier gave a useful method for finding this reactance from test data, and so it has been called after his name. Then, it was found that this reactance did not check the reactance required to determine the momentary short circuit current, so that this latter, or "transient" reactance, was separately considered. It was thought that the transient reactance was equal to the Potier reactance plus an element due to the field. Next, it was learned that the difference between the transient reactance and the field reactance is normally much smaller than the Potier reactance. Thus, the true armature leakage reactance which, added to the field reactance, accounts for the transient reactance, came to be more accurately calculated and generally used as distinct from the Potier reactance. The Potier reactance was then recognized to be larger than the leakage reactance, because of the drop in voltage under load, due to the increased saturation of the pole body resulting from increased field leakage. For correct calculation of voltage regulation by the Potier diagram, it was necessary to use a fictitiously high value of leakage reactance.

The authors of this paper have now shown that this increment of leakage reactance due to pole saturation, which is a part of the Potier reactance, varies markedly with machine saturation, thus fairly completing our understanding of the nature of all of these machine reactances.

It is evident that the actual value of Potier reactance is only useful for problems of calculating regulation, and so is not pertinent to ordinary machine performance guarantees. The importance of the paper lies in the improved understanding it gives of the actual flux densities existing throughout the magnetic circuit, thus enabling designers to more intelligently proportion the parts of a machine to utilize the materials effectively.

C. C. Shutt (Westinghouse Elec. and Mfg. Co., East Pittsburgh, Pa.): This paper presents some interesting data on the variation of Potier's reactance. However, a review of a considerable amount of test data does not seem to justify the conclusion that Potier's reactance as tested at any practical value of pole saturation may be used as a reliable measure of the armature leakage reactance.

One type of data not presented in the paper is the variation of Potier's reactance with armature current for constant values of voltage. A set of test curves was taken on a standard commercial motor rated 200 horsepower at 450 rpm. This machine had about 75 per cent of its saturation in the pole at open circuit rated voltage.

Figure 1 of this discussion shows the results of these tests, which covered a fairly wide range of current and voltage. Two noticeable points shown by the tests on this machine are:

1. Over the range tested the variation in Potier reactance is only 22 per cent.
2. The Potier reactance, at the lowest value tested, is 240 per cent of the calculated armature leakage reactance.

The same data can be plotted as reactance against voltage for constant armature current.

As stated in the paper, it is desirable to have some means of testing for the armature leakage reactance. Both Alger and Kilgore have pointed out that the value which they calculate is based on arbitrarily classifying a certain portion of the total flux in the machine as armature leakage reactance. The most practical means of checking this value appears to be that used by Alger of measuring the direct axis synchronous reactance and subtracting the properly calculated reactance of armature reaction. This is not a direct test and involves subtraction of a number from another, which is not much larger, to obtain a relatively small number for the answer. But the average absolute variation as given in Alger's paper between calculated and test results for 100 machines is only 0.6 per cent.

Table I of the present paper gives data on 5 machines. Here the average absolute variation by the high field current method is 28 per cent while by subtracting X_{ad} from the tested value of X_d gives an average variation of only 3 per cent.

Another interesting feature brought out by a study of the curves is that for a constant voltage back of Potier reactance; the value of this reactance is practically constant for a variation in the armature current. The variation in the reactance under these conditions is shown by the dashed line K in figure 1 of this discussion.

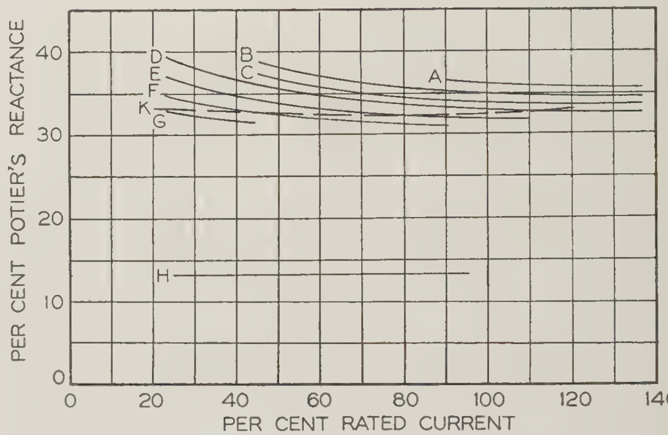


Fig. 1. Variation of Potier's reactance with armature current at various voltages

Voltages: A—57 per cent; B—68 per cent; C—79.5 per cent; D—91 per cent; E—102.5 per cent; F—113.5 per cent; G—125 per cent; K—Constant back of Potier reactance; H—Curve for x_l by Kilgore's method.

This curve is derived as follows: Calculate the voltage back of Potier reactance for any point. Using the point of 100 per cent current and 91 per cent terminal voltage, it is found that the voltage back of Potier reactance is 124 per cent. Consider this voltage constant and calculate the terminal voltage for different values of armature current. With a couple of trials the calculated value of terminal voltage can be made to check with the value of Potier reactance as shown on the curve for that value of voltage and armature current.

T. A. Rogers (University of California, Berkeley): Between 1895 and 1905 there were many papers on the subject of alternator regulation discussing the theoretical aspects of the problem and suggesting some methods for experimentally determining the factors to be used in computing the voltage drop under load. Practically all of the methods considered the leakage reactance of the armature windings in some form or other. Of the many methods proposed ("The Regulation of Alternators," D. B. Rushmore, Intl. Elec. Congress *Trans.*, St. Louis, 1904, v. 1, p. 729) that of Potier probably has received the most attention until recent years.

In 1900 Potier pointed out that if one shifted the no load saturation curve downward and to the right by given amounts, it would approximate very closely the zero power factor curve for overexcitation. That is, the no load saturation curve and the zero power factor curves are practically parallel to each other. These latter curves may be drawn for any current, just so long as that current is constant. In addition, by shifting the no load saturation

curve upward and to the left, Potier indicated that it would successively coincide with the underexcited zero power factor curves. The horizontal shift, he states, is due to the effects of armature reaction, the vertical shift to dispersion (leakage).

Two coefficients are suggested in the determination of the excitation of a machine. One of them, λ, has the dimensions of reactance, and is used in conjunction with the armature current to determine the voltage drop due to the leakage fluxes. The other, α, has the dimensions of turns (zero dimensions), and, when multiplied by the phase current, gives the magnitude of the demagnetizing armature ampere turns referred to the rotor field.

Potier proposed his theories only as an attempt to determine the excitation of a loaded alternator, and not to introduce a method of determining armature leakage reactance. He made no mention of armature or pole leakage fluxes, or of leakage reactances, as such. He merely pointed out that magnetic leakage in the machine influenced the vertical displacement of the no load saturation curves, and that the magnitude of the shift is affected by saturation in the poles. He states further that the length of the air gap, and the depth and enclosure of the slots, modify the leakage flux, and hence the vertical displacement. Although one might assume from his discussion that armature leakage flux was implied in the treatment, his writings fail to disclose specific reference to that quantity or to armature leakage reactance.

Many writers have interpreted Potier's theory as a means of determining armature leakage reactance in addition to its value in calculating excitation. In spite of the fact that Rothert pointed out in 1902 that the Potier reactance coefficient was not true armature leakage reactance but contained, in addition, the components of pole leakage occurring under load, many authors of textbooks continue to cite Potier's triangle as the only method which will give the true armature leakage reactance.

It is quite evident, as pointed out by the authors and conclusively indicated by their test results, that the substitution of the Potier reactance for the armature leakage reactance of a machine introduces large errors in the performance calculations. As discussed in an earlier paper by Crary, March, and Shildneck, the value of the air gap voltage is directly dependent upon the correct values of armature resistance and armature leakage reactance, and it is this voltage that serves as a measure of the amount of saturation occurring within the machine. An error of 75 or 100 per cent in the value of leakage reactance used may introduce an error of 200 per cent or more in the calculated value of field amperes necessary to overcome the effect of saturation within the machine.

Furthermore, leakage reactance, because of the calculation of additional field ampere turns required for the saturated machine, affects the torque angle. The angle always is decreased because of saturation. However, depending upon the power factor, replacing armature leakage reactance by Potier reactance may decrease the apparent air gap flux, decrease the apparent saturation, and consequently increase the torque angle. Both excitation and angle thus are influenced appreciably by the value for amature leakage reactance.

This paper provides not only a procedure for experimentally approaching armature leakage reactance but also a rather complete physical picture of the component parts of that fictitious quantity termed Potier reactance. These results should assist even those who are most reluctant in discarding Potier reactance in machine theory and substituting armature leakage reactance which is not only susceptible to an exact definition but also may be calculated.

B. L. Robertson (University of California, Berkeley): The first point of major significance in the paper is that the so-called Potier reactance is not armature leakage reactance but is something quite different. This fact has long been known, but Potier reactance still is very commonly employed in textbooks as true armature leakage reactance, in some instances without the apparent knowledge that these 2 reactances are not the same thing. Potier reactance may include armature leakage reactance as a component, and, as shown by the procedure introduced by the authors, may approach the latter as a limiting value. Nevertheless, Potier reactance as usually found is far too great in magnitude to be substituted for armature leakage reactance with any accuracy.

The continued use of Potier reactance as the substitute for armature leakage reactance is based upon the fact that whereas armature leakage reactance is calculable but difficult of test, Potier reactance is not calculable but is comparatively easy to test. Potier reactance, however, does not possess the nice physical significance which lies behind armature leakage reactance, and it is for this reason that Potier reactance is defined only by stating its method of being found, rather than by picturing it also in terms of physical concepts.

Usual statements are to the effect that in determining Potier reactance one should work at some point above the knee of the saturation curve. Rated voltage is most often taken as that point, although for no quite definite reason. As shown very clearly by the authors, it obviously makes a considerable difference just where the Potier triangle is located.

In the determination of the leakage reactance triangle, the authors do not state explicitly the manner by which they obtain the zero power factor curves, although one should infer that actual runs with circulating currents were taken. Treatments on locating such curves make use of several methods, some of which are for constructions using only one, or perhaps 2, test or calculated points. Because the saturation curve *may not* be shifted parallel to itself to give the zero power factor curve, and because the proposals for obtaining this latter curve do not all agree with each other, it might be well for the authors to indicate definitely their method.

In determining the load saturation curves the authors rely upon the assumption that armature leakage reactance is not affected by saturation, and further imply (there is no attending statement to this) that the direct synchronous reactance is unsaturated. Working out on the curves at such high degrees of saturation as are shown it is hard to believe that these quantities are not so affected, in which case the leakage reactance triangles will be altered progressively, ultimately to change the Potier reactance curves. In this respect, assumption 3 and conclusion 4 appear to require further discussion. The conclusion seems to be more a restatement of the assumption than a conclusion.

Since the authors rely upon the approach of Potier reactance to the *calculated* armature reactance (this, of course, to show the value of their treatment), it would seem that the second conclusion should contain also a recommendation that with data available the calculated armature reactance be used. Certainly, calculation is quicker and more convenient than test.

I think it is of considerable value to have this method which the authors give for approaching armature leakage reactance by test. The only difficulties I see in its use lie in excitation problems and field heating when one is attempting to obtain very high saturation, particularly with a machine which does not saturate easily.

L. A. Kilgore (Westinghouse Elec. and Mfg. Co., E. Pittsburgh, Pa.): This paper presents some interesting data on Potier's reactance. The conclusion is reached that Potier's reactance varies so widely as to be of limited use and also that the Potier's reactance approaches the leakage reactance at high values of voltage.

Although it is true that the Potier's reactance does vary, it is still a very useful quantity. It forms the basis of the A.I.-E.E. method of calculating the zero power factor saturation curves from the no load curve and one or more zero power factor test points.

For accurate calculations it is possible for the designer to determine the stator and rotor saturation separately at the actual operating conditions. Even here some approximation is necessary since it is seldom possible to determine the complete flux distribution in the teeth. The stator saturation under load may be determined approximately as at no load for a voltage equal to terminal voltage plus half the stator leakage. The rotor saturation is determined by the actual flux in the pole which is the sum of the gap flux on the direct axis and the field leakage. The ampere turns effective in producing field leakage are determined by adding vectorially the armature demagnetizing and the sum of the gap and stator saturation ampere turns.

For others who use machine characteristics and for rapid design calculations, the Potier's reactance is very useful since it gives a method of estimating the total saturation under load from the no load saturation curve.

The Potier's reactance is not entirely empirical since the voltage back of Potier's reactance may be visualized as a flux somewhere between the flux in the stator and the flux in the rotor, which gives an approximate measure of the total saturation. If all the saturation is in the stator x_p is approximately $^1/_2 x_l$; if it is all in the rotor $x_p = x_l + xf$, where x_l = stator leakage and xq = rotor leakage reactances. When the saturation at no load in the stator is

F_s and in the rotor F_r, the Potier's reactance may be approximated as x_p =

$$^1/_2 x_l + (^1/_2 x_l + x_f)\left(\frac{F_r}{F_s + F_r}\right).$$

The Potier reactance decreases at higher voltages because of saturation of the leakage paths and in some machines it increases considerably below rated voltage because the ratio of rotor to stator saturation is increasing. This last effect is especially true of machines with gaps under the poles or partially filled dovetail slots. The authors have concluded that the Potier's reactance approaches the leakage reactance for the reason that field leakage due to the armature demagnetizing approaches zero. Actually this component of the field leakage is reduced by the saturation of the pole tip but it does not approach zero. The stator leakage is also reduced appreciably by saturation at high densities. Thus it would seem that although the Potier's reactance may be as low as the stator leakage and even lower in some cases, it cannot be said theoretically to approach the leakage reactance. The data which the authors give does not show any accurate agreement between the minimum values of x_p and the unsaturated value of x_l, and the data which the writer was able to collect gave even less consistent results. Therefore, it does not seem practical to use this as any test for the leakage reactance.

The test curves on 30 machines of all types were studies to determine the variations in Potier's reactance. These results might be summarized by saying that the ratio of the reactance at 90 per cent voltage to that at 110 per cent voltage ranged from 1.05 to 1.2 for salient pole machines with relatively large gaps under the pole and for salient pole machines with little or no gap under the pole, the ratio ranged from 1.0 to 1.15. For turbine generators the variation was, in general, much less and in several cases the Potier's reactance decreased at lower voltages.

Charles Kingsley, Jr. (Massachusetts Institute of Technology, Cambridge): This paper shows that the Potier reactance measured at high saturation is probably approximately equal to the armature leakage reactance; that is, at high values of saturation, the Potier reactance approaches the value of leakage reactance calculated by certain design formulas. Unfortunately, there are at present no entirely satisfactory tests by which the accuracy of these design formulas can be established.

The Potier method assumes that the zero power factor characteristic at any given constant value of armature current is a curve equal to the open circuit characteristic shifted vertically by a voltage drop and horizontally by a magnetomotive force, each of which is proportional to the armature current and is independent of saturation. As is shown in the paper, this is not strictly correct, largely due to the effects of field leakage.

In references 3 and 4 the armature leakage reactance drop is defined as the voltage drop due to certain component fluxes which include all of the component fluxes producing fundamental reactive, armature voltage and which probably are affected only slightly by saturation. Hence, if the Potier reactance were constant, it should be approximately equal to the armature leakage reactance as defined in references 3 and 4, since it too would be that part of the total reactive effect of armature current which would be independent of saturation. It is shown in the paper that at high values of saturation, although the field leakage may be large, its effect on the Potier reactance becomes small and the Potier reactance approaches a constant value. It therefore seems reasonable to conclude that the highly saturated value of Potier reactance is approximately equal to the leakage reactance. This is probably the best test for leakage reactance which is at present known.

L. A. March and **S. B. Crary:** C. C. Shutt has shown, for a given set of test data, the variation of Potier reactance with change in armature current, with voltage as a parameter. We would judge from his results that the test data were not for high degrees of saturation. In this respect, it is unfortunate that he did not present the saturation curves from which the data were obtained, so that this question could be answered and his test results compared with that presented in the paper.

L. A. Kilgore states "The authors have concluded that the Potier's reactance approaches the leakage reactance for the reason that field leakage due to the armature demagnetizing approaches zero. Actually this component of the field leakage is reduced by the saturation of the pole tip but it does not approach zero." It is important in discussing this point to make a definite distinction between saturation magnetomotive force and flux. We have attempted to show that the *additional* field leakage flux at high values of saturation caused by the addition of a field magnetomotive force equal to $x_{ad}i_d$, becomes increasingly less in magnitude, as well as a smaller proportion of the total. It is true that there is a large amount of total field leakage flux producing saturation under these conditions. However, the additional increment of field leakage flux caused by the magnetomotive force $x_{ad}i_d$ becomes less, and is small although the saturation produced by this increment of flux may be large. This is shown by the small vertical and the large horizontal displacements respectively of the curves a and c of figures 1 and 2.

We cannot agree with Kilgore that under load the stator tooth saturation is proportional to a voltage equal to the terminal voltage plus half the stator leakage. We have been able to calculate, with good accuracy, zero power factor characteristics assuming the stator saturation to be proportional to the voltage back of leakage reactance. Accordingly, for a machine with all the saturation in the stator, we consider that $x_p = x_l$. For any other condition $x_p = x_l +$ *a factor which is a function of the load saturation* (neglecting possible changes in wave form, which may affect the final required magnetomotive force).

B. L. Robertson has asked as to the method used in obtaining the test data. The zero power factor characteristics were obtained by operating the machines as overexcited synchronous condensers and obtaining enough points to draw a curve. Additional machines were used to supply the losses, and to provide zero power factor lagging kilovolt-amperes. The open circuit characteristics were obtained in the regular manner. Machines actually were tested using values of excitation as high as 10 per unit (1,000 per cent of normal). The rotors were watched carefully for heating, and the high values of excitation held only as long as necessary, with ample time allowed between tests.

We have made attempts to calculate the decrease in leakage reactance due to tooth saturation. A sufficient portion of the leakage flux path is in air, so that even though the iron becomes highly saturated, the decrease in the total permeance will be but slight. In addition, the teeth are not saturated uniformly over the entire pole pitch. If an attempt is made to calculate the effect of saturation on slot leakage permeance, it will be found that at a density of 150,000 lines per square inch (over the middle $^1/_3$ of the pole pitch) the decrease will be approximately 5 per cent. As a result, it seems reasonable to conclude that in salient pole synchronous machines leakage reactance is sensibly independent of saturation for practical operating flux densities.

Protective Signaling

Discussion and author's closure of a paper by P. M. Farmer published in the June 1935 issue, pages 617–23, and presented for oral discussion at the selected subjects session of the summer convention, Ithaca, N. Y., June 27, 1935.

Wm. A. Del Mar (Habirshaw Cable and Wire Co., Yonkers, N. Y.): I should like to inquire whether any device has been developed to protect libraries from theft of books. A few years ago I tried inserting a magnet in the back of a book and winding a wire in several turns around a door frame in the hope that the magnet passing through would induce enough current in the wire to give a signal. The wire was plugged to the microphone socket of a radio set and sure enough, the loud speaker clicked whenever the magnet passed through the door. Unfortunately, it did the same when a bunch of keys was passed through the door. It seemed impossible to distinguish reliably between a book thief and John Doe, Member A.I.E.E., with a bunch of keys in his pocket. However, I do not believe that the possibilities of such an arrangement were exhausted and I should like to hear whether the author knows of any success having been attained in the matter of devices for protection against theft of library books.

P. M. Farmer: Commercial burglary alarm service does not ordinarily include the type of protection that would be demanded by the service mentioned by W. A. Del Mar. His approach to the problem seems to be

a most logical one and it is possible that the detector circuit could be made to distinguish between magnetized metal such as would be used in the book and nonmagnetized metal such as the keys and other objects that might be carried in the pocket of the individual "under suspicion." The detector would have to be quite sensitive, and speed of motion through the detector loop would be a factor of course.

D-C Cleanup in Insulating Oils

Discussion and authors' closure of a paper by J. B. Whitehead and S. H. Shevki, published in the June 1935 issue, pages 603–9, and presented for oral discussion at the selected subjects session of the summer convention, Ithaca, N. Y., June 27, 1935.

R. W. Atkinson (Gen. Cable Corp., Perth Amboy, N. J.): When we first began to be interested in the conductivity and dielectric loss of oils, and for a good many years after that, it was thought that these properties were inherent in the material itself. Thus, one type of oil would be considered as having inherently high conductivity and power factor and another type as having inherently low conductivity.

As time went on, however, it began to be recognized that the conductivity of these oils is not due to the main body of the oil itself, but to a relatively small portion of it which is unlike the bulk of the oil. Because the unlike portion has characteristics which are different from those of the bulk of the oil, the term "impurities" is applied to it, although it may differ chemically from the bulk of the oil only in some relatively minor respect, such as the addition of an atom of oxygen to the molecule or the absence of an atom of hydrogen from it. The essential point of difference that makes the impurities conducting is the fact that they contain bonds that are free or easily freed whereas the bulk of the oil does not.

In the early stages of oil purification it was considered fortunate that certain refining processes, such as filtration through fullers' earth, reduced the conductivity of the oil. It is now recognized, of course, that the improvement is not in any way fortuitous, but is due merely to the fact that the impurities adhere to the filtering medium and that what makes them adhere is the same thing that makes them conducting in the first place.

There has been steady progress in removal of the impurities from oils, to such a point that now there are any number of commercial oils, from different sources and made by widely different processes, which have extremely low loss. Indeed, for present day cable of any but the lowest voltages, oils are used whose losses are too small to be measured with apparatus initially available.

The oils discussed by the authors represent increasing refinement and thoroughness of removal of impurities. The almost infinitesimal conductance remaining may no longer be due entirely to impurities in the same sense as before, but may be due in part to free ions present by reason of radioactivity or similar causes. Thus, this type of conductivity is due essentially to the same reason as before, that is, to the presence of a relatively small number of particles different from the rest.

E. W. Greenfield (Johns Hopkins Univ., Baltimore, Md.): In view of the evidence presented in this and other papers by Dr. J. B. Whitehead, on the existence of space charges within the dielectric medium and the consequent variation and distortion of the normal linear electric gradient from electrode to electrode, it is well to give some thought to the correlation of measurements on oil samples varying widely in interelectrode spacing.

The volumetric distribution of space charge as related to interelectrode separation is apparently not a simple function of stress so that dielectric measurements of oils made on very thin samples may yield quite different results from those made on relatively larger samples.

I bring this point up as a result of the increasing tendency for measurements to be made on exceedingly small specimens of oil using quite thin sections between electrodes and then using the results of these measurements to interpret the behavior of insulating liquids in normal bulk. This is the more important due to the fact that the study of chemically pure insulating liquids of known relatively simple structure must be carried out at present with very small samples for the most part because larger quantities are unavailable.

What I believe is needed in this respect, are studies of insulating liquids along the lines presented by Doctor Whitehead where the measurements are carried over a wide range of electrode separation but at each spacing the stress maintained or varied by change of applied voltage only. Such studies should throw more light on the volumetric distribution of the space charge and its behavior as a function of stress.

Wm. A. Del Mar (Habirshaw Wire and Cable Co., Yonkers, N. Y.): A few years ago we made an experiment along the lines described in this paper but using oil-impregnated paper instead of oil, as in the authors' experiments. The idea was to deionize an impregnated paper cable by subjecting it to direct potential for a considerable period. Oil contains ions of both polarities, but tests had shown that the ions of molecular dimensions are mostly positive.

It was therefore decided to try to "wash out" the positive ions by repelling them from the conductor of a cable. Six $^1/_2$ inch copper rods were each covered with 5 layers of butted 5-mil wood pulp paper and impregnated with paraffin base oil containing 15 per cent rosin and covered with shielding tape. They were kept immersed in this oil, maintained at a temperature of about 120 degrees centigrade and treated as follows:

Samples A, B, C, and D were connected to d-c supply, the conductors being positive and the shields negative.

Samples E and F were not connected but were immersed in the same oil.

The initial voltage was 220 volts, as preliminary tests had indicated that "cleanup" of oil occurred at quite low voltages, but after 30 days, no change of power factor having occurred, the voltage was raised to 2,000 volts, which was maintained for 75 hours.

The power factor of all samples remained substantially unchanged throughout the test. In other words, no deionization occurred which affected the power factor of the insulation.

The samples were then subjected to a-c voltage-time tests, at average stresses, starting with 364 volts per mil and working toward 500 volts per mil.

In general the untreated dummies, E and F, gave slightly better lives than the supposedly deionized samples.

I am interested to know if the authors can explain the failure of this experiment to improve the insulation.

Oscar Hess (Simplex Wire and Cable Co., Boston, Mass.): The authors stated in their paper that the application of a d-c potential across the oil cell decreases the conductivity of the oil. I should like to ask Doctor Whitehead whether it is possible to make this improvement permanent by removing the oil containing the space charges which accumulated during the cleanup near the electrodes.

H. H. Race (General Elec. Co., Schenectady, N. Y.): There are 2 comments which I should like to make regarding this paper. The first is concerned with practical implications. At the 1931 A.I.E.E. winter convention we presented a paper on insulating oils (see "Some Electrical Characteristics of Cable Oils," abstract, ELEC. ENGG., Aug. 1931, p. 673) in which we concluded from variable frequency, variable temperature dielectric loss measurements that in commercial insulating oils at power frequencies and normal operating temperatures the major cause of dielectric loss is some form of ionic conduction. Doctor J. B. Whitehead's work using an entirely different experimental approach confirms this conclusion.

Secondly, from a theoretical point of view it would be very valuable to obtain more information concerning the nature of the ions responsible for observed conduction phenomena. I should like to ask Doctor Whitehead whether he is planning to extend these studies in an attempt to determine quantitative data for such properties of the ions as their sources, range of sizes, charges and mobilities.

J. B. Whitehead: Referring to Oscar Hess's question, no method has been devised for making permanent the improvement in an oil due to the application of continuous potential. If the original conductivity is due to electrolytic ions, theoretically it should be possible to remove these and so reduce the conductivity and loss. There are, however, other causes of conductivity such as radioactive and cosmic ray influence, particularly as related to the contact of the metal with the electrodes. It is my feeling that this is the more important factor and from its nature we would not expect a permanent cleanup following the application of continued potential.

As to W. A. Del Mar's question, I think that the barrier action of the paper is the principal reason why impregnated paper is

not subject to d-c cleanup. The space charges may not separate sufficiently far to cause an over-all polarization.

Doctor H. H. Race raises the question of the date at which conduction was recognized as a cause of dielectric loss in oils. This was first suggested by the work of Tank in 1915. In 1929 Dr. J. B. Whitehead and R. H. Marvin at the A.I.E.E. winter convention ("Anomalous Conduction as a Cause of Dielectric Absorption," A.I.E.E. TRANS., v. 48, April 1929, p. 299–314) reported loss measurements under alternating stress, together with short-time and long-time conductivity measurements and stated that anomalous conductivity is sufficient to account for all the loss in insulating oils. In a subsequent paper at the 1931 A.I.E.E. winter convention, Doctor Whitehead presented further, more intimate measurements concerning the origin of loss (see "The Conductivity of Insulating Oils—II," A.I.E.E. TRANS., v. 50, June 1931, p. 692–8). In these measurements, it was shown that for many oils, the initial conductivity over a period of a second was sufficient to account for all the loss. In certain oils a d-c current curve decaying sharply within the first few thousandths of a second indicated the presence of a polar component. In these cases it was shown that the total measured loss was accounted for principally by a conduction component, but with an added component of loss due to the polar property referred to. It was also shown that the conductivity of liquids computed from d-c measurements one minute after the application of voltage would account for the alternating loss only in the cases of oils relatively impure and of conductivity so great as to obscure the short-time properties referred to above.

Definitions of Power and Related Quantities

Discussion of a paper by H. L. Curtis and F. B. Silsbee published in the April 1935 issue, pages 394–404, and presented for oral discussion at the instruments and measurements session of the summer convention, Ithaca, N. Y., June 25, 1935.

G. M. L. Sommerman (Amer. Steel and Wire Co., Worcester, Mass.): The writers are to be commended on their very complete treatment of the subject of power definitions. While some of the terms and definitions may not meet with extended use, it is well to have a complete set available. It seems proper that the subject of terminology should be kept open for discussion until a general reading and digestion of the paper has taken place.

In figure 2 and throughout the paper the power vector F is called "fictitious" power. The name "nonactive" power for this vector seems preferable to "fictitious" power The writer has no objection to the use of the adjective "fictitious" in scientific writing where such use implies that the noun modified is treated as existing for purposes of discussion only. However, in the present case, the vector F exists just as surely as do vectors D, N, V, and U. What is really meant is that F does not contain any active power. Vector N has been named "nonreactive power" because it represents all the power except the reactive power, Q, i. e.,

$N = U - Q$ vectorially.

Q is perpendicular to both components of N, i. e., N is not a function of Q. In the same manner, F should be called "nonactive power," since it represents all the power except the active power, P, i. e.,

$$F = U - P$$

P is perpendicular to both components of F, i. e., F is not a function of P. The name "nonactive" is also more mnemonic than is "fictitious."

It might be argued that this would be justification for calling the vector V "nondistortion power" since these vectors are also mutually perpendicular. It is true that the adjective "vector" is more generic than specific, and might well be substituted by another word. However, this name has been used a great deal in the past; moreover, the vector V depends on the amount of distortion in many cases. In cases where harmonics appear in the voltage wave only or in the current wave only, i. e., a sinusoidal voltage applied to an over-excited transformer, there exists a finite D of which P and Q are not functions. In such cases the name "nondistortion power" for V is truly applicable.

The use of symmetrical components in the unbalanced 3-phase cases, pages 401 and 402, seems to be the only logical and definite method for treating such cases. It should be noted that, for the sinusoidal case, a unique expression for the apparent power can be obtained directly from the components

$$U = 3(E_pI_p + E_nI_n + E_zI_z)$$

for 3 phases, where E and I are effective values, and p, n, and z refer to the positive, negative, and zero sequence sets, respectively.

For the most complicated case, that of an unbalanced polyphase system with nonsinusoidal voltage and current waves, it is convenient to introduce a double subscript notation, thus: E_{fg}, I_{fg}; where the first subscript refers to the sequence and the second subscript refers to the order of the harmonic. Using this notation, the definite quantity designated on page 402 as the symmetrical apparent power becomes, for the 3-phase case

$$U_s = 3\left\{\begin{matrix} E_{p1}I_{p1} + E_{n1}I_{n1} + E_{z1}I_{z1} + \\ E_{p2}I_{p2} + E_{n2}I_{n2} + E_{z2}I_{z2} + \\ \vdots \qquad \vdots \qquad \vdots \\ E_{pn}I_{pn} + E_{nn}I_{nn} + E_{zn}I_{zn} \end{matrix}\right\}$$

It is obvious that this expression can be less than the apparent power. Thus, if any E or any I is zero, its cofactor is absent in the expression for U_s, whereas, the cofactor, if it exists, actually contributes to the apparent power. To avoid this difficulty, E_p may be defined as

$$E_p = \sqrt{E_{p1}^2 + E_{p2}^2 + \ldots + E_{pn}^2}$$

and

$$I_p = \sqrt{I_{p1}^2 + I_{p2}^2 + \ldots + I_{pn}^2}$$

and similarly for E_n, I_n, E_z, and I_z. If these quantities are substituted in

$$U_s' = 3(E_pI_p + E_nI_n + E_zI_z)$$

one obtains an expression for symmetrical apparent power which also contains all quantities actually existing regardless of the existence of their cofactors.

With the increasing interest manifest in the application of tensor analysis to electrical engineering problems, it might be well to point out that the voltages and currents with the double subscript notation mentioned above may be regarded as tensors of the second rank. It is interesting to see that the long expression for U_s for the m-phase, nonsinusoidal case becomes simply

$$U_s = mE_{\alpha\beta}I^{\alpha\beta}$$

in the tensor notation.

W. H. Pratt (General Elec. Co., West Lynn, Mass.): This is a very effective summary of the complexity that results when a rigorous extension of the ideas in connection with the simple sinusoidal flow of energy is carried out for nonsinusoidal and polyphase conditions.

Most of the quantities defined cannot be very generally useful for they do not identify but merely summarize.

The designers of apparatus and circuits need to know the magnitude and phase relationships of the currents and voltages of the particular frequencies, i. e., harmonics that appear, but these are not identified in this system of units.

The operating man must in general ignore the quantities of other than fundamental frequency for only those of fundamental frequency are within his control.

For commercial purposes, quantities that broadly summarize conditions and correlate circuit conditions as found with circuit possibilities would be most useful; thus "limiting power" as the basis of the definition of polyphase power factor would seem to be most useful.

In the activity of many electric circuits, the electromotive force and current are accurately represented by a fundamental frequency and its harmonics for the electromotive forces are actually the resultant of a system of identifiable component harmonic frequencies.

In other circuits such as those in which the magnetic qualities of ferromagnetic substances play an important part or where electron discharges are prominent, a Fourier series can only imperfectly represent the situation, perfect within any desired degree of approximation for certain analyses but quite imperfect as a means of solving certain other problems.

For this reason, quantities that are based on a Fourier analysis cannot be considered as having a truly fundamental standing unless they happen to fully identify a characteristic of the circuit.

True power does not suffer from this limitation. It is undeniably a very fundamental and definite physical conception, and the writer feels that it is unfortunate that the word should be freely used as a component part of the names of other quantities that have only a superficial resemblance to power. This remark applies even to "reactive power."

The writer also objects to the use of the word "vector" where the vector characteristic is present, only in the representation not in the nature of the quantity. The geometrical relationship that appears in the diagrams is a convenient way of represent-

ing a combination in which $C = \sqrt{A^2 + B^2}$, but the quantities here used are not by nature vectors.

As a more specific illustration of the writer's views, the quantity that in this paper is called "reactive power" has great importance when sinusoidal conditions are under discussion. Its importance seems to be due to its proportionality to the derivative of power, but with the extension to periodic quantities in general this derivative relationship is lost and with it all practical significance to the quantity as defined in the paper, except when sinusoidal conditions are approximated.

W. V. Lyon (Massachusetts Inst. of Tech., Cambridge): In this paper there is presented a proposal to define 17 different kinds of power. Eight of these definitions relate to single-phase circuits while the remainder relate to polyphase circuits. The very number of these definitions is arresting and invites discussion as to the need for this particular differentiation. The writer is of the opinion that the sole test by which the proposed definitions will stand or fall is whether they define useful quantities, and not whether they appear to form a "complete and logical set." An electrical quantity may be useful because it has physical reality such as do the instantaneous potential, the instantaneous current and the instantaneous power. It may be useful in the solution of the varied problems that arise both in theoretical investigations and in practice. Finally it may be useful as an aid in establishing equitable rates for the sale of electric energy. From this point of view of usefulness the writer wishes to discuss briefly some of the proposed definitions and to point out certain of the desirable and undesirable characteristics that they possess.

Instantaneous power possesses the following very important characteristic. The total instantaneous power supplied to any network equals the algebraic sum of the instantaneous powers supplied to the individual branches of the network. That is, the whole is equal to the sum of its parts. This is true regardless of the arrangement of the branches or of the wave forms of the potentials and currents that are involved. Since active power is the average value of the instantaneous power, taken over a complete cycle, it also possesses this same important characteristic.

Reactive power is usually associated with the reactance in the circuit although there are exceptions to this relationship. For example, the reactive power taken by a synchronous motor may be capacitive rather than inductive. Again a resistance which varies cyclically may take reactive power whereas an inductance which varies cyclically may take active power as well as reactive power. Under sinusoidal conditions the reactive power is often linked with the "amplitude of the alternating component of the power resulting from the reactance of the circuit," as in paragraph III, 4, *h* of this paper. This conception readily leads to confusion, as the writer has shown ("Reactive Power and Power Factor," W. V. Lyon, A.I.E.E. TRANS., v. 52, Sept.–Dec. 1933, p. 763–70), when the reactive powers supplied to the individual branches of a network are combined in order to determine the total reactive power supplied to the network. The best physical conception of reactive power is given in paragraph III, 4, *i*, although this is not universal. In the sinusoidal case reactive power is best defined as $EI \sin \theta$. It then has the important characteristic that the total reactive power input at the terminals of any network equals the algebraic sum of the reactive powers supplied to the individual branches of the network. Should it be considered desirable to extend the definition of reactive power to the nonsinusoidal case, as the authors propose, the reactive power would still possess the foregoing characteristic. In spite of this great advantage the writer questions the usefulness of this extension and believes that before it is adopted, carefully selected numerical illustrations should be presented which indicate its probable usefulness in connection with the calculation of power network problems. The measurement of the quantity should also be considered as well as its possible effect on the rates at which electric energy is sold. If the extension of the definition of reactive power to the nonsinusoidal case cannot be justified by its usefulness the proposal to so extend it will in effect be rejected by common consent even though "the possibility of the further extension of power concepts would (thereby) vanish." However, let us assume that the extension of the definition is accepted in order that the discussion can continue.

In a single-phase circuit the distortion power is the third of the 3 mutually perpendicular components into which the apparent power (volt-amperes) is divided. The other components are the active power and the reactive power. The distortion power is due to the presence of harmonics in the potential or current. If the potential and current are both sinusoidal the distortion power is zero. However, if a *nonsinusoidal* potential is impressed on a constant resistance the distortion power is also zero. If a nonsinusoidal potential is impressed on a constant inductance (of zero resistance) the power, although it results wholly from reactance, is not wholly reactive but has a distortion component. For example, if a potential consisting of a fundamental component E_1 and a third harmonic component E_3 is applied to a resistanceless inductance having a fundamental reactance X_1, the active power is zero, the reactive power is $\frac{E_1^2}{X_1} + \frac{E_3^2}{3X_1}$ and the distortion power is $\frac{2}{3}\frac{E_1E_3}{X_1}$. The authors present no evidence as to the usefulness of the distortion power either in regard to its bearing on the calculation of power networks or on the establishment of more equitable energy rates. The direct measurement of the distortion power presents some difficulty.

It should also be noted that the proposed definition of distortion power places a certain restriction upon its usefulness inasmuch as it ignores the *frequencies* of the harmonics in the potential and current, whereas the heating losses in alternating-current machines depend upon the frequency as well as upon the amplitude of the potential and current.

The division of the single-phase apparent power (volt-amperes) into 3 mutually perpendicular components corresponds to the resolution of a force into such components. In the latter case the resolution has proved an invaluable aid when it is desired to combine a number of forces, inasmuch as the resultant component along one of the mutually perpendicular axes equals the sum of the components of the individual forces along that axis. As the writer has pointed out this additive law is also true in regard to the active power and to the reactive power. Most unfortunately, however, it is not true in regard to the distortion power. That is to say, the distortion power measured at the terminals of a single-phase network does not in general equal the sum of the distortion powers in the individual branches of the network. In the writer's opinion this is a vital defect. For example, consider 2 parallel branches to which is applied a nonsinusoidal potential. If the impedance at fundamental frequency of the first branch (*a*) is $1 + j\,1$ units and of the second branch (*b*) is $1 - j\,1$ units, and the applied potential consists of a fundamental of one unit and a third harmonic of one unit, the following values of the component powers are obtained:

$$P_a = 0.6 \text{ unit} \qquad P_b = 1.4 \text{ units}$$
$$Q_a = -0.8 \text{ unit} \qquad Q_b = 0.8 \text{ unit}$$
$$D_a = \sqrt{0.2} \text{ unit} \qquad D_b = \sqrt{0.2} \text{ unit}$$

These parallel branches are equivalent to a nonreactive resistance at all frequencies. Thus $Q_a + Q_b = 0$. This being true the resulting distortion power calculated at the terminals is zero, whereas the sum of the distortion powers in the 2 branches is $2\sqrt{0.2}$ unit. Of what significance then is the sum of the distortion powers in the various branches of a single phase of a polyphase network?

Finally let us consider 2 simple illustrations of the application of the definitions of arithmetic apparent power, algebraic apparent power, and vector power. The circuit in the first illustration consists of 3 impedances each having the same ohm value but with angles of zero, + 60 degrees, and −60 degrees. These impedances are connected between the lines and neutral of a balanced 3-phase system in such a phase order that the currents in the lines and in the neutral conductor are each *I* amperes. If the line-to-neutral potentials are sinusoidal and have values of *E* volts, the total active power is 2 *EI* and the total reactive power is zero. There is no distortion power. The arithmetic apparent power is 3 *EI*, the algebraic apparent power is 2.73 *EI*, while the vector power is 2 *EI*. The vector power factor is unity, the algebraic apparent power factor is 0.732, while the arithmetic apparent power factor is 0.667. Which of these factors should be used in computing the charge for energy according to a power contract?

The second illustration relates to a balanced 4-phase, 4-wire system which has 2 equal lamp loads connected across opposite pairs of adjacent line terminals. The line potentials are each *E* volts and the line currents are each *I* amperes. In this case the active power is 2 *EI* and the reactive power is zero. Thus the vector power is 2 *EI*, but arithmetic and algebraic apparent powers are each 2.83 *EI*. The vector power factor is unity as is customary in the case of a lamp load. Both the arithmetic and algebraic apparent power factors, however, are 0.707, which is a most unusual rating for a lamp load and might well be the cause of considerable friction in the matter of power contracts.

News
Of Institute and Related Activities

Pacific Coast Holds 23d Annual Convention at Seattle

FOR the third time since Pacific Coast conventions of the Institute were initiated in 1910, the Seattle Section (first in 1916 and again in 1925) played host to a national convention of the Institute; this occasion was the 23d Pacific Coast convention, held at the Olympic Hotel in Seattle, Wash., from Tuesday morning through Friday, August 27–30, 1935. (For those who will check the mathematics involved: no Pacific Coast conventions were held in 1917, 1918, or 1933.)

Although the convention committee was set up and ready for business Monday afternoon at 3 o'clock, advance registrations were few, and activities did not start in earnest until Tuesday morning, when the registration rush caused a slight delay in the opening of the initial session. However, at about 10:15 a.m. Prof. E. A. Loew, chairman of the general convention committee, called to order the 130 members and guests present in the meeting hall, and introduced Dr. Lee Paul Sieg, president of the University of Washington. Following Doctor Sieg's cordial welcoming remarks, President E. B. Meyer was called upon by Professor Loew. President Meyer in his brief response paid tribute to the several convention committees for the evident thoroughness of arrangements made; he also spoke briefly of the scope of Institute activities available to members, urging each individual member to participate directly in as many such activities as possible, not only in order that he might make a contribution toward professional development, but that he might also receive the maximum benefit from his membership.

Analysis of Attendance at 1935 Pacific Coast Convention

	Location					
Classification	Seattle	Dist. 9*	Dist. 8	Dist. 10	Misc.	Totals
Members	56	50	23	9	7	145
Men Guests	25	2	4	1	1	33
Women Guests	32	19	10	1	2	64
Students	14	9	3		1	27
Totals	127	80	40	11	11	269

* Outside of Seattle, Wash.

Attendance is analyzed in the accompanying tabulation. Members were present from all 7 Sections in the Pacific Coast territory (Districts 8 and 9 and the western portion of District 10), as were representatives from 12 of the 14 Student Branches in the territory. National Institute officers present included President E. B. Meyer, Newark, N. J.; vice presidents F. O. McMillan of Corvallis, Ore., and N. B. Hinson of Los Angeles, Calif.; and past-presidents H. V. Carpenter of Pullman, Wash., and R. W. Sorensen of Pasadena, Calif.

Technical Sessions

Two of the 5 general technical sessions were held Thursday, one in the morning and one in the afternoon; others were held Wednesday, Thursday, and Friday mornings. These sessions accommodated the presentation and active discussion of all 16 of the formal technical papers embraced in the detailed program of this meeting as published on page 784 of ELECTRICAL ENGINEERING for July 1935, which program also gives reference to the pages in the April, May, June, and July issues of ELECTRICAL ENGINEERING on which 8 of these papers were published; the 8 papers not so designated were published in the August issue. Taking advantage of the flexibility provided by the unified publication plan, the committee in charge of the technical program for the Seattle convention scheduled for presentation and discussion several papers previously presented at eastern conventions.

Dr. L. P. Sieg (right) president of the University of Washington, Seattle, welcoming E. B. Meyer, president of the A.I.E.E., to the Northwest

In addition to the regular technical papers, 3 special illustrated talks were pr[e]sented:

1. PROGRESS OF THE GRAND COULEE DEVEL[OP]MENT TO DATE, by Alzin F. Darland (A'20, M'2[…]) Bureau of Reclamation, Coulee Dam, Wash.
2. ELECTRICAL FEATURES OF THE BONNEVILL[E] PROJECT, by L. E. Kurtichanof, U.S. Army Eng[i]neering Corps, Portland, Ore.
3. OFF-PEAK CONTROL OF WATER HEATER LOA[DS], by F. M. Starr (A'30), General Electric Co[m]pany, Schenectady, N. Y.

Presiding over the several technica[l] sessions were Prof. E. A. Loew, general con[]vention chairman, and past chairman of th[e] Seattle Section; F. J. Bartholomew, pas[t] chairman of the Vancouver, B. C., Section; Vice President F. O. McMillan; Vice Presi[]dent R. W. Sorensen; and Dr. C. E. Mag[]nusson, past chairman of the Seattle Section.

Student Technical Sessions

Consistent with the effective policy tha[t] has been followed for several years in con[]nection with Pacific Coast conventions, 2 en[]tire afternoon technical sessions were de[]voted to the presentation of 10 Studen[t] papers, the titles and authors of which wer[e] as published on page 784 of ELECTRICA[L] ENGINEERING for July 1935, except fo[r] elimination of the paper scheduled to hav[e] been presented by Stephen S. Stevens.

Conference on Student Activities

In addition to these Student technica[l] sessions, the Branch chairmen, Studen[t] counselors, vice presidents, and severa[l] others interested in Student activities at[]tended the regular annual Student confer[]ence that was held Wednesday evenin[g] following a dinner meeting of the group. This conference was presided over by Prof. O. E. Osburn of Washington State College, Pullman, counselor-chairman for District [] for the preceding year. In his opening re[]marks, Chairman Osburn sought to empha[]size the importance of the responsibility tha[t] rests upon the Student Branch chairmen and particularly upon the several Studen[t] counselors in connection with the successfu[l] development of Student activities.

The conference was given over to [a] thorough-going discussion of various sug[]gestions for improving the effectiveness [of] activities in and among the Studen[t] Branches represented. The principal topi[c] of discussion had to do with practical way[s] and means of enabling Enrolled Student[s] to establish effective contact with th[e] realities of their chosen profession. Vic[e] President Hinson in brief remarks pointe[d] out that such contact must be establishe[d] sooner or later anyway, and that it was dis[]tinctly to the advantage of the young engi[]neer to go as far as possible in that directio[n] before graduation instead of leaving it unt[il] after graduation. It was the general con[]sensus of opinion that 2 particularly effec[]tive channels are provided through the Stu[]

dent Branches—first, the opportunity for personal participation in Institute affairs and the establishment of personal contact with practicing engineers; second, the opportunity for direct contact with, and the gradually developing understanding of, important current technical advances and broader professional problems through the use of material published by the Institute in its monthly journal ELECTRICAL ENGINEERING. Although it was suggested that

At the speakers' table, during the annual banquet held on Thursday evening

From left to right: Vice President F. O. McMillan, Convention Chairman E. A. Loew, President E. B. Meyer, Toastmaster Joseph Hellenthal, National Secretary H. H Henline, and Vice President N. B. Hinson

more material of special interest to Students perhaps could to advantage be developed for publication, it was definitely a prevailing consensus of opinion that there is more such material being published currently than is being put to full use.

In this connection, one of the Branch chairmen said pointedly that "there is a real need in a great many instances for a proper and effective stimulation of initial Student interest, and that it was in such instances that the Branch counselors could indeed render a valuable service in assisting the Student to overcome his 'starting inertia' and in guiding the direction of the Student's initial efforts to read and understand the technical papers and discussions which his Institute affiliation brings to him through ELECTRICAL ENGINEERING." This sentiment was reflected widely in comments of other Branch chairmen, in the remarks of some of the counselors, and was emphasized by Vice President (former counselor) Mc-

Chairman A. E. Loew (left) and Vice Chairman L. B. Robinson of the 1935 Pacific Coast convention committee

Millan, who called attention frankly to the fact that responsibility for the success or failure and for the degree of effectiveness of the program of each Student Branch rests squarely upon the counselor in each case.

The question of Student sessions at future conventions also was discussed. Looking forward to the 1936 annual summer convention that will be held in southern California under the auspices of the Los Angeles Section, considerable attention was given to the development of preliminary plans for sessions at that convention to be devoted to Student papers.

Of the 14 Student Branches in Districts 8, 9, and the western part of 10, 12 were represented by Student chairmen or vice chairmen; and 11 out of the 14 Branch counselors also were present at the conference. Branch chairmen present included: Felix Berra, University of Arizona, Tucson; H. P. Blanchard, Stanford University, Calif.; Louis Brewer, Montana State College, Bozeman; Glenn G. Davis, University of Utah, Salt Lake City; W. A. Gish, Oregon State College, Corvallis; R. A. Greulich, University of Nevada, Reno; James H. Miller, University of Idaho, Moscow; J. T. Mullin, University of Santa Clara, Calif.; John D. Oliphant, University of Southern California, Los Angeles; Winfield Pullen, Jr., University of Washington, Seattle; Edward Simmons, California Institute of Technology, Pasadena; George L. Zimmerman, Washington State College, Pullman.

Branch counselors present were: A. L. Albert, Oregon State College, Corvallis; P. A. Biegler, University of Southern California, Los Angeles; W. A. Hillebrand, University of California, Berkeley; J. H. Johnson, University of Idaho, Moscow; O. E. Osburn, Washington State College, Pullman; E. F. Peterson, University of Santa Clara, Calif.; G. R. Shuck, University of Washington, Seattle; H. H. Skilling, Stanford University, Calif.; R. W. Sorensen, California Institute of Technology, Pasadena; A. L. Taylor, University of Utah, Salt Lake City; and J. A. Thaler, Montana State College, Bozeman.

At group conferences of the counselor-delegates, held immediately after the general conference, District Student counselor chairmen for the ensuing administration year were selected: for District 8, Prof. E. F. Peterson; for District 9, Prof. G. R. Shuck.

ENTERTAINMENT

General entertainment features opened with a reception and informal dance held Tuesday evening, August 27, in the Junior Ballroom of the Olympic Hotel; embraced a variety of features of special interest to both men and women; and with one exception closed with the annual banquet held Thursday evening in the Spanish Ballroom where 175 participants were entertained with song, dance, and orchestral numbers during the banquet. The banquet was a highly informal and thoroughly successful affair presided over by past chairman Joseph Hellenthal of the Seattle Section, featuring brief remarks from general chairman E. A. Loew of the convention committee, and President E. B. Meyer, and largely dominated by the golf committee in making its awards of a galaxy of valuable prizes.

Principal feature of the entertainment especially for women was the bridge luncheon held Thursday afternoon at the Sand Point Golf and Country Club, Seattle, and enjoyed by 49 women of whom the following were reported as prize winners: Mrs. Reinier Beeuwkes, Mrs. Charles Cross, Mrs. C. H. Hoge, Mrs. Ray Rader, Mrs C. E. Rogers, and Mrs. Wellington Rupp, all of Seattle, and Mrs. J. C. Henkle, Mrs. H. H. Schoolfield, and Mrs. V. B. Wilfley, all of Portland, Ore., and Mrs. M. H. Brewer of Belgrade, Mont. Other events included a boat trip participated in by 60 women, and a boulevard tour around the city followed by tea enjoyed by 31.

Supplementing miscellaneous inspection trips, the principal feature of the entertainment program for men was a salmon fishing contest held in Elliott Bay of Seattle Harbor, under the sponsorship of Seattle's

J. B. Fisken (left) of Spokane, Wash., congratulates H. H. Schoolfield of Portland, Ore., upon his winning the prized Fisken golf trophy for 1935

salmon-minded "Mystic Knights of the Sea." The organizer and leader of this novel event was C. E. Rogers, past chairman of the Seattle Section. Each a guest of a Mystic Knight, some 28 Institute members attending the convention arose with the aid of a commanding telephone bell at 2:15 Wednesday morning, August 28 (after having retired at midnight or later following the reception) and gathered for breakfast promptly at 2:45 a.m. at Rippe's Cafe. By 4 o'clock, the party was afloat in 28 boats, each trolling 2 lines. Dawn revealed some 200 other boats on the same quest. A check-up of 56 Mystic Knights and guests on the dock shortly after 7 a.m. revealed that the trophy cup and claim to supremacy had been won hands down by E. L. Bettannier of Pasadena, Calif., and

A group of the Student Branch chairmen who attended the Pacific Coast convention at Seattle

From left to right: G. L. Zimmerman, Washington State College, Pullman; James Miller, University of Idaho, Moscow; R. M. Morton, University of British Columbia, Vancouver; Edward Simmons, California Institute of Technology, Pasadena; J. T. Mullin, University of Santa Clara, Calif; and Louis Brewer, Montana State College, Bozeman

his Mystic Knight host A. E. Cross of Seattle, who landed the only salmon caught by the party.

Golf

The 15th annual competition for the John B. Fisken Pacific Coast A.I.E.E. golf trophy and the related competitions were held Thursday afternoon, August 29, at the Sand Point Golf and Country Club, with a field of 75 entrants. To provide the contestants with as much of an opportunity as possible, the committee had arranged 6 different competitions in addition to that for the Fisken cup, all dependent upon the tournament results. Competition was open only to members and guests officially registered at the convention. Subject to the committee's announced rule that no contestant could win more than one prize, and subject to the committee's rules for preferential selection, the winners of the several events were as follows:

1. John B. Fisken cup competition (medal play on handicap): winners, H. H. Schoolfield, Portland, Ore., 90–22–68; first runner-up, R. H. Dearborn, Corvallis, Ore., 83–14–69; second runner-up, R. W. Mastick, Seattle, Wash., 84–14–70.
2. Other winners of valuable prizes, in the order of the magnitude of their proficiency as determined by the committee were: (1) A. J. Schmitz, Seattle; (2) W. D. McDonald, Seattle; (3) E. L. Breene, Seattle; (4) W. D. Shannon, Seattle; (all winners in guest competition); (5) F. D. Carroll, Seattle; (6) C. R. Boyle, Seattle; (7) J. C. Henkle, Portland; (8) R. E. Thatcher, Seattle; (9) G. R. Murphy, San Francisco; (10) E. L. Criger, Seattle; (11) Peter Diederich, Glendale, Calif.; (12) V. B. Wilfley, Portland; (13) J. H. Seigfried, Kennewick, Wash.; (14) N. H. Callard, San Francisco; (15) Max Bitts (Student) Seattle; (15) J. I. Caldwell, Seattle; (16) J. G. Miles, Seattle; (17) C. H. Cutter, Seattle; and 6 or 8 others each of whom received one golf ball for each of several birdies made.

The Fisken trophy is a cup, originally donated by the Portland Section for annual competition (for members in good standing only) and named in honor of J. B. Fisken, hydroelectric pioneer of the Northwest. This year's tournament was the 15th competition for the Fisken trophy; winners are:

1920—C. L. Wernicke, Portland
1921—C. P. Osborne, Portland
1922—J. B. Fisken, Spokane
1923—S. J. Lisberger, San Francisco
1924—K. E. Van Kuran, Los Angeles
1925—W. C. Heston, San Francisco
1926—P. M. Downing, San Francisco
1927—C. E. Heath, Los Angeles
1928—G. D. Luther, Seattle
1929—E. W. Rockwell, Los Angeles
1930—W. F. Hynes, Portland
1931—M. S. Barnes, San Francisco
1932—J. E. Underhill, Vancouver, B. C.
1933—No convention
1934—H. W. Flye, San Francisco
1935—H. H. Schoolfield, Portland

Committees

Committees and committee workers responsible for the success of the Seattle convention were:

General Convention Committee—E. A. Loew, *chairman;* L. B. Robinson, G. H. Walker, A. M. Bohnert, Walter Brenton, F. J. Bartholomew, Fred Garrison, E. B. Hansen, G. L. Hoard, R. H. Hull, R. E. Kistler, F. C. Lindvall, F. O. McMillan, R. U. Muffley, E. O. Osburn, H. T. Plumb, G. E. Quinan, Wellington Rupp, C. E. Rogers, R. W. Sorensen, and J. A. Thaler.

Salmon fishing champion E. L. Bettannier of Pasadena, Calif., exhibiting the result of 232 man-hours of fishing

Meetings and Papers—G. L. Hoard, *chairman* Reinier Beeuwkes, Joseph Hellenthal, K. L. Howe and C. E. Rogers.

Finance—L. B. Robinson, *chairman;* C. E. Rogers and G. H. Rogers.

Publicity—R. E. Kistler, *chairman;* J. H. Kelley A. I. Launder, C. M. Lubcke, and Ray Rader.

Hotel and Registration—E. B. Hansen, *chairman;* H. O. Blair, M. T. Crawford, E. R. Hannibal, W. S. Hill, C. A. Lund, and G. R. Shuck.

Trips and Transportation—Wellington Rupp, *chairman;* R. O. Bach, Charles Cross, W. E. Conroy, W. S. McCrea, E. L. White, and Laurence Wylie.

Golf—G. E. Quinan, *chairman;* E. L. Crider, E. J. Descamp, R. W. Mastick, and T. S. Wood

Fishing Contest—C. E. Rogers, *chairman;* G. O Cranmer, C. A. Foley, Ray Hasselo, and C. L. Johnson.

Entertainment—R. U. Muffley, *chairman;* F. W. Carlson, E. S. Code, C. J. Hawkes, C. H. Hodge, G. Smith, and C. R. Wallis.

Ladies' Entertainment—Mrs. J. Hellenthal, *general chairman;* Mrs. R. E. Kistler, Mrs. E. A. Loew, Mrs. C. E. Rogers, and Mrs. G. H. Walker.

Engineering Examiners to Meet. The 16th annual convention of the National Council of State Boards of Engineering Examiners will be held at Columbus, Ohio, October 23–25, 1935, with headquarters at the Deshler-Wallick Hotel. The program calls for business sessions during the mornings and afternoons of Wednesday and Thursday, October 23 and 24, with committee meetings held late on Wednesday afternoon. The annual dinner, to which women are invited, will be held on Wednesday night, and the "professional engineers banquet" on Thursday night. There will be a sightseeing trip Friday morning. Business to be transacted during the convention will include reports of the following standing committees: accredited engineering schools, uniform examinations for registration, legal, National Bureau of Engineering Registration, Engineers' Council for Professional Development, constitution, and the special committee on reciprocity and certification of engineers. Perry T. Ford, who is secretary of the Ohio State board, is chairman of the convention committee.

Membership—

Mr. Institute Member:

By this time you have received our fall letter asking you to help your Section membership committee in obtaining new members by sending in the name of one person who, you feel, should be invited to join the Institute. Your individual co-operation in this activity in the past has been of the greatest helpfulness in the work and without your part in it we would not be able to return to you a substantial record of progress. If you have not yet replied to the letter, will you not do so promptly?

Everett S. Lee

Chairman National Membership Committee

Great Lakes District Meeting at Purdue

The Great Lakes District of the A.I.E.E. will hold a meeting at Purdue University, West Lafayette, Ind., on Thursday and Friday, October 24 and 25, 1935. Three technical sessions will provide an interesting variety of timely subject matter and a Student technical session will be held on Friday morning, at which 15 of the best papers selected from the various Student Branches within the District will be presented. The social and recreational features which will round out the program consist of a dinner, dance, and inspection trips to the factory of the Duncan Electric Manufacturing Company in Lafayette, and the various buildings and laboratories of the university.

It is interesting to note that Purdue University has one of the finest electrical engineering laboratories in the country. For many years they have specialized in extra high potential research, which provides a very appropriate setting for the session on high potential measurements. The majority of the technical papers for this session and the entire program except the Student session and the paper entitled "Characteristics of Luminous Tube Circuits" are published in this issue and the September issue of ELECTRICAL ENGINEERING. Copies of the latter paper may be made available by the author at the meeting. For the technical program and more complete details regarding various features of the meeting see ELECTRICAL ENGINEERING for September 1935, pages 1005–7. All meetings, with the exception of the dinner meeting, will be held in the new electrical engineering building on the North Campus, which is directly on routes 52 and 152 between Chicago and Indianapolis.

Take advantage of the opportunities which the program affords and attend this meeting on October 24 and 25. An additional feature will be the university homecoming program and football game with Carnegie Institute of Technology on Saturday.

Electrical Insulation Committee to Meet

The 8th annual meeting of the committee on electrical insulation, division of engineering and industrial research, National Research Council, will be held at Pittsfield, Mass., October 17–19, with the General Electric Company as host. The technical program is in 3 sessions in which 20 papers in the field of dielectric research and applications to the problems of electrical insulation will be presented.

Dr. J. B. Whitehead (A'00, M'08, F'12, Life Member, and junior past-president), Johns Hopkins University, Baltimore, Md., is chairman of the committee, and W. F. Davidson (A'14, F'26), Brooklyn Edison Company, Brooklyn, N. Y., is secretary.

18 Schools Request Accrediting by E.C.P.D.

Eighteen colleges and universities in the New England and Middle Atlantic States offering courses leading to an engineering degree have requested the Engineers' Council for Professional Development to consider their engineering curricula for accrediting. The group includes many of the large and important engineering schools in these 2 regions.

Engineers' Council for Professional Development is a conference of engineering bodies representing the technical, educational, and legislative interests of engineers. As part of its plan to enhance the professional status of engineers E.C.P.D. hopes to promote higher standards of education by studying engineering curricula and accrediting those worthy of recognition. A list of approved institutions will be published for the assistance of those who wish to make use of it.

As announced last June the plan, similar to that followed by the medical and legal professions, will be put in effect this fall in the New England and Middle Atlantic States, and extended later to other regions of the United States. Both qualitative and quantitative criteria will be used as a basis for accrediting. Qualitative criteria will be evaluated through visits of inspection by regional committees of qualified individuals. Quantitative criteria will be evaluated through data obtained from catalogs and by questionnaires.

The questionnaires which the committee spent over a year developing and which were tried out experimentally in a limited number of schools, are now being distributed to the institutions requesting con-

Future AIEE Meetings

Great Lakes District Meeting,
West Lafayette, Ind., Oct. 24–25, 1935

Winter Convention,
New York, N. Y., Jan. 28–31, 1936

North Eastern District Meeting,
New Haven, Conn., May 1936

Summer Convention,
Huntington Hotel, Pasadena, Calif., June 22–26, 1936

Middle Eastern District Meeting,
Akron, Ohio (date to be determined)

At the Institute's Recent Pacific Coast Convention—the Annual Banquet

A scene at the annual convention banquet held as part of the Institute's 23d Pacific Coast convention at Seattle, Wash., August 27–30, 1935. At this highly successful affair, held on Thursday evening, August 29, 175 persons were in attendance

sideration. The material thus obtained will be analyzed and studied as it is received and will be presented to the regional committees for their consideration prior to visiting the institutions.

The prompt acceptance of this plan by so many institutions is an excellent indication of the interest with which this program of E.C.P.D. is being received. A number of additional institutions expressed interest in the matter but found it necessary to defer actual requests until the return of their officers in September.

The plan being followed in the accrediting procedure was outlined in ELECTRICAL ENGINEERING for February 1935, pages 249–50; March 1935, page 343; and August 1935, page 895.

A.S.M.E. Nominates Officers. Officers of The American Society of Mechanical Engineers to serve for the year 1936 were recently nominated. W. L. Batt, president of the SKF Industries, Inc., was nominated for the office of president. Mr. Batt, a graduate of Purdue University, is also a director of the Air Preheater Corporation, and a director of the Hudson Insurance Company, New York. He is probably best known to members of the Institute for his activities on United Engineering Trustees, Inc., being at present chairman of the coordination committee of engineering societies. For his interest and activity in promoting commercial relations with Sweden, King Gustav V, of Sweden, in 1926, conferred upon Mr. Batt the Order of Vasa, and in 1933 the Royal Order of the North Star. Newly nominated vice presidents of the A.S.M.E. are Alex D. Bailey, assistant chief operating engineer of the Commonwealth Edison Company, Chicago, Ill.; J. A. Hunter, professor of mechanical engineering at the University of Colorado, Boulder; W. A. Shoudy, associate professor of mechanical engineering at Columbia University, New York, N. Y., and a practicing consulting engineer; and R. L. Sackett, dean of the school of engineering at Pennsylvania State College, State College. Newly nominated managers are W. L. Dudley, vice president and chief engineer, Western Blower Company, Seattle, Wash.; W. C. Lindemann, works manager and secretary, A. J. Lindemann and Hoverson Company, Milwaukee, Wis., and J. W. Parker, vice president and chief engineer, the Detroit (Mich.) Edison Company.

Porcelain Insulator Bodies Studied. An investigation undertaken to determine the relation and correlation between the electrical and the mechanical properties of porcelain insulator bodies furnished by manufacturers of high voltage insulators, and of similar bodies produced under laboratory conditions, has been conducted by the engineering experiment station of the University of Illinois in co-operation with the Utilities Research Commission of Chicago, Ill. Until January 1, 1936, or until the supply available for free distribution is exhausted, copies of this bulletin, No. 273, may be obtained, without charge, upon application to Engineering Experiment Station, Urbana, Ill.

Santa Fe's Mighty Diesel-Electric Locomotive

WHAT is reported to be the most powerful Diesel-electric locomotive yet constructed was recently delivered to the Atchison, Topeka, and Santa Fe Railway Company, the "Santa Fe" line. Rated 3,600 horsepower, with a weight of 240 tons and an approximate over-all length of 127 feet, this new locomotive is actually a multiple unit of 2 identical sections which can be operated either singly or together. The units are arranged for double-end operation. Motive power of each unit is supplied by 2 Winton V-type 12-cylinder high-compression 2-cycle oil engines, each rated at 900 horsepower, and weighing less than 20 pounds per horsepower. Among the many new features incorporated in the locomotive is a lightweight, compact, automatic steam generating unit for heating and air conditioning in the cars of the train. The locomotive, built by the Electro-Motive Corporation, subsidiary of General Motors Corporation, is unique in appearance, having, in addition to a streamlined exterior, an attractive handling of color scheme. Named the "Super Chief," the locomotive will be subjected to exhaustive tests; if these prove successful it will haul the road's fastest train, "The Chief," between Chicago and California on a schedule faster than at present.

Microphotography for Scientific Publication

Microphotographic, other photographic, and related methods of reproduction of printed and typewritten material have for some time been under consideration as a means of making information more widely available. The dissemination of scientific information by these methods is being studied by the documentation division of "Science Service," Washington, D. C., which points out in its document 72, dated August 16, 1935, that the first step in dissemination is publication in some manner of the research results, and the second step is the incorporation of references to published research results into usable bibliographies. This project is for the benefit of the scientific world, and in that respect differs from the major concern of "Science Service," which is an institution for the popularization of science.

The documentation division of "Science Service" states that it will first devote its energies and resources to:

1. Development of mechanisms useful in microphotographic and other photographic duplication and in bibliography.
2. Development of a method of publication for those scientific papers and monographs that cannot now secure prompt or complete issuance.
3. Co-operation with libraries in making available by photographic methods the literature of the past.
4. Investigation of the broad problem of scientific bibliography and useful mechanisms.

As one part of the plan, printed or typewritten material could be microphotographed on a 35 millimeter film which would be the master negative. For distribution, either film positives for use in reading machines, or 6 by 8 inch photographic prints suitable for reading without optical aid, could be made.

As a possibility for the future, experiments are being conducted on the publication of scientific journals by microphotography. The printing of some 200 pages of typewritten manuscripts $8^1/_2$ by 11 inches in size on a piece of film the size of a library catalog card, 3 by 5 inches, is believed to be a technical possibility, although a suitable reading machine for these microphotographs has yet to be developed. (Details of this plan are given in document 46, free from "Science Service," 2101 Constitution Avenue, Washington, D. C.)

Investigations also are under way by the documentation division upon problems connected with scientific bibliography. It is considered that photoelectric means might be used for picking out the desired subject classification from a bibliographical file, which might be a roll of film. (Detailed discussion of bibliographical procedure is contained in documents 58 and 61, free from "Science Service," 2101 Constitution Avenue, Washington, D. C.)

Standards for Large Rivets. A proposed American Standard for large rivets, $^1/_2$ inch nominal diameter and larger, has recently been completed by subcommittee No. 1 of the sectional committee on standardization of bolts, nuts, and rivet proportions. This proposed standard, a second revision, is now being distributed for criticism and comments. Copies are available upon application. All communications should be addressed to C. B. LePage, The American Society of Mechanical Engineers, 29 West 39th Street, New York, N. Y.

S.P.E.E. Elects Officers. At the recent annual meeting of the Society for the Promotion of Engineering Education, D. S. Anderson (A'01), dean, college of engineering, Tulane University, was elected president for the coming year. P. H. Daggett (A'08), dean, college of engineering, Rutgers University, and S. B. Earle, dean of engineering and director of the engineering experiment station, Clemson Agricultural College, S. C., were elected vice presidents. It was voted to hold the 1936 annual meeting of the S.P.E.E. at the University of Wisconsin, Madison.

Doctor Hotchkiss Elected Rensselaer President. Dr. William Otis Hotchkiss, who for the past 10 years has been president of Michigan College of Mining and Technology, Houghton, has been elected to the presidency of Rensselaer Polytechnic Institute, Troy, N. Y. Doctor Hotchkiss, who was born at Eau Claire, Wis., in 1878, and who studied at the University of Wisconsin, has had a long and varied career as an engineer and geologist, as well as an educator. During his 10 years at the Michigan college, he added and built up strong courses in several branches of engineering.

Wanted: Skilled Labor. A report in pamphlet form entitled "Wanted: Skilled Labor" has been prepared by the National Industrial Conference Board, Inc., which reveals information on the existing and potential shortage of skilled labor in the metal working industries. The report is based on an extensively circulated questionnaire. This 6 by 9 inch pamphlet, 37 pages in length, can be obtained from the National Industrial Conference Board, Inc., 247 Park Avenue, New York, N. Y., at a cost of $1.

Tests on Night Visibility on Highways. A paper entitled "Some Visibility Tests on Lighted and Unlighted Highways" by Parry Moon (F'34) and R. C. Waring (A'19, M'27) has been published in the *Journal* of the Franklin Institute, volume 219, March 1935, pages 285–314. The paper discusses the results of over 5,000 tests made to determine the effects of various factors on the distance at which the driver of a motor vehicle can see a pedestrian. The work was done as part of the Massachusetts Highway Accident Survey, a project of the Civil Works Administration. The principal conclusions are:

(*a*). The safety of a pedestrian walking along a highway at night can be greatly increased by the use of a small area of white (such as a handkerchief) or by reflector buttons. The visibility distance is increased approximately 50 per cent by the former and 100 per cent by the latter.

(*b*). Maximum safe speeds at night on unlighted highways are approximately 30 miles per hour when there is glare from passing cars and 40 miles per hour when there is no glare.

(*c*). Type of pavement and speed of car have little effect on visibility distance on unlighted roads.

(*d*). An increase in headlamp candle power above 32 seems advisable.

(*e*). Depressing the headlamp beams of passing cars reduces visibility distance.

(*f*). Highway lighting does not increase visibility distance appreciably unless the average luminosity of the pavement is above the chromatic threshold [order of 0.05 lumen(foot²)].

Air Conditioning Association Moves. Headquarters of the Air Conditioning Manufacturers' Association have been moved to Washington, D. C., and offices opened in the Southern Building there.

American Engineering Council

Effects of Federal Legislation

Excerpts from the current "news letter" of the American Engineering Council follow:

In 4 months Congress will reassemble. In the interval between now and the first of January 1936, it is the task both of industry and of the federal agencies to adjust themselves to the heavy load of legislation. Both Washington and the country are reviewing with soberness the results of emergency legislation passed during the previous sessions of Congress. There are certain to be difficulties in the way of putting some of the legislation into effect.

There continues to be much confusion in Washington and, in spite of public statements to the contrary, a growing feeling on the part of those responsible for administering the legislation that there is a big difference between having new ideas and putting those new ideas into action.

The executive secretary of A.E.C. has returned from trips both to the South and to the North within the last 4 or 5 weeks. It is made evident that the "emergency" needs have passed and it is realized that what has been made possible through new legislation now has to be put into action and paid for.

Much of the new legislation, concerning as it does, natural resources, transportation, and construction, directly affects engineers and engineering development. The regulatory legislation covering utility holding companies, motor carriers, the soft coal industry, and "hot oil"; the increased powers of the T.V.A. and A.A.A., all tend to centralize authority in Washington over industrial practices and procedure.

The legislation dealing with labor, especially that trending to establish the procedure for collective bargaining by labor, as well as the "social security" system of old age pensions and unemployment reserves bring new problems to engineers and to industries employing engineers, in so far as the centralization of regulatory centralization in Washington tends to open channels for constructive study of industrial practices and procedures. There will be many opportunities within industry for men trained in engineering to apply the engineering method of analysis.

Probably in the next 10 years, many new fields of opportunity for engineers will be opened, not in the government, but in private industry, because of the need of more intensive analysis of manufacturing costs, of relation of machinery to reduction of labor costs, and of manufacturing methods. Undoubtedly also, there will be an increase in the use of power per man.

Letters to the Editor

CONTRIBUTIONS to these columns are invited from Institute members and subscribers. They should be concise and may deal with technical papers, articles published in previous issues, or other subjects of some general interest and professional importance. ELECTRICAL ENGINEERING will endeavor to publish as many letters as possible, but of necessity reserves the right to publish them in whole or in part, or to reject them entirely.

ALL letters submitted for consideration should be the original typewritten copy, double spaced. Any illustrations submitted should be in duplicate, one copy to be an inked drawing but without lettering, and other to be lettered. Captions should be furnished for all illustrations.

STATEMENTS in these letters are expressly understood to be made by the writers; publication here in no wise constitutes endorsement or recognition by the American Institute of Electrical Engineers.

Copper and Aluminum Cable Fusing Time-Current Formulas

To the Editor:

The following formulas for calculating the time-current fusing characteristics of copper and aluminum cables for short time intervals are based upon several general assumptions but have been found to be sufficiently accurate for most purposes.

The assumption of constant current is most convenient and while most conditions in practice will involve some variation in the amount of current these cases may usually be referred to the constant current data. The time required to bring the conductor to the melting point is divided into the time required to raise the temperature to the melting point and the time required to store the heat of fusion.

The time required to reach the fusing temperature is:

Neglecting losses

$$s_1' = 2.3\,\frac{h(y+t_0)\log_{10}\dfrac{y+t_f}{y+t_0}}{I^2 r_0}\ \text{(seconds)} \quad \textbf{(1)}$$

Including losses

$$s_1 = \frac{2.3\,hH}{I^2\dfrac{r_0 H}{y+t_0} - 1}\,\log_{10}\frac{\left(\dfrac{t_f - t_a}{t_0 - t_a}\right)\left(I^2\dfrac{r_0 H(y+t_f)}{(y+t_f)(t-t_a)} - 1\right)}{I^2\dfrac{r_0 H}{t_0 - t_a} - 1}\ \text{(seconds)} \quad \textbf{(2)}$$

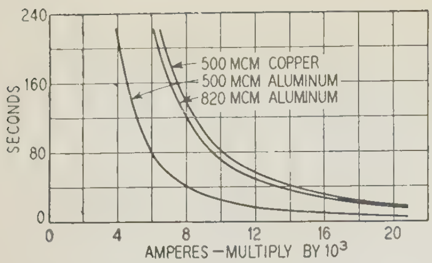


Fig. 1

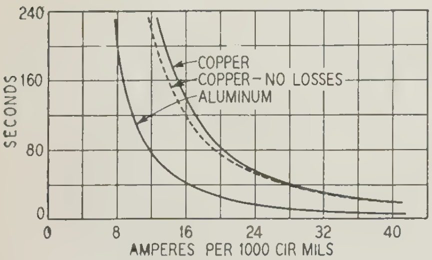


Fig. 2

The time required to store the heat of fusion is:

Neglecting losses

$$s_2' = \frac{h_f}{I^2 r_0 \dfrac{y + t_f}{y + t_0}} \quad \text{(seconds)} \tag{3}$$

Including losses

$$s_2 = \frac{h_f H}{(t_f - t_a)\left(I^2 \dfrac{r_0 H(y + t_f)}{(y + t_0)(t_f - t_a)} - 1\right)} \quad \text{(seconds)} \tag{4}$$

The total time required to reach the molten state is the sum of s_1 and s_2.

SYMBOLS

I = current in amperes
r_0 = initial conductor resistance in ohms per foot
t_0 = initial conductor temperature in degrees centigrade
t_a = ambient temperature of surrounding medium in degrees centigrade
t_f = fusion temperature of conductor metal in degrees centigrade
h = specific heat of conductor in watt-seconds per foot of cable per degree centigrade
= 200 times pounds per foot for copper
= 418 times pounds per foot for aluminum
h_f = heat of fusion of conductor in watt-seconds per foot of cable
= 1.44×10^6 times pounds per foot for copper
= 2.63×10^6 times pounds per foot for aluminum
H = thermal resistivity of cable insulation and covering in degrees centigrade per watt per foot of cable
y = constant for conductor metal determined by temperature resistance coefficient

Table I

	Copper	Aluminum
Constants		
t_f, fusion temp., deg. C	1080	655
y, temp. constant	234	228
Electrical conductivity, %	100	61
Thermal conductivity, % (silver)	89	48
500 MCM Copper; 500 MCM Aluminum		
Current carrying capacity, %	100	61
h, specific heat, w-s/ft/°C	308	187
h_f, latent heat of fusion, w-s/ft	221×10^3	118×10^3
H, thermal res., 600 volt cable	0.952	0.952
s' (10,000 amp)	75.6	24.5
s (10,000 amp)	82.0	25.0
500 MCM Copper; 820 MCM Aluminum		
Current carrying capacity, %	100	100
h, specific heat, w-s/ft/°C	308	306
h_f, latent heat of fusion, w-s/ft	221×10^3	194×10^3
H, thermal res., 600 volt cable	0.952	0.829
s' (10,000 amp)	75.6	65.8
s (10,000 amp)	82.0	72.0

The curves in figures 1 and 2 were calculated for 600 volt cable using the thermal resistivity given in the table assuming that the cable covering was maintained at the ambient temperature. These values check quite closely with some of the published test values.

Very truly yours,

H. J. REEVES (A'30)
Paul & Reeves,
West 1030 First Ave.,
Spokane, Wash.

Titles of Engineers

To the Editor:

The article on "Structure of the Electrical Engineering Profession" by Theodore J. Hoover in ELECTRICAL ENGINEERING, July 1935, pages 695–9, outlines a problem that may well be an early one for the Engineers' Council for Professional Development to consider. That is, the determination of approved professional titles and making these titles a part of the proposed professional certification. However, it would seem that the list of professional titles should be short and carefully selected rather than permitting the compounding of titles of numerous descriptive adjectives. As admitted by Dean Hoover the use of compound titles would result in several thousand combinations and, to my mind, be almost as confusing as the present situation.

At one time it was sufficient to classify an engineer as an "electrical engineer" or a "civil engineer," etc. This is admittedly not adequate now, but for example, the title of "communication engineer" might be suitable although the individual's interest may be in any one of several branches such as acoustical, communications, sound, telegraph, telephone, or telephone equipment. The distinction between professional titles and present position titles further avoids the necessity of compound detail professional titles. The present position title often includes the engineering function or identifies the limited field of the individual's interest. This is usually a specific title established by the engineer, his firm, or the institution with which he is connected, and gives sufficient latitude for designation of restricted activity or individuality.

Any step leading to the establishment of recognized professional titles and a distinction between professional titles and position titles deserves the support of the entire engineering profession.

Very truly yours,

FLOYD B. KNISKERN (A'19, M'27)
Engr., Circuit Breaker Division, Westinghouse Elec. and Mfg. Co., E. Pittsburgh, Pa.

Complex Hyperbolic Functions and Their Inverse

To the Editor:

The hyperbolic functions of complex variables, sinh $(X + j\theta)$, cosh $(X + j\theta)$, and tanh $(X + j\theta)$, present themselves prominently in numerous problems pertaining to long distance transmission of electrical energy, filter networks, attenuation equalizers used in communication systems, and generally in the solution of recurrent circuits both balanced and unbalanced.

In the excellent Kennelly tables of complex hyperbolic functions ("Tables of Complex Hyperbolic and Circular Functions," by A. E. Kennelly, Harvard University Press, 1914) the real member X of the complex variable is in steps of 0.05 and the j or quadrature member q is in steps of 0.05 of a quadrant, that is, in steps of 4.5 degrees for θ. A single interpolation is necessary if either the real member X or the quadrature member θ (i. e., q) is not included in the table, and a dual interpolation is needed to obtain the vector value of the desired function when the 2 members X and θ are not included. Thus the vector value of sinh (1.25 + j27 degrees), for example, may be obtained directly by using the proper table, either in the complex form $(U + jV)$ or in the polar form $r \angle \gamma$. A dual interpolation, however, is necessary to obtain the vector value of sinh (1.23 + j29 degrees) for example, because the complex quantity (1.23 + j29 degrees) is not included in the tables.

The well-known formulas

$$\sinh (X + j\theta) = \sqrt{\sinh^2 X + \sin^2\theta} \angle \tan^{-1} (\tan\theta / \tanh X) \tag{1}$$

$$\cosh (X + j\theta) = \sqrt{\sinh^2 X + \sin^2\theta} \angle \tan^{-1} (\tan\theta \cdot \tanh X) \tag{2}$$

and

$$\tanh(X + j\theta) = \frac{\sinh (X + j\theta)}{\cosh (X + j\theta)} \tag{3}$$

afford a rapid and convenient scheme of evaluating either one of these functions by considering the numerical value of equation 1 as the hypothenuse of a right triangle whose sides are sinh X and sin θ, and the numerical value of the function cosh X, represented by the radical in equation 2 as the hypothenuse of a right triangle whose sides are sinh X and cos θ.

Applying, therefore, the trigonometric relations between the sides of a right triangle and the angle between the hypothenuse and either side, the numerical value of either sinh $(X + j\theta)$ or cosh $(X + j\theta)$ may be obtained by only 2 divisions instead of taking the square root of the sum of 2 squares. The angle associated with the numerical value of the function as a vector may be obtained by the single division (tan θ/tanh X) for the hyperbolic sine, and by the single multiplication (tanh $X \cdot$ tan θ) for the hyperbolic cosine. The complete vector value of either one of the 2 functions may therefore be obtained by 3 divisions. This scheme of evaluating hyperbolic functions of complex variables is the fundamental principle embodied in the Keuffel and Esser log log vector slide rule devised and described by the author in his

paper "Vector Calculating Devices" (see A.I.E.E. JOURNAL, volume 47, May 1928, pages 336–40).

The use of the Kennelly tables for the determination of the complex variable from the known vector value of the hyperbolic sine, or of hyperbolic cosine or of the hyperbolic tangent of the complex variable, consists in scanning over the columns of the proper table to locate the known vector value of the function in terms of its numerical value r and the angle γ associated with it side by side in adjacent columns, if the known function is in the polar form $r\angle\gamma$; or its 2 components (U and V) side by side in adjacent columns, if the known function is in terms of a complex number $(U + jV)$.

Thus to determine $(X + j\theta)$, when sinh $(X + j\theta)$ is equal to $1.665\angle 30.99$ degrees $= 1.427 + 0.857$, for example, we scan over the columns of the proper $r\angle\gamma$ table or the proper $(U + jV)$ table to locate the particular association of $r = 1.665$ with the angle $\gamma = 30.99$ degrees side by side in adjacent columns, or the particular association of $U = 1.477$ and $V = 0.857$ side by side in adjacent columns to determine $(X + jq) = 1.25 + j0.3$, that is $(X + j\theta) = 1.25 + j27$ degrees.

Because of the comparatively large steps in the values of X and of θ, however, the particular vector value of the function is frequently not found in the tables. It is therefore usually necessary and frequently more convenient to determine the unknown complex variable $(X + j\theta)$ from the known vector value of its hyperbolic function, either the sine, cosine, or the tangent (whichever happens to be known) by direct calculation.

EVALUATION OF $\sinh^{-1}(U_s + jV_s)$

It may be shown by a graphical method, as Kennelly has done, or by means of the hyperbolic and trigonometric relations involved, that if

$$\sinh(X + j\theta) = A\angle\alpha = U_s + jV_s,$$

where $A\angle\alpha = U_s + jV_s$ is known, the complex quantity $(X + j\theta)$ may be determined by

$$\sin\theta = \frac{\sqrt{U_s^2 + (1 + V_s)^2} - \sqrt{U_s^2 + 1 - V_s)^2}}{2} = \frac{M_s + N_s}{2} \qquad (4)$$

and

$$\sinh X = \frac{U_s}{\cos\theta}, \qquad (5)$$

where the subscript s to the quantities U, V, M, and N indicates that they pertain to a sine function.

Since the radicals in equation 4 may be thought of as hypothenuses of right triangles whose sides are U_s and $(1 + V_s)$ for one, and U_s and $(1 - V_s)$ for the other, each may be calculated by only 2 divisions instead of by the square root of the sum of 2 squares. Thus

$$\sqrt{U_s^2 + (1 + V_s)^2} = \frac{U}{\sin\rho} = M_s \qquad (6)$$

where

$$\rho = \tan^{-1}\left(\frac{U_s}{1 + V_s}\right) \qquad (7)$$

and

$$\sqrt{U_s^2 + (1 - V_s)^2} = \frac{U_s}{\sin\phi} = N_s \qquad (8)$$

where

$$\phi = \tan^{-1}\left(\frac{U_s}{1 - V_s}\right) \qquad (9)$$

A slide rule having both trigonometric and hyperbolic scales such as the Keuffel and Esser log log vector rule may therefore be used conveniently and effectively to determine directly the value of θ by carrying out the divisions indicated in the last 4 expressions, and the value of X by the division indicated in equation 5.

To illustrate the above, let

$$\sinh(X + j\theta) = 0.662\angle 68.15 \text{ degrees} = 0.246 + j0.613$$

where

$U_s = 0.246$ and $V_s = 0.613$

By equations 6 and 7 there results $M_s = 1.63$; by equations 8 and 9 there results $N_s = 0.46$; whence by equation 4 there results $\theta = 35.8$ degrees, and by equation 5, $X = 0.3$.

EVALUATION OF $\cosh^{-1}(U_s + jV_s) = (X + j\theta)$

It may be shown in a manner similar to that for the evaluation of $\sinh^{-1}(U_s + jV_s)$ that for

$\cosh(X + j\theta) = \quad \angle\beta = (U_c + jV_c)$

where $B\angle\beta = U_c + jV_c =$ is known, the complex quantity $(X + j\theta)$ may be determined by

$$\cos\theta = \frac{\sqrt{V_c^2 + (1 + U_c)^2} - \sqrt{V_c^2 + (1 - U_c)^2}}{2} = \frac{M_c - N_c}{2} \qquad (10)$$

and

$$\sinh X \frac{V_c}{\sin\theta}. \qquad (11)$$

From the similarity of expressions 10 and 11 to expressions 4 and 5, respectively, it may readily be seen that the same scheme of calculation may be used to determine the complex quantity $(X + j\theta)$ when the vector value of cosh $(X + j\theta)$ is known as when the hyperbolic sine is known. Thus for

$\cosh(X + j\theta) = 1.2\angle 25.75$ degrees $= 1.081 + j0.522$,

where $U_c = 1.081$ and $V_c = 0.522$, there results $M_c = 2.15$, and $N_c = 0.527$. Whence by equation 10 there results $\theta = 36$ degrees and by equation 11, $X = 0.802$

EVALUATION OF $\tanh^{-1}(U + jV) = (X + j\theta)$

The complex quantity $X + j\theta$ may be determined from the components U and V of the vector value of its hyperbolic tangent by the following formula obtained by graphical construction and given on page 171 of Kennelly's tables

$$X + j\theta = \frac{1}{2}\log_e\left\{\frac{\sqrt{(1 + U)^2 + V^2}}{\sqrt{(1 - U)^2 + V^2}}\right\} + j\left\{\frac{180 \text{ degrees} - \tan^{-1}[(U + 1)/V] + \tan^{-1}[(U - 1)/V]}{2}\right\}$$

The values of X and θ respectively may, however, be calculated by the following simpler and more convenient formulas obtained from the hyperbolic and trigonometric relations involved

$$\tanh 2X = \frac{2U}{1 + D^2} \qquad (12)$$

and

$$\tan \frac{2V}{1 - D^2} \qquad (13)$$

In these formulas the quantity D is magnitude of the vector value of the hyperbolic tangent of $(X + j\theta)$, U is the real component of D, and V the j or quadrature component, as indicated by the following relation

$$\tanh(X + j\theta) = D\angle\delta = (U + jV) \qquad (14)$$

To illustrate the above, let

$$\tanh(X + j\theta) = 0.876\angle 24.07 \text{ degrees} = 0.8 + j0.357$$

in which $D = 0.876$, $U = 0.8$, and $V = 0.357$.

By equation 13, there results

$$\tanh 2X = \frac{2 \times 0.8}{1 + 0.767} = 0.906$$

whence

$2X = 1.5$ and $X = 0.75$

By equation 13 there results

$$\tan 2\theta = \frac{2 \times 0.357}{1 - 0.767} = 3.064$$

whence

$2\theta = 71.9$ degrees and $\theta = 35.95$ degrees

The calculations were made by means of the Keuffel and Esser log log vector slide rule which has both trigonometric and hyperbolic scales.

Very truly yours,

M. P. WEINBACH (A'22, M'31)
Professor of Elec. Engg.,
University of Missouri, Columbia

Registration of Engineers

To the Editor:

The paper entitled "Registration of Engineers" by D. B. Steinman in the August 1935 issue of ELECTRICAL ENGINEERING, pages 876–81, represents the point of view which is causing the continual extension of the field of government to the regulation of every conceivable activity of mankind. No one regulatory activity of this kind can be said to be unbearably oppressive, but in the aggregate the bureaucratic organization thus created is becoming an intolerable burden on this country.

The statement that the registration boards are made up of engineers of high standing may be true at the present time, but how long the boards will remain so constituted under the constant pressure of political influence is another matter. The writer doubts very much whether engineers with creative ability will care to serve for any length of time on such boards. Only engineers who have such ability coupled with the rare quality of good common sense should be allowed to sit in judgment on the fellow members of their profession. The enhancement of the standing of the

profession by legal restriction may be questioned in view of the fact that such restriction has long been established for stationary engineers, marine engineers and other mechanical trades, while the greatest living American electrical engineer chooses to be called by the unregistered and unlicensed title of "professor." The only respect to which the engineering profession is entitled is that which it gains through the creative work of its members. It is in this direction that the efforts of the profession may most profitably be directed.

The reason for the registration of the medical and legal professions is that their clientele is almost entirely composed of those who have no other means of protection against the quacks and the incompetent. The services of the engineer are given almost exclusively to those who are quite as competent to evaluate them as are any of the legally constituted boards.

The insistence that engineering is one profession and that all members should be subject to the same requirements, coupled with the requirement of a written examination, indicate that the examination must be so general in character that it could be passed more easily by a recent graduate than by an engineer with many years' experience in specialized work. It would appear that in an effort to fit the profession into a Procrustean bed, many useful members might be lopped off.

The above are the personal opinions of the undersigned, and were written without consultation with members of any organization with which he is connected.

Very truly yours,

H. T. Faus (A'24, M'34)
61 Nahant St.,
Lynn, Mass.

Armature Reactions in Unloaded Single Phase Generators

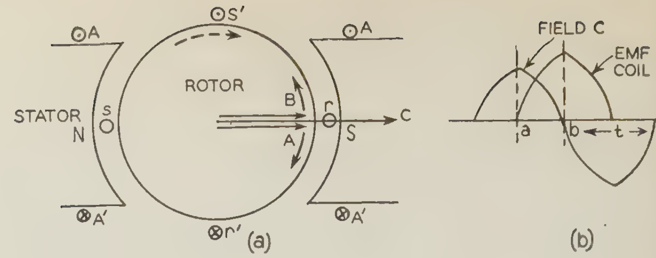
To the Editor:

This interesting study on Ferraris' theory applied to a study of armature reactions in unloaded single-phase synchronous generators, was prompted by the query of a junior electrical engineering student and it was this: Why not excite the field of a single-phase generator by alternating current and at synchronous speed take direct current from any 2 collector rings? We tried it and it did not work, but we also tried some other things which are explained in this letter.

Ferraris' theory states that a single-phase a-c field C or flux, N-S, figure 1(a), pulsating in time, can be resolved into 2 rotating fields A and B. Each is of constant magnitude and is equal to $^1/_2$ the maximum value of the pulsating field C. Field A rotates clockwise at synchronous speed with respect to C and is called the forward gliding component ($f.g.c.$) of C. Field B rotates counterclockwise at synchronous speed with respect to C and is called the backward gliding component ($b.g.c.$) of C. Evidently $A = B$.

The object of this communication is to present a physical concept of single-phase armature reactions, or flux, from Ferraris' viewpoint. It has been the writer's experience that single-phase reactions have

Fig. 1



been a stumbling block to many students from any physical viewpoint.

Studies were made on 2 machines, number 1 and number 2. Generator number 1 is a 4 pole synchronous converter (a revolving armature generator). Generator number 2 is a 4 pole revolving field generator.

Each generator was directly connected to a shunt motor whose speed could be varied. In this paper both generators are assumed to have but 2 poles; this makes the synchronous speed $n = f$ the applied frequency. Each armature is assumed to have but one coil, rs, figures 1 and 2.

Instead of impressing single-phase current across the armature of number 1, or across the stator of number 2, exactly the same flux effect was produced by impressing 250-volt 25-cycle single-phase alternating current across the d-c field of number 1, or across the d-c rotor of number 2.

The first half of the paper discusses instantaneous electromotive forces (e) for generator number 1 at zero speed, for different armature positions. The discussion is unchanged for generator number 2 and is therefore omitted. The symbols and abbreviations used are as follows:

Symbols and Abbreviations

C = pulsating field or flux, in space phase with field pole axis N-S
A = clockwise or forward gliding (or rotating) component of C
B = counterclockwise or backward gliding component of C
e_{rs} = instantaneous electromotive force in armature coil in position rs
$e_{r's'}$ = instantaneous electromotive force in armature coil in position $r's'$
E = effective electromotive force
E_m = maximum cyclic electromotive force
n = synchronous speed in revolutions per second
n' = speed of rotating member in revolutions per second
f = applied frequency
α = proportional to
I_f = field current
t = time in seconds
$f.g.c.$ = forward gliding component of field C
$b.g.c.$ = backward gliding component of field C

(a). At Instant a, Figure 1(b)

In this case, 25 cycle field current I_f produces field C; C pulsates back and forth along field axis 25 times a second ($n = 25$), no matter what speed n' may be. C also pulsates along horizontal axis in space 25 times a second, no matter what n' may be, because the field is always at rest. Pulsating flux $C = +$ maximum at instant a, figure 1(b); $+$ flux is assumed to be from left toward right in figure 1(a). Field current I_f is also $+$ maximum and is in time phase with C (this neglects hysteretic lag). The 2 rotating fluxes A and B are in space phase with C at instant a and $A = B = {}^1/_2C$.

First. Assume armature coil to be in position rs. Conductor r is cut clockwise by A, this induces an instantaneous electromotive force *into* the paper; conductor r is cut counterclockwise by B, this induces an instantaneous electromotive force *out* of the paper; total instantaneous electromotive force in conductor r is 0; similarly total instantaneous electromotive force in conductor s is 0. Therefore total instantaneous electromotive force in coil rs $(e_{rs}) = 0$.

Second. By hand, move armature coil clockwise into position $r's'$. Conductors r' and s' are cutting 0 flux since A and B are in horizontal position, at instant a. Therefore, total instantaneous electromotive force in coil $r's'$, i. e., $e_{r's'} = 0$.

(b). At Instant b, Figure 1(b)

Third. By hand, move armature coil back into position rs. Conductors r and s are cutting zero flux because A and B are in vertical position. Therefore total instantaneous electromotive force in coil rs, i. e., $e_{rs} = 0$.

Fourth. By hand, move armature coil clockwise into position $r's'$. Conductor r' is cut clockwise by arrow *heads* of A, this induces an instantaneous electromotive force *into* the paper and at instant b, it has a maximum value; conductor r' is cut counterclockwise by arrow *tails* of B, this induces an instantaneous electromotive force *into* the paper, it has a maximum value. Therefore total instantaneous electromotive force in conductor r', i. e., $e_{r'}$ = sum of these 2 electromotive forces, and $e_{r'}$ = maximum value; similarly $e_{s'}$ = maximum value. Therefore, total instantaneous electromotive force in coil $r's'$, i. e., $e_{r's'} =$ $+$ maximum at instant b ($= E_{r's}$).

Conclusion

1. Instantaneous electromotive force in coil rs, i. e., $e_{rs} = 0$, at instant a. Instantaneous electromotive force in armature coil $= 0$ in any position of armature at instant a. Instantaneous electromotive force in coils rs, i. e., $e_{rs} = 0$, at instant b. Instantaneous electromotive force in coil $r's'$, i. e., $e^{r's'} = +$ maximum at instant b, or instantaneous electromotive force in armature due to field C = 90 degrees behind C in time, figure 1(b).

2. From the viewpoint of transformer action, (a) and (b) above may be stated thus:
   (a). With armature coil in position rs the primary and secondary axes are at right angles and $e_{rs} = 0$ at all times; $E_{eff.} = 0$ at all times.
   (b). With coil in position $r's'$ the primary and secondary axes coincide and $e_{r's'} = E_m$ at instant b: $E_{r's'eff.}$ = maximum at all times.

The second half of this paper deals only with effective values of electromotive force (E).

The observations used to substantiate the theory were obtained by connecting in parallel, to any 2 collector rings of generator number 1 or to any 2 terminals of generator number 2: (a) a 0-0.25, 0-center d-c voltmeter; (b) an a-c voltmeter of suitable

range; and (c) an oscilloscope. In this manner single-phase observations were made on 2 polyphase generators.

Each generator was studied when: (a) rotor speed $n' = 0$; (b) $n' = n$; (c) $0 < n' < n$; (d) $n' = 2n$; and (e) $n < n' < 2n$.

Generator Number 1 With Revolving Armature, Figure 1

Case (a), $n' = 0$. Armature is at rest. See conclusion 2 above.

Observations: (1) armature coil in horizontal position rs.

(a). $E_{d\text{-}c} = 0$; needle does not vibrate.
(b). $E_{a\text{-}c} = 0$.
(c). Oscilloscope shows nothing.

Observations: (2). Armature coil in vertical position $r's'$.

(a). $E_{d\text{-}c} = 0$; needle vibrates 25 times a second.
(b). $E_{a\text{-}c} = 3$ volts $\alpha (A + B)n \; \alpha \; (A + B)25$. See ($b$) "*fourth*," above.
(c). Oscilloscope showed 25 cycle stationary wave; length of 1 cycle = 4 inches.

Observations: (3). Armature coil moved by hand from position rs to $r's'$ back to rs.

(a). $E_{d\text{-}c}$ reads to left of zero, then to right of zero (due to higher rate of cutting by B than by A and *vice versa*). Its magnitude and rapidity of reversal vary with the speed by which the coil is moved back and forth.
(b). $E_{a\text{-}c}$ reads 0 to 3 back to 0 volts (from zero transformer action, to maximum, back to zero). Time required to move pointer from 0 to 3 = time required to move coil from position rs to position $r's'$.
(c). Oscilloscope shows nothing, then a 25 cycle stationary wave, then nothing. Time required to change from nothing to stationary wave = time required to move coil from position rs to position $r's'$.

Case (b), $n' = n$. Armature is driven clockwise at synchronous speed.

E_1 (due to *f.g.c.* A) $\alpha \; (n - n') \; A = 0$. Therefore $E_{d\text{-}c} = 0$; see "third" final conclusion.

E_2 (due to *b.g.c.* B) $\alpha \; (n + n')B \; \alpha \; B \times 50$. *F.g.c.* A is at rest with respect to ar-

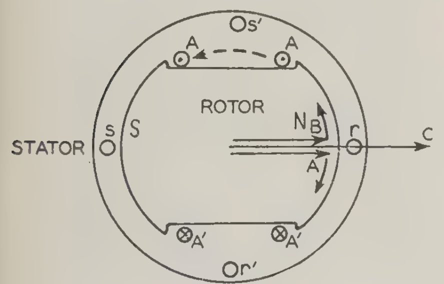


Fig. 2

mature; *b.g.c.* B passes surface of armature at speed $n + n'$.

Observations:

(a). $E_{d\text{-}c} = 0$; needle vibrates 50 times a second (due to speed $n + n'$ of *b.g.c.* B).
(b). $E_{a\text{-}c}$ (= 3 volts), $\alpha \; B \times 50$, but $B \times 50 = (A + B)$ 25; see observation 2(b) above.
(c). Oscilloscope shows a 50 cycle stationary wave; length of 1 cycle = 2 inches.

Case (c), $n' < n$

Observations:

(a). $E_{d\text{-}c}$; needle goes to right and to left of zero and more slowly as n' approaches n. This is due to the decreasing relative speed of field A and coil rs. Needle vibrates $(n + n')$ or < 50 times per second.
(b). $E_{a\text{-}c} =$ 3–4 volts; $E_{a\text{-}c}$ is made up of $E_1 \; \alpha \; A \times (n - n')$ and $E_2 \; \alpha \; B \times (n + n')$ and $E_{a-c} = \sqrt{E_1^2 + E_2^2}$. (See "Principles of Alternating Currents," first edition, page 84, McGraw-Hill Book Co.)
(c). Oscilloscope shows a moving wave of constantly changing amplitude and frequency as n' varies from 0 to n and as frequency varies from 25 to 50; 1 cycle of wavelength changes from 4 inches ($n' = 0$) to 2 inches ($n' = n$).

Case (d), $n' = 2n$. Armature is driven clockwise at 2 times synchronous speed.

E_1 due to *f.g.c.* $A \; \alpha \; (n' - n)A \; \alpha \; 25 \times A$; $f = 50 - 25 = 25$ cycles.

E_2 due to *b.g.c.* $B \; \alpha \; (n' + n)B \; \alpha \; 75 \times B$; $f = 50 + 25 = 75$ cycles.

Observations:

(a). $E_{d-c} = 0$; needle vibrates 75 times per second due to B and pulsates 3 times per second due to the 75 cycle field B being superimposed on the 25 cycle field A.
(b). $E_{a-c} =$ about 6 volts; $E = \sqrt{E_1^2 + E_2^2}$.
(c). Oscilloscope shows a 75 cycle stationary wave; length of 1 cycle = 4/3 inch (= 4 inches × 25/75); this wave is superimposed on a 25 cycle wave; if oscilloscope is synchronized for 25 cycles,

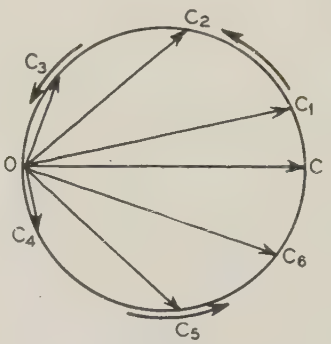


Fig. 3

than at $n' = 2n$ the 75 cycle wave is stationary, as well as the 25 cycle wave, and it is superimposed on the 25 cycle wave; see case (d), observation (a).

Case (e), $n < n' < 2n$.

Observations:

(a). $E_{d\text{-}c} = 0$; needle moves back and forth slowly on either side of zero as n' increases from n. This is due to the relative speed of field A and coil rs. Needle vibrates $(n + n')$ times a second due to speed of *b g.c.* B relative to coil rs; and $(n' - n)$ times a second due to speed of *f.g.c.* A.
(b). $E_{a\text{-}c} =$ 3–6 volts. Compare with observation (b) case (d).
(c). Oscilloscope, when $n' = 49$, shows a 74 cycle wave (49 + 25) due to *b.g.c.* B superimposed on a 24 cycle wave 49–25) due to *f.g.c.* A. Since oscilloscope is synchronized for 25 cycles, the resultant 74 cycle wave moves 1 cycle per second, of 25 cycle wave length, to left and moves up and down, because its axis is the 24 cycle wave which is not in synchronism with the oscilloscope.

General Observations:

First. Oscilloscope shows stationary wave at $n' = 0$, $n' = 12^1/_2$, $n' = 25$, and $n' = 50$ and length of 1 cycle is 4 inches, 3 inches, 2 inches, and 4/3 inch, respectively.
Second. When n' increases, wave moves forward; when n' decreases, wave moves backward and in each case these waves become stationary at the frequencies stated in the first general observation.

Generator Number 2 With Revolving Field, Figure 2

Case (a), $n' = 0$. (See Figure 2).

In this case, 25 cycle field current, I_f produces field C; C pulsates back and forth along field axis 25 times a second, no matter what speed n' may be; C pulsates along horizontal axis in space 25 times a second, only when $n' = 0$; see [(a) at instant a].

Number 2 generator is identical throughout cases (b), (c), (d), and (e), with number 1 generator except where $E_{a\text{-}c} = 3$ volts for number 1, $E_{a\text{-}c} = 16.5$ volts for number 2. This is due to the fact that C is larger in generator number 2 and so are A and B.

Final Conclusions

First. In number 1 generator, figure 1.

(a). Field C pulsates along field axis, no matter what n' may be.
(b). Field C pulsates along horizontal axis in space no matter what n' may be.
(c). *F.g.c.* A glides clockwise past field at speed n, no matter what n' may be.
(d). *F.g.c.* A glides clockwise in space at speed n, no matter what n' may be.
(e). *B.g.c.* B glides counterclockwise past field at speed n, no matter what n' may be.
(f). *B.g.c.* B glides counterclockwise in space at speed n, no matter what n' may be.

Second. In number 2 generator, figure 2.

(a). Field C pulsates along field axis, no matter what n' may be.
(b). Field C pulsates along horizontal axis in space only when $n' = 0$.
(c). *F.g.c.* A glides clockwise past field at speed n, no matter what n' may be.
(d). *F.g.c.* A is at rest in space, when $n' = n$.
(e). *B.g.c.* B glides counterclockwise past field at speed n, no matter what n' may be.
(f). *B.g.c.* B glides counterclockwise in space at speed $2n (= n + n)$, when $n' = n$.

If A is at rest in space at $n' = n$ see (d) above and B glides counterclockwise in space at speed $2n$ see (f) above the resultant of a stationary vector A + a rotating vector B gives a resultant C in space shown in figure 3.

The resultant C follows, in a counterclockwise direction, the locus of a circle and at $n' = n$ assumes successively the values OC, OC_1, OC_2, OC_3, OC_4, OC_5, OC_6, back to OC, 50 times each second in space, but only 25 times a second with respect to the rotor.

In number 1 generator, resultant C pulsates back and forth along a horizontal axis in space, 25 times a second. This is the big difference between the armature reactions in the 2 unloaded generators.

They both pulsate, back and forth along their respective field axes, 25 times a second.

Third. About obtaining direct current or when $f = 0$: only when $n' = n$ is frequency of $A = 0$ ($f = n - n'$), but then *f.g.c.* A has zero rate of cutting and therefore $E_{d\text{-}c} = 0$; see generator number 1, case (b) $n' = n$.

Fourth. It can easily be shown that if the generator has 3 windings 120 degrees apart, the 3 *f.g.c.* fields A coincide, producing 3 times the magnitude of field A, discussed in this paper, and the 3 *b.g.c.* fields B cancel each other and do not exist in the fundamental wave. This is a frequently used method of studying armature reactions in 3-phase generators.

Fifth. This same Ferraris' theory could be used in a similar manner in studying single-phase synchronous motors.

Sixth. Ferraris' theory is often used as an easy though somewhat unsatisfactory method of studying single-phase induction motors.

Seventh. This study of armature reactions could have been made using transformer (or pulsational) electromotive forces and rotational electromotive forces. Such a method is to be preferred in studying what takes place in single-phase induction motors.

Very truly yours,

J. L. Beaver (A'14, F'26)
Professor of Elec. Engg.,
Lehigh University, Bethlehem, Pa.

Personal Items

E. B. Meyer (A'05, F'27, and president) chief engineer, Public Service Electric and Gas Company, Newark, N. J., has been appointed chairman of the Institute's executive committee for the year 1935–36. Mr. Meyer, who was born at Newark in 1882, is a graduate of Pratt Institute, Brooklyn, N. Y., completing the electrical engineering course in 1903. In that year he entered the employ of the Public Service Corporation of New Jersey as engineering assistant, and was promoted through successive positions to become assistant chief engineer in 1919. Three years later he was appointed chief engineer of the Public Service Production Company at its formation, and in 1929 was made a vice president. With the merger the following year of this company and United Engineers and Constructors, Inc., Mr. Meyer was appointed a vice president of the latter in the capacity of executive and engineering head of the Newark office. Recently he was appointed chief engineer of the Public Service Electric and Gas Company. Mr. Meyer has served on many Institute committees in the past, and was a director 1928–31, and a vice president 1932–34. He is now a member of the Edison medal committee, and is Institute representative on the American Engineering Council and the Charles A. Coffin fellowship and research fund committee. Other societies of which he is a member include The American Society of Mechanical Engineers, the New York Electrical Society, Inc., and the American Transit Association. Mr. Meyer is the author of a number of technical articles, and the book "Underground Transmission and Distribution."

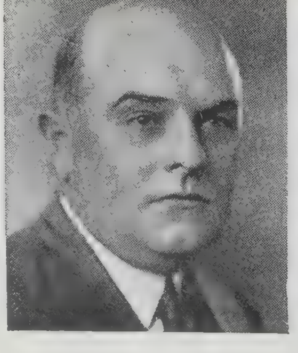
J. B. Whitehead (A'00, F'12, Life Member, and junior past-president) dean of the school of engineering, The Johns Hopkins University, Baltimore, Md., has been appointed chairman of the committee on Institute policy for the year 1935–36. Doctor Whitehead, who was born at Norfolk, Va., August 18, 1872, studied at The Johns Hopkins University, from which he received the degrees of electrical engineer (1893), bachelor of arts (1898), and doctor of philosophy (1902). He first was employed by the Westinghouse Electric and Manufacturing Company, and for a short time was with the Niagara Falls Power Company and the Pittsburgh Reduction Company. In 1898 he returned to the university as an instructor in electrical engineering, and became associate professor of applied electricity in 1905. He became professor in 1919, and since 1925 has been dean. In addition to teaching he has carried on a large consulting practice. He is well known for his work on high voltage insulation, and has presented a number of papers on this subject before the Institute. Doctor Whitehead has been active in the work of the Institute, and was a manager 1924–28 and president 1933–34. In addition to his present appointment as committee chairman he is a member of the executive, electrophysics, and Edison medal committees, and is a representative on the John Fritz medal board of award, and on the division of engineering and industrial research of the National Research Council. Among other committees on which he has served are electrochemistry and electrometallurgy, of which he was a member 1924–33, and research, of which he was a member 1920–33, and chairman 1922–27.

W. R. Smith (M'18, F'30) transmission construction engineer, Public Service Electric and Gas Company, Newark, N. J., has been appointed chairman of the Institute's committees on technical program and award of Institute prizes for the year 1935–36. Mr. Smith was born at Charleston, S. C., in 1885 and was graduated from Clemson Agricultural and Mechanical College in 1906 with the degree of bachelor of science in electrical and mechanical engineering. He remained at the college for a year as secretary and engineering assistant to the department head before entering the apprentice course of the Westinghouse Electric and Manufacturing Company, and from 1908 to 1913 was with the Hartford Suspension Company at Jersey City, N. J. In 1914 he engaged in construction work with the Public Service Electric Company at Newark, and was made field engineer in 1917. When the Public Service Construction Company was formed in 1922 he was made superintendent of the electrical construction department, becoming managing electrical engineer in charge of the electrical engineering department 2 years later. From 1927 until recently he was assistant chief engineer of United Engineers and Constructors, Inc., Newark. Mr. Smith has been a member of the Institute's board of examiners since 1930, and has served as chairman since 1934. He is also serving on the committees on publication and co-ordination of Institute activities. Mr. Smith is a member also of the Illuminating Engineering Society and the Edison Electric Institute.

O. W. Eshbach (A'17, M'30) special assistant, personnel department, American Telephone and Telegraph Company, New York, N. Y., has been appointed chairman of the Institute's committees on education and Student Branches for the year 1935–36. Mr. Eshbach was born at Pennsburg, Pa., April 13, 1893. He is a graduate of Lehigh University, from which he received the degrees of electrical engineer (1915) and master of science (1920). During the year 1916–17 he was an instructor in electrical engineering at the university, then entered the U.S. Army, subsequently being commissioned a lieutenant in the Signal Corps. In 1919 he returned to the university as an instructor, and the following year was made assistant professor. In 1923 he was made assistant engineer with the Bell Telephone Company of Pennsylvania, his work being associated with the employment and training of engineering graduates. He accepted his present position in 1925, his duties involving educational and employment work, and is nonresident instructor in electrical engineering at Massachusetts Institute of Technology and instructor in the graduate night school at Brooklyn Polytechnic Institute. Mr. Eshbach has been a member of the Institute's committee on education since 1931, and is now a member of the committees on technical program, publication, and Sections. He is a member of the Society for the Promotion of Engineering Education and other organizations, and in 1932 was director of the survey of adult technical education in the New York industrial area, on which subject he presented a paper before the Institute.

A. M. MacCutcheon (A'12, F'26, and past director) engineering vice president, The Reliance Electric and Engineering Company, Cleveland, Ohio, has been appointed chairman of the Institute's committee on the Lamme medal for the year 1935–36. Mr. MacCutcheon was born at Stockport, N. Y., December 31, 1881, and taught mathematics and science in high schools for 3 years following his graduation from Albany State Normal School in 1901. He then entered Columbia University, completing the electrical engineering course in 1908. From 1909 to 1914 he was employed by the Crocker-Wheeler Company at Ampere, N. J., where he was successively in charge of engineering estimates, all estimates and proposals, and the drafting room. In 1914 he took charge of all new design work for The Reliance Electric and Engineering Company, and was appointed chief engineer in 1917. From 1917 to 1919 he served in the U.S. Navy, becoming lieutenant in charge of fire control on the U. S. S. Louisiana. A

E. B. MEYER



J. B. WHITEHEAD

A. M. MAC CUTCHEON

W. R. SMITH

O. W. ESHBACH

R. N. CONWELL

year after his return to civil life he was elected a director of the company, and in 1923 was appointed vice president in charge of engineering. Mr. MacCutcheon has prepared many technical papers, and is recognized as an authority on motor applications for steel mill auxiliaries. He is also a member of the Institute's standards committee, on which he has served since 1922, and of which he was chairman 1931–34, and is Institute representative on both the electrical standards committee and the council of the American Standards Association. Mr. MacCutcheon has served on a number of other committees, including general power applications, of which he was chairman 1925–28, electrical machinery, applications to iron and steel production and meetings and papers (now technical program), and was representative on the U.S. national committee of the International Electrotechnical Commission 1931–35. He has also been active in the Association of Iron and Steel Electrical Engineers and other organizations of which he is a member.

R. N. Conwell (A'15, F'31) transmission and substation engineer, Public Service Electric and Gas Company, Newark, N. J., has been appointed chairman of the Institute's committee on constitution and bylaws for the year 1935–36. Mr. Conwell, who was born at Anderson, Ind., February 10, 1885, is a graduate of Purdue University and George Washington University. In 1911, following a year as engineer in charge of acceptance tests for the sewer department of Washington, D. C., he was employed as a cadet engineer by the Public Service Electric Company at Newark, later becoming laboratory assistant and chief of the materials division of the testing laboratory. He was appointed assistant engineer in the engineering department, with charge of station and substation design and system protection in 1917, and in 1922 was appointed transmission engineer in the distribution department with charge of transmission design and system load forecasting and planning. While in this capacity he redesigned and reconstructed the transmission system of the company. Since 1925 he has held his present position. Many technical papers have been prepared by him, and he is the holder of a number of patents, including that of a type of inverse current relay. Mr. Conwell has also served the Institute on many other committees, and is now a member of the committees on technical program and award of Institute prizes, having been chairman of both during 1933–35, publication, power transmission and distribution, and co-ordination of Institute activities, and is representative on the Alfred Noble prize committee and the American committee on marking of obstructions to air navigation. In addition to serving on many of the Institute's committees, Mr. Conwell has been a member of committees of the former National Electric Light Association (now Edison Electric Institute), American Standards Association, American Committee on Inductive Co-ordination, National Fire Protection Association, and Association of Edison Illuminating Companies.

D. W. Roper (A'93, F'14, and member for life) assistant electrical engineer, Commonwealth Edison Company, Chicago, Ill., retired recently. Mr. Roper was born at Grafton, Ill., and received the degree of mechanical engineer at Cornell University in 1893. Following experience in the student course of the General Electric Company and with the Missouri Edison Electric Company, St. Louis, Mr. Roper in 1903 entered the engineering department of the Chicago Edison Company, a company later consolidated into the present Commonwealth Edison Company. In 1904 he was made assistant to chief operating engineer, and had charge of engineering, construction, and operation of underground cables. In 1911 overhead systems were added to his responsibilities. From 1915 to 1933 he was superintendent of the street lighting department, being appointed assistant electrical engineer in charge of the research division of the engineering department. Mr. Roper is a well-known authority on underground cables, and has presented a number of papers to the Institute on this subject. He has been a member of the Institute's committee on research since 1924, and other committees on which he has served include power transmission and distribution, 1917–35, and U.S. National Committee of the International Electrotechnical Commission, 1923–30.

M. S. Coover (A'16, M'32) former professor of electrical engineering at the University of Colorado, Boulder, has been appointed professor and head of the department of electrical engineering at Iowa State College, Ames, where he succeeds F. E. Johnson (A'13, F'31). Professor Coover was born at Shippensburg, Pa., and was graduated from Rensselaer Polytechnic Institute in 1914 with the degree of electrical engineer. Following graduation he was in the employ of the New York Central Railroad and in 1915 went with the Montana Power Company until the outbreak of the World War. Shortly after his release from army service (1919) he was appointed instructor in electrical engineering at the University of Colorado. The following year he was appointed assistant professor, and 2 years later became associate professor. He was appointed professor in 1930. Professor Coover served on the automatic stations committee of the Institute, 1927–29, and was secretary of the North Central District 1929–33. He is also a member of the Society for the Promotion of Engineering Education and served on the electrical engineering committee 1933–34. During summer vacations at Colorado his activities as a consultant and investigator on electrical engineering problems have taken him to widely separated parts of the United States.

I. E. Moultrop (A'10, F'29, and past vice president) chief engineer and superintendent of the construction bureau of The Edison Electric Illuminating Company of Boston, Mass., has retired from active service after 43 years with the company. Mr. Moultrop was born at Marlboro, Mass., in 1865 and became an apprentice with the Whitter Machine Company in 1882. He became head draftsman of this company, which was a pioneer in the manufacture of electric elevators, and resigned in 1892 to become chief draftsman of The Edison Electric Illuminating Company of Boston. In 1897 he was appointed mechanical engineer, serving in that capacity until 1913 when he was appointed assistant superintendent of the construction bureau. His appointment to chief engineer was made in 1926, the supervision of the construction bureau being added subsequently. Throughout his period of service he has had a prominent part in all the construction work of the company, both mechanical and electrical, including also transmission and distribution. Mr. Moultrop's work has been recognized by the award of the Elliott Cresson medal by the Franklin Institute and the honorary degree of mechanical engineer by Stevens Institute of Technology. Recently he was appointed chairman of the Boston area committee in the drive to raise funds for Engineering Index, Inc. A past chairman of the Boston Section, he has been very active in the affairs of the Institute, and was a director 1926–30 and a vice president 1930–32. He is the author of several Institute papers, and has been a member of the power generation committee since 1916 and of the automatic stations committee since 1930, serving also on others. Mr. Moultrop is a member and past officer of The American Society of Mechanical Engineers, and is president of the Engineers' Club of Boston.

F. P. Cox (A'01, F'13) manager of the West Lynn, Mass., works of the General Electric Company, retired on September 1, 1935. Mr. Cox graduated from Rose Polytechnic Institute, Terre Haute, Ind., in 1887, and since 1889 has been with the General

Electric Company and its predecessors, having charge of meter and instrument design and development from 1894 to 1920, when he was appointed manager. A number of patents were granted to him for his work, which included prepayment and rate meters. Mr. Cox was a member of the Institute's standards committee 1915–22 and 1923–28, and served also on the instruments and measurements committee, 1917–24, and on a joint power factor committee 1919–20.

F. E. Johnson (A'13, F'31) who has been head of the department of electrical engineering of Iowa State College, Ames, has accepted the position of dean of the college of engineering at the University of Missouri, Columbia. Dean Johnson was born at Le Roy, Mich., and is a graduate of the University of Michigan, having received the degrees of bachelor of arts, in 1906 and of electrical engineer in 1909. Following some experience in electrical construction work he became an instructor at Rice Institute, Houston, Tex., in 1912 and 3 years later went to the University of Kansas, Lawrence, where he was successively assistant professor, associate professor, and professor, being appointed head of the department of electrical engineering in 1928. In 1930 he resigned to become head of the department at Iowa State College. During the period 1933–35 Dean Johnson served as a member of the Institute's commmittee on education.

J. T. Mountain (A'04, F'20) Chicago, Ill., has retired from the Commonwealth Edison Company after 36 years with the company. Mr. Mountain entered the employ of the Chicago Edison Company following his graduation from the electrical engineering course at the University of Michigan in 1899. He served as load dispatcher and chief load dispatcher from 1904 to 1910, when he was made assistant to the chief operating engineer. He held this position until recently, when he was appointed assistant service manager. In addition to his general interest in operating affairs he concerned himself especially with lamp matters and policies, and was a member of the lamp committee of the Association of Edison Illuminating Companies.

R. H. Tapscott (A'18, F'29, and vice president) vice president of the New York Edison Company, New York, N. Y., has been elected to the board of the New York Edison Company and to the executive committee of the United Electric Light and Power Company. Mr. Tapscott, in addition to serving the Institute as a vice president, is also chairman of the finance committee and a member of executive, headquarters, and co-ordination of Institute activities committees.

J. D. Ross (A'08, F'12) superintendent of lighting, City of Seattle, Wash., has been appointed a member for a 5 year term of the Securities and Exchange Commission by President Roosevelt. Mr. Ross was born at Chatham, Ont., Can., and was educated in Canada. For a short time he was engaged in mining, and in 1903 was employed on the design, construction, and operation of the municipal electric plant at Seattle, engaging also in other consulting and supervising work. Since that time Mr. Ross has been superintendent of the system. Recently he was appointed chief consulting engineer of the power division just organized by the Public Works Administration. He was a member of the Institute's committee on power stations (now power generation) 1918–19 and 1920–21.

R. C. Hummel (A'29) equipment engineer, West Coast Telephone Company, Everett, Wash., has been awarded the 1934 A. I. E. E. North West District prize for best paper for his paper "The Design and Operation of an Automatic All Relay Telephone

R. C. HUMMEL

Switching System for Small Communities." Mr. Hummel was born at Lopez Island, Wash., and has been engaged in telephone work since 1918, when he was employed at switchboard maintenance by the Puget Sound Telephone Company, predecessor of the West Coast Telephone Company. During the year 1923–24 he was design and production engineer with the North Electric Manufacturing Company, Galion, Ohio, where he worked on the design of the first magneto machine switching telephone exchange placed in commercial service and on power supervisory equipment of various types. In 1924 Mr Hummel assumed his present duties as equipment engineer.

A. H. Kehoe (A'12, F'25) vice president, New York Edison Company, New York, N. Y., has been elected to the board of the United Electric Light and Power Company, New York. Mr. Kehoe is a member of the Institute's committee on standards, and was recently elected second vice president of the New York Electrical Society to serve for the year 1935–36.

N. J. Darling (M'34) who is manager of the River Works of the General Electric Company at Lynn, Mass., has also assumed management of the works at West Lynn following the retirement of F. P. Cox (A'01, F'13). Mr. Darling graduated from Cornell University in 1907, and was entered in the student course of the General Electric Company at Schenectady, N. Y., until 1909. He returned to the company at Erie, Pa., in 1915, and 3 years later was appointed assistant works manager there. Since 1922 he has been manager of the River Works at Lynn. Mr. Darling was first elected to membership in the Institute in 1922, and was a member of the committee on education during 1926–28.

W. B. Jackson (A'97, F'13, past vice president and Life Member) rate engineer of the New York Edison Company, New York, N. Y., since 1920, has been made rate consultant. Colonel Jackson graduated from Pennsylvania State College in 1890, and was engaged by several companies prior to his becoming a member of the consulting firm of D. C. and W. B. Jackson in Chicago, Ill. He was made rate engineer of the New York Edison Company in 1920, shortly after his return from service in the army. Colonel Jackson was a manager of the Institute, 1912–15, and a vice president 1918–19, and is now an Institute representative on the commission of the Washington award. He is a member and past officer of a number of organizations.

L. S. Keith (A'06) consulting engineer, Chicago, Ill., has been elected secretary of the Western Society of Engineers. Mr. Keith, who graduated from Massachusetts Institute of Technology in 1900, has been a consultant on public utility and industrial matters since 1922, and has been connected with the Cook Electric Company, Chicago, as vice president and director. Recently he was a member of the staff of the administrator of the work and rehabilitation division of the Illinois Emergency Relief Commission, resigning to assume his position with the Western Society of Engineers, which he already has served as treasurer and vice president.

F. M. Terry (A'22, M'28) executive assistant, New York Edison Company, New York, N. Y., has been appointed rate engineer to succeed W. B. Jackson (A'97, F'13, past vice president and Life Member). Mr. Terry, who was graduated from Union College in 1920, was employed in the test department of the New York Edison Company in 1921, following a year with the General Electric Company at Schenectady, N. Y. In 1923 he was appointed assistant to the rate engineer, and became chief assistant in 1927. Since 1932 he has been executive assistant.

G. E. McCarn (M'22) Denver Colo., has retired as chief engineer of The Mountain States Telephone and Telegraph Company. Mr. McCarn was born at Plattsville, Wis., and was employed in the engineering department of The Colorado Telephone Company in 1899, becoming assistant chief engineer of the company 4 years later. In 1911 he became general plant superintendent in charge of the engineering department of its successor, The Mountain States Telephone and Telegraph Company, and in 1921 was appointed chief engineer.

Russell Hastings (A'14) electrical engineer, Boston, Mass., who for the past 15 years has been engaged in public utility rate work, has joined the staff of the Edison Electric Illuminating Company of Boston as rate engineer. Mr. Hastings, a graduate of Massachusetts Institute of Technology, was employed by the Westinghouse Electric and Manufacturing Company and the Boston Elevated Railway Company before he engaged in his present line of activity; in this work he has been in association with A. S. Knight, through whose offices he has been closely identified with The Edison Electric Illuminating Company.

W. J. Canada (A'20, M'20) formerly head of the engineering department of the National Electrical Manufacturers' Association, New York, N. Y., has entered upon a new activity as special engineering representative with offices at New York for the Chase-Shawmut Company and the Wiremold Company. Mr. Canada, who has given special attention to safety work, has previously been connected with the National Fire Protection Association, Rocky Mountain Fire Underwriters' Association, and Bureau of Standards, and was a member of the Institute's committee on safety codes from 1924 to 1928.

J. T. Nichols (M'30) former manager of the research laboratory of the American Sheet and Tin Plate Company, Pittsburgh, Pa., has been appointed special representative for the company, and is now in San Francisco, Calif. Mr. Nichols, in connection with his past work, developed an automatic optical pyrometer for measuring the temperature of glowing objects. He is a member of the American Society of Refrigerating Engineers, the Engineers Society of Western Pennsylvania, and other organizations, and is the author of several technical papers.

G. T. Shoemaker (M'20) vice president and electrical engineer, United Light and Power Engineering and Construction Company, Davenport, Ia., has assumed the duties of vice president of the Kansas City (Mo.) Power and Light Company. Mr. Shoemaker, a graduate of Purdue University became electrical engineer for the United company in 1913, and since then has been engaged in the engineering of various steam power and hydraulic plants, as well as transmission lines and distribution systems.

Magnus Bjorndal (A'25, M'34) former chief engineer of The Daven Company, Newark, N. J., is the founder and president of Tech Laboratories, Jersey City, N. J., a new company which will specialize in the manufacture of precision electrical resistance instruments and allied products. Mr. Bjorndal, before joining The Daven Company recently, was chief engineer of Hardwick, Hindle Incorporated, Newark, where he was in charge of the design and production of various electrical equipment.

C. E. Wilson (A'16) vice president of the General Electric Company, Schenectady, N. Y., has been made chairman of the board of Houses, Incorporated. Houses, Incorporated, is a company newly organized by the General Electric Company to cooperate with others in the development of houses of any type which seems worthy and promising, to conduct research work, and to assist in the management and financing of such enterprises, being concerned primarily with the interior mechanism of the house.

H. V. Putman (A'23, M'32) since 1931 manager of the transformer engineering department of the Westinghouse Electric and Manufacturing Company at Sharon, Pa., is now manager of the switchgear management unit of the company at East Pittsburgh, Pa. Mr. Putman is the author of a number of Institute papers, and has been appointed a member of the power transmission and distribution committee for the year 1935–36. He served as a member of the electrical machinery committee 1931–35.

Matthew Luckiesh (A'11, M'15) director, lighting research laboratory, General Electric Company, Cleveland, Ohio, has had the honorary degree of doctor of engineering conferred upon him by Purdue University, which has conferred this degree only 6 times in 61 years. The citation in regard to Doctor Luckiesh is "whose skillful moulding of the candles of science has furnished new light for the darkened places where men live."

Elihu Thomson (A'84, F'13, HM'28, past-president and member for life) consulting engineer, General Electric Company, and director, Thomson Research Laboratory, Lynn, Mass., was elected an honorary president oft he International Electrotechnical Commission at the meeting in Brussels. Recently Doctor Thomson received the award of the V. D. I. medal of honor (Electrical Engineering, May 1935, page 574).

O. E. Allende (A'25, M'31) electrical engineer and assistant to the general manager, Electric Company of Costa Rica, (Delaware) and local manager, Compania Electrica de Turrialba, Cartago, Costa Rica, C. A., has been appointed by the Chilean government civil attaché to the Chilean Legation for all the Central American nations, this appointment being in addition to his regular position.

F. A. Merrick (A'07) president, Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa., has been decorated with the Order of the Rising Sun, third class, by Emperor Hirohito of Japan in appreciation of technical assistance rendered to that country's electrical industry and railway lines. The presentation was made by Communications Minister Tokonami.

J. C. Burkholder (M'30) former partner of the firm of Burkholder and Kelley, Toronto, Ont., Can., is now vice president and chief engineer of Burlec Limited, a company newly formed at Scarboro Junction, Ont., for the manufacture of line and special telephone and telegraph apparatus. Mr. Burkholder was at one time engaged with the Canadian National Railways in developing a method of telephoning from a moving train.

Sidney Hosmer (A'97, F'12, and member for life) vice president and general manager, The Edison Electric Illuminating Company of Boston, Mass., has assumed the duties of acting head of the construction bureau, following the retirement of I. E. Moultrop (A'10, F'29, and past vice president). Mr. Hosmer has been a vice president of the company since 1926, and was appointed general manager in 1932.

J. W. Barker (M'26, F'30) dean, school of engineering, Columbia University, New York, N. Y., is serving as New York area chairman in the drive to raise funds for Engineering Index, Inc. Dean Barker has been serving on the Institute's committees on education, technical program, publication, and production and application of light, having been chairman of the latter since 1933.

William Spraragen (A'17, M'26) president, Spraragen Engineering Corporation, New York, N. Y., and former secretary of the National Research Council, has been appointed a member of the welding research committee sponsored by Engineering Foundation. Mr. Spraragen has served on the Institute's committee on electric welding since 1927.

G. M. Armbrust (A'11, F'33) assistant electrical engineer, Commonwealth Edison Company, Chicago, Ill., has been given charge of the new plant engineering division of the company, formed from the combination of the design and field engineering divisions. Mr. Armbrust was recently appointed a member of the Institute's power transmission and distribution committee.

H. C. Wolf (A'16, M'23) formerly vice president of the Consolidated Gas and Electric Company, New York, N. Y., has become president of the Central Indiana Gas Company, Muncie, the operations of which cover gas, electric, and water properties in the central and southern portions of the state. Mr. Wolf is a former director of the Institute's Baltimore Section.

W. E. Wyss (A'35) who has been in the test department of the General Electric Company at Schenectady, N. Y., has been sent to Washington, D. C., as a student patent attorney in the patent department of the company. He is co-author of a paper on induction motor starting which is scheduled for presentation at the coming Great Lakes District meeting of the Institute.

N. M. DuChemin (A'29) formerly general superintendent of the West Lynn, Mass., works of the General Electric Company, has assumed the duties of assistant manager in charge of operations. He was educated at London University, and graduated from the General Electric engineering school in 1917. He was made general superintendent at West Lynn in 1924.

E. E. TURKINGTON (A'08) electrical engineer, Associated Factory Mutual Fire Insurance Companies, Boston, Mass., has been appointed chairman of a newly formed subcommittee of the electrical committee of the National Fire Protection Association, the subcommittee being concerned with the subject of small circuit breakers.

C. H. CHAMPLAIN (A'28) general works manager, Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa., has been forced to retire by illness. He had been with the company for many years, becoming manager of the plant at Sharon, Pa., in 1922 and general works manager at East Pittsburgh more recently.

W. J. MAHAN (A'25) electrical inspector in the building department of the city of New Haven, Conn., has been appointed chairman of a newly formed subcommittee of the electrical committee of the National Fire Protection Association, the new subcommittee being concerned with the subject of fuses.

R. H. COMBS (A' 13, M'18) president and general manager, Prest-O-Lite Storage Battery Company, Ltd., Toronto, Ont., Can., has been elected a member of the council of the Association of Professional Engineers of Ontario, representing the mechanical branch, and also has been elected a member of the executive council of the association.

H. M. HOBART (A'94, F'12, and member for life) consulting engineer, Schenectady, N. Y., has been appointed a member of the welding research committee sponsored by Engineering Foundation. Mr. Hobart is chairman of the Institute's committee on electric welding, and a member of the technical program committee.

J. P. BARTON (A'30) former sales engineer in the electrical sheet division of the Empire Sheet and Tin Plate Company, Mansfield, Ohio, has accepted a similar position with the American Sheet and Tin Plate Company, Pittsburgh, Pa. Mr. Barton, a member of several other organizations, is the author of 2 Institute papers.

J. C. LANGDELL (A'07) meter engineer, Commonwealth and Southern Corporation, Jackson, Mich., has been appointed a member of the new subcommittee on small circuit breakers formed under the electrical committee of the National Fire Protection Association.

H. N. PYE (A'15, M'27) chief engineer, Southeastern Underwriters Association, Atlanta, Ga., has been appointed a member of the new subcommittee on small circuit breakers formed under the electrical committee of the National Fire Protection Association.

R. B. SHEPARD (A'25, M'26) electrical engineer, Underwriters' Laboratories, Inc., New York, N. Y., has been appointed a member of the new subcommittee on small circuit breakers formed under the electrical committee of the National Fire Protection Association.

A. D. STODDARD (A'16) chief engineer, Haliburton Oil Well Cementing Company, Duncan, Okla., has been made a vice president of the company. Mr. Stoddard, who has been with the company for several years, will be in charge of manufacturing and engineering.

C. A. ADAMS (A'94, F'13, past-president and member for life) Lawrence professor of engineering, Harvard University, Cambridge, Mass., has been appointed chairman of Engineering Foundation's welding research committee. He is also a member of the Institute's committee on electric welding.

S. M. KINTNER (A'02, M'03) vice president in charge of research, Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa., has been appointed chairman of the Pittsburgh area committee in the drive to raise funds for Engineering Index, Inc. Mr. Kintner has been a member of several Institute committees.

C. F. SCOTT (A'92, F'25, HM'29, past-president and member for life) professor of electrical engineering emeritus, Yale University, New Haven, Conn., has been appointed a member of the board which will supervise registration and examination of all types of engineers in Connecticut.

E. G. CULLWICK (A'26, M'33) who for a short period was with the electrical engineering branch of the Military College of Science at London, England, has returned to Vancouver, B. C., Can., as associate professor of electrical engineering at the University of British Columbia.

J. E. YARMACK (A'35) has joined the Diehl Manufacturing Company, the electrical division of the Singer Manufacturing Company, at Elizabethport, N. J., as research and development engineer in charge of the laboratory, development, and experimental shops.

A. F. DARLAND (A'20, M'29) former superintendent of electrical construction and design in the department of public utilities, Tacoma, Wash., is now field engineer with the Department of Interior reclamation service on the Grand Coulee (Wash.) Columbia River project.

W. J. LEWIS (A'33) has resigned as assistant electrical engineer with the Cincinnati (Ohio) Street Railway Company to take a position as engineer with R. Roy Holden, Inc., Chicago, Ill., manufacturers of overhead equipment for trackless trolley lines.

E. A. MOYLE (A'26) who has been distribution engineer for the Texas Power and Light Company at Tyler, is now consulting and chief engineer of the S. W. Scales electric Company, distributor at Greenville, Texas, in which Mr. Moyle has a part interest.

C. A. TUDBURY (A'35) is now a junior electrical engineer with the New England Power Engineering and Service Corporation, Worcester, Mass. A paper by him recently received a prize in a technical paper contest between the Lynn and Boston Sections.

W. D. CASSIN (A'23) Philadelphia, Pa., former sales engineer with the Westinghouse Electric and Manufacuring Company, is now in the engineering department of the Pennsylvania State Public Service Commission, Philadelphia.

D. S. JACOBUS (A'03, and member for life) advisory engineer, Babcock and Wilcox Company, New York, N. Y., has been appointed a member of the welding research committee sponsored by Engineering Foundation.

M. M. BRANDON (A'34) associate electrical engineer, Underwriters' Laboratories, Inc., New York, N. Y., has been appointed a member of the new subcommittee on fuses formed under the electrical committee of the National Fire Protection Association.

L. W. GOING (A'19) chief electrical inspector for the city of Portland, Ore., has been appointed a member of the new subcommittee on fuses formed under the electrical committee of the National Fire Protection Association.

C. F. HIMES (A'34) formerly in the Mound Valley School at Hominy, Okla., is now with the Eagle Mills at Edmond, Okla., as engineer. Mr. Himes received the A.I.E.E. South West District prize for Branch paper in 1933.

S. K. WALDORF (A'27) who has been a fellow at The Johns Hopkins University, Baltimore, Md., is now with the Pennsylvania Water and Power Company, Baltimore. Mr. Waldorf has presented several papers to the Institute.

ALEX DOW (A'93, F'13, and member for life) president, Detroit Edison Company, Detroit, Mich., has received the honorary doctor of science degree from the University of Detroit. He is the author of a 1934 Institute paper on the schooling of engineers.

L. E. MESSINGER (A'30) president and managing director, Canadian Line Materials Limited, Scarboro Junction, Ont., Can., is president of the newly formed Burlec Limited, a manufacturing company at Scarboro Junction.

SIDNEY SIMPSON (A'12, M'27) formerly deputy electrical locomotive superintendent, Eastern Bengal Railroad, Kanchrapara, Bengal, India, is now chief electrical engineer of the North Western Railway, Lahore, Punjab.

RAYMOND RUGGE (A'30) formerly in the engineering department of the Curtiss Wright Airplane Company, Robertson, Mo., is now municipal engineer and superintendent of the electric light plant at Larned, Kan.

P. M. ROSS (A'34) formerly in the commercial engineering department of the

Frigidaire Corporation, Dayton, Ohio, is now laboratory engineer with the insulator division of the Ohio Brass Company at Barberton.

H. B. Dates (A'98, F'32, and member for life) professor of electrical engineering, Case School of Applied Science, Cleveland, Ohio, has been elected vice president of the Illuminating Engineering Society to serve a 2 year term.

S. W. Stoddard (A'28, M'30) who has been superintendent of the northeastern division of the New England Power Association at Lawrence, Mass., is now central division superintendent at Worcester, Mass.

Peter Fries, Jr. (A'34) New York, N. Y., has been appointed associate editor of The Justinian, legal newspaper of Brooklyn Law School of St. Lawrence University, for the year 1935–36.

W. A. F. Zarth (A'23) who was formerly employed by Southern Dairies, Inc., Washington, D. C., has become affiliated with Dictograph Products Company, Inc., Jamaica, N. Y., as production engineer.

Samuel Ferguson (A'02) president and chairman of the board of directors, Hartford Electric Light Company, Hartford, Conn., has been re-elected as a trustee of the Edison Electric Institute.

N. A. Rollins (M'27) station installation engineer, Commonwealth Edison Company, Chicago, Ill., has been given supervision of the line installation and service investigation sections.

J. W. Keeney (A'22) who has been superintendent of the southern division of the New England Power Association at Providence, R. I., is now superintendent of the northeastern division at Lawrence, Mass.

L. H. Hendrixson (A'32) formerly with the Southern California Telephone Company, Los Angeles, is now connected with the bureau of power and light of the City of Los Angeles.

W. L. Abbott (A'01, F'13, and member for life) chief engineer, Commonwealth Edison Company, Chicago, Ill., is serving as area chairman in Chicago in the drive to raise funds for Engineering Index, Inc.

J. H. Herron (M'35) president, James H. Herron Company, Cleveland, Ohio, is sponsoring a committee in Cleveland in the drive to raise funds for Engineering Index, Inc.

Chester Stevens (A'32) former plant superintendent for the Producers Cold Storage Company, Shelbina, Mo., is now plant manager for the Independence Produce Company, Independence, Iowa.

C. G. Grimes (A'33) lieutenant, U.S. Navy, who has been electrical officer of the U. S. S. New Mexico, been has transferred to the bureau of engineering of the Navy Department at Washington, D. C.

R. Y. Miner (A'35) former engineer with Calibron Products, Inc., West Orange, N. J., has accepted a position as design engineer with Arma Engineering Company, Inc., Brooklyn, N. Y.

E. J. Withers (A'34) formerly employed by the Shell Petroleum Company, East Chicago, Ind., has taken a position as switchboard operator with the Inland Steel Company at East Chicago.

William Waterman (A'31) former chief engineer, Lang Radio Corporation, Brooklyn, N. Y., is now a radio engineer with the Victor division of the R.C.A. Manufacturing Company at Camden, N. J.

E. C. Morse (A'07, M'13) former manager, co-operative merchandising department, American Bemberg Corporation, New York, N. Y., is now general director for Associated Wool Industries, N. Y.

R. H. Hughes (A'20, M'30) assistant vice president, New York (N. Y.) Telephone Company, has been elected president of the New York Electrical Society to serve for the year 1935–36.

G. F. Fowler (A'29) member of technical staff, Bell Telephone Laboratories, Inc., New York, N. Y., has been elected treasurer of the New York Electrical Society to serve for the year 1935–36.

W. F. Oliver (A'23) who has been foreign wire relations supervisor with the Southern Bell Telephone and Telegraph Company at Atlanta, Ga., has been transferred to Baton Rouge, La.

W. G. Kelley (A'08, F'26) plant design engineer, Commonwealth Edison Company, Chicago, Ill., has been given supervision of the drafting and equipment sections.

E. A. Loew (A'08, M'13) professor of electrical engineering at the University of Washington, Seattle, was recently appointed acting dean of the college of engineering.

J. H. Hunt (A'07, M'13) patent section, General Motors Corporation, Detroit, Mich., has been elected president of the Detroit Engineering Society.

C. T. Mess (A'27, M'29) former electrical engineer of the California Railroad Commission, San Francisco, has been appointed valuation engineer for the commission.

J. P. E. Arberry (A'30) is now connected with the Pittsburgh Plate Glass Company, Pittsburgh, Pa., as assistant electrical engineer.

A. H. Lane (A'27) who is employed by the American Telephone and Telegraph Company, has been transferred from New Haven, Conn., to New York, N. Y.

J. P. Schroeder (A'31) Oakland, Calif., is now employed in the engineering department of the Yuba Manufacturing Company, Benicia.

Paul Boodberg (A'30) junior engineer, Pennsylvania Power and Light Company, has been transferred from Pottsville, Pa., to Hazleton.

H. M. Duphorne (A'29) toll plant extension engineer, Southwestern Bell Telephone Company, has been transferred from Oklahoma City, Okla., to Little Rock, Ark.

F. C. Moak (A'29) professional engineer of Saugerties, N. Y., is now connected with the Arma Manufacturing Company, Brooklyn, N. Y.

J. R. McCaa (A'34) is now employed in the mechanical engineering department of the York Ice Machinery Corporation, York, Pa.

W. T. White (A'33) assistant, U.S. Coast and Geodetic Survey, who has been at Corpus Christi, Texas, is now at Burlington, N. J.

J. J. Rose (A'33) electrical draftsman, New York and Queens Electric Light and Power Company, Flushing, N. Y., has been appointed chief draftsman.

J. G. Pleasants (A'32) production foreman, Proctor and Gamble Company, who has been at Cincinnati, Ohio, is now at Staten Island, N. Y.

P. J. Boesen (A'23) is now with the Public Service Commission of West Virginia at Charleston.

W. A. Stelzer (A'34) is now employed in the U.S. Engineer Department at Fort Peck, Mont.

L. R. Rockholt (A'33) is now junior electrical engineer with the Bowie Switch Company, San Francisco, Calif.

A. N. Budden (A'26) of Montreal, Que., Can., is now with the Dominion Engineering Company, Limited, Toronto, Ont.

G. H. Ulrich, Jr. (A'34) is now a draftsman in the engineering department of the Link Belt Company, Chicago, Ill.

D. F. Hild (A'33) is now area manager at Edmore for the Central West Public Service Company of North Dakota.

H. R. Anderson (A'27) of Huron, S. D., is now connected with the U.S. Engineers at Fort Peck, Mont.

T. C. Dearlove (A'25) is now employed by the Cemco Electrical Manufacturing Company in Vancouver, B. C., Can.

E. O. Lunn (A'32) is now employed as mechanic and electrician at Bradian Mines Limited, Bralorne, B. C., Can.

T. T. Woodson (A'33) is now in the test department of the General Electric Company at Schenectady, N. Y.

F. L. Ost (A'29) who has been at Vesteras, Sweden, is now employed by the Svenska Elektriska Aktiebolaget at Ludvika.

W. P. Stevens (A'28) has engaged in consulting engineering in Palestine, Texas.

Obituary

William Charles Gotshall (A'01, M'02, and Life Member) consulting and constructing engineer, New York, N. Y., and former president and chief engineer of the New York and Port Chester Railroad, died August 20, 1935. Mr. Gotshall was born at St. Louis, Mo., May 9, 1870. Following studies at Washington University he was employed by the St. Louis and Eastern Railroad Company and the Missouri Power and Light Company, and subsequently rebuilt and operated a number of electric railways. As chief engineer of the Union Railway Company, St. Louis, he undertook the rehabilitation of the system, which involved the first introduction of the 3 wire system on railways. In 1897 he was in charge of the rebuilding of the Second Avenue Railway in New York, which was converted from horsepower to a conduit system. Following this he became president and chief engineer of the New York and Port Chester Railroad, which he designed and constructed. This was the first high speed electric railway in the United States built entirely on its own right of way without grade crossings, which involved the introduction of many improvements in high speed electric traction. This work was completed in 1912, and subsequently Mr. Gotshall was engaged in other railroad work in the United States, Europe, and Africa. He also took part in several archaeological expeditions and was the author of a number of articles on electric railways as well as a book on their economics. He was a member of the American Society of Civil Engineers, the American Association for the Advancement of Science, and other organizations.

Otto Wallace Walter (A'21, M'29) assistant professor of electrical engineering, College of the City of New York, N. Y., was killed in an automobile accident July 7, 1935. He was born at Beatrice, Neb., December 28, 1892, and received the degrees of bachelor of arts (1920), bachelor of science (1921), and electrical engineer (1926) at the University of Oklahoma, Norman, where he was an instructor and assistant professor from 1919 to 1926. During the year 1926–27 he was an instructor at Massachusetts Institute of Technology, Cambridge, where he received the degree of master of science in electrical engineering. The following year he returned to the University of Oklahoma as associate professor of electrical engineering, and in 1928 accepted the position of assistant chief engineer with the Hall Electric Heating Company, Inc., Philadelphia, Pa. He resigned as research engineer of the company in 1931 to resume teaching as assistant professor at The College of the City of New York.

Jay Houghton Hall (A'05, M'12) assistant chief engineer, Electric Controller and Manufacturing Company, Cleveland, Ohio, died in August 1935 while on a vacation trip. Mr. Hall was born at Momence, Ill., April 9, 1875, and studied electrical engineering at Rose Polytechnic Institute, from which he received the degree of bachelor of science in 1893. Subsequently he received the degrees of master of science in 1906 and electrical engineer in 1908. His first position was in charge of an isolated light plant, following which, in 1899, he became a draftsman in electrical department of the Carnegie Steel Company at Munhall, Pa. For a short time in 1901 he was in charge of the electric signal system of the Chicago and Eastern Illinois Railroad Company, and from 1901 to 1904 was chief draftsman and assistant superintendent for the Youngstown Engineering Company, Youngstown, Ohio. In 1904 he became chief draftsman for the Electric Controller and Manufacturing Company, and since then had held successively the positions of assistant engineer, sales engineer, manager of the New York sales office, assistant to the president, and electrical engineer, with the exception of the period 1915–17, when he was at Morganton, N. C. Mr. Hall received a number of patents on motor controlling apparatus, and was a member of the institute's committee of applications to iron and steel production in 1926–27.

Carl Borgmann (A'24) manual telephone equipment engineer, Bell Telephone Laboratories, Inc., New York, N. Y., died suddenly August 20, 1935. He was born at Christiania, Norway, January 31, 1881, and graduated from the Technical College there in 1900. Until 1902 he was in charge of telephone equipment design with a manufacturing company, then came to the United States and was employed by the Mark Manufacturing Company at Evanston, Ill. In 1903 he became a supervisor in the drafting department of the Western Electric Company, Chicago, Ill., and in 1907 became inspector of installation. From 1909 to 1919 he was equipment engineer and since 1919 had been manuel telephone equipment engineer at New York; in 1925 the laboratories of this company became the Bell Telephone Laborabories, Inc. In 1930 he was a member of the group of engineers sent abroad to study European communications.

Charles Martin Clark (A'96, and member for life) director of many utility companies and former president of The Bradstreet Company, New York, N. Y., died July 24, 1935. Mr. Clark was born at Boston, Mass., November 5, 1873, and received the degree of electrical engineer from the Columbia School of Mines in 1897. From 1904 to 1926 he was treasurer of The Bradstreet Company, becoming president in 1926. Among the many companies of which he was a director were American Power and Light Company, Electric Power and Light Company, United Gas Corporation, The Washington Water Power Company, Florida Power and Light, and Dun and Bradstreet, Inc.

Mortimer Dickinson Gould (M'25) Buffalo, N. Y., died on June 8, 1935. He was born at Buffalo, February 28, 1879, and was graduated from Stevens Institute of Technology in 1905. Until 1907 he was engaged in construction work with the Niagara, Lockport and Ontario Power Company, and then entered the engineering department of the Rochester (N. Y.) Gas and Electric Company. From 1911 to 1917 he was in charge of engineering, construction and operation for the Livingston Niagara Power Company at Avon, N. Y., and in 1917 joined the Westinghouse Electric and Manufacturing Company, becoming manager of the engineering division of the Buffalo office of the company, which position he held until recently.

Edward K. Shelton (A'12) designing engineer, General Electric Company, Pittsfield, Mass., died on August 31, 1935. He was born at Manhattan, Kan., January 7, 1890, and graduated from the electrical engineering course at the University of Washington in the class of 1910 with the degree of bachelor of science in electrical engineering. In that year he entered the employ of the General Electric Company in the testing department at Schenectady, N. Y. In 1913 he was transferred to the lightning arrester department in the Pittsfield works, and since 1920 had been in charge of the design and development of capacitors.

Aldis H. Wurts (A'21) attorney with Cotton, Franklin, Wright, and Gordon, New York, N. Y., died August 2, 1935. He was born at Chatham, Ohio, August 9, 1890, and received the degrees of bachelor of arts at Western Reserve University in 1913, bachelor of science in electrical engineering at University of California in 1914, and bachelor of laws at Harvard University in 1917. Following military service he engaged in the practice of law with M. B. and H. H. Johnson in Cleveland, Ohio, and in 1922 became associated with the firm of Cotton and Franklin in New York.

Joseph Hugh McHugh (A'31) electrical inspector, City of St. Paul, Minn., died in January 1935 according to word recently received at Institute headquarters. He was born at Minneapolis, May 24, 1884, and was engaged in electrical construction work from 1900 to 1908, and as a theatrical electrician from 1908 to 1918. Since that year he had been an electrical inspector.

Harley C. Schulze (A'24) results engineer, The Ohio Power Company, Philo, Ohio, died March 21, 1935, according to word just received at Institute headquaters. Mr. Schulze was born at Hubbard, Ia.,

September 25, 1899, and was a graduate of Iowa State College, from which he received the degree of bachelor of science in electrical engineering. In 1921 he entered the testing department of the General Electric Company at Schenectady, N. Y., where he was employed until he became efficiency engineer at the Philo plant of the Ohio Power Company in 1926.

Membership

Applications for Election

Applications have been received at headquarters from the following candidates for election to membership in the Institute. If the applicant has applied for direct admission to a grade higher than Associate, the grade follows immediately after the name. Any member objecting to the election of any of these candidates should so inform the national secretary before Oct. 31, 1935, or Nov. 30, 1935, if the applicant resides outside of the United States or Canada.

Angell, C. H., Westinghouse Elec. & Mfg. Co. St. Louis, Mo.
Baker, A. W. (Member), Am. Transit Assoc., New York, N. Y.
Bell, F. E. (Member), T.V.A., Joe Wheeler Dam, Ala.
Bertolet, W. B., Metropolitan Edison Co., Easton, Pa.
Briner, J. F., Jr., Federal Pwr. Comm., Washington, D. C.
Brooking, J. M. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Counts, R. E. L., Federal Pwr. Comm., Washington, D. C.
Cross, G. F., Hudson Bay Mining & Smelting Co., Flin Flon, Manitoba, Can.
Dawson, J. W. (Member), Westinghouse Elec. & Mfg. Co., E. Pittsburgh, Pa.
Edgar, C. K., Dept. of Water & Pwr., City of Los Angeles, Calif.
Eslick, E. (Member), Federal Pwr. Comm., Washington, D. C.
Gray, L. A., Simplex Wire & Cable Co., Cambridge, Mass.
Hansen, A., Pacific Tel. & Tel. Co., Seattle, Wash.
Harrison, P. W. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Heitman, C. E. Jr. (Member), Edward G. Budd Mfg. Co., Philadelphia, Pa.
Henkel, L. C. (Member), 220 West 7th St., Hanford, Calif.
Hiscox, W. L. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Howe, C. D., Pacific Tel. & Tel. Co., Seattle, Wash.
Jones, G. G. (Member), Am. Tel. & Tel. Co., St. Louis, Mo.
Logan, J. T., Ga. Pwr Co., Atlanta.
Maddock, G. F. (Member), Federal Pwr. Comm., Washington, D. C.
Masters, M. H., Ford Motor Co., Somerville, Mass.
McClure, F. J., 9332 N.E. Skidmore St., Portland, Ore.
Millard, F. P. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Morris, M., N. Y. Edison Co., Inc., New York, N. Y.
Morrow, B. E. (Fellow), Consumers Pwr. Co., Jackson, Mich.
Oklund, A. L., Milwaukee Sch. of Engg., Milwaukee, Wis.
Perkinpine, R. W. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Pollard, E. I., Westinghouse Elec. & Mfg. Co., E. Pittsburgh, Pa.
Richardson, A. F., Jr. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Richardson, G. E., Am. Steel & Wire Co., Worcester, Mass.
Rosene, V. E., Bell Tel. Lab., New York, N. Y.
Smith, G. R. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Smith, L. A. (Member), Western Union Tel. Co., New York, N. Y.
Sprague, C. H., Intl. Corresp. Schools, Scranton, Pa.
Stockhus, C. R., Union Elec. Lt. & Pwr. Co., St. Louis, Mo.
Tenney, G. C., "Electrical West," San Francisco, Calif.
Thompson, S. H., Federal Pwr. Comm., Washington, D. C.
Thomson, J. L., Independent Subway System, New York, N. Y.
Vaile, R. B., Jr., Iowa State College, Ames.
Weston, P. O., St. Louis Rockbestos Products Corp., St. Louis, Mo.

41 Domestic

Addresses Wanted

A list of members whose mail has been returned by the postal authorities is given below, with the addresses as it now appears on the Institute record. Any member knowing of corrections to these addresses will kindly communicate them at once to the office of the secretary at 33 West 39th St., New York, N. Y.

Bock, F. S., 1642 W. Broad St., Richmond, Va.
Brune, Otto, 214 White St., Waverly, Mass.
Chiofalo, J., 203 Graham Ave., Brooklyn, N. Y.
Cole, Fred H., 1116 Washington Blvd., Oak Park, Ill.
Crite, Mitchel, 32 E. 126th St., New York, N. Y.
Ghosh, K. C., c/o Compagnia Generale Di Elettricita, 34 Via Borgognone, Milan, Italy.
Golikoff, A., Main P. O. Gen. Del., Moscow, U.S.S.R.
Greene, F. M., 656—50th St., Brooklyn, N. Y.
Kimball, Gordon S., 154 Elmer Ave., Schenectady, N. Y.
Nelson, Charles J., 1515 N. Lotus Ave., Chicago, Ill.
Rozelle, P. M., 2018 Chestnut St., Harrisburg, Pa.
Schellberg, Kenneth O., 4115—51st St., S., Seattle, Wash.
Spiegel, William F., 7 Stegman Court, Jersey City, N. J.
Vance, Paul E., c/o Marietta Mfg. Co., Point Pleasant, W. Va.
Walker, Jas. E. L., 773 N. Jefferson St., Milwaukee, Wis.
Whittemore, Geo. W., 151 Ridgewood Ave., Glenwood, N. J.
Williams, Ellis Richard, 97 W. 8th St., Wyoming, Pa.

17 Addresses Wanted

Engineering Literature

New Books in the Societies Library

Among the new books received at the Engineering Societies Library, New York, recently, are the following which have been selected because of their possible interest to the electrical engineer. Unless otherwise specified, books listed have been presented gratis by the publishers. The Institute assumes no responsibility for statements made in the following outlines, information for which is taken from the preface of the book in question.

COMBUSTION ENGINE FUELS and HEAT, $1.75; STATIONARY DIESEL ENGINES, $1.75; DIESEL ENGINE DETAILS and MANAGEMENT, $2.00. By J. Vanderdoes, L. H. Morrison and C. T. Baker. Scranton, Pa., International Textbook Co., 1935. Illus., diagrs., charts, tables, 8x5 in., lea. The first discusses the nature of heat, its effects, thermodynamics, heat and work, combustion, liquid and gaseous fuels, and the principles of the internal-combustion engine. The second describes the various types and their construction. The third describes the parts of these engines and their auxiliaries, and explains the operation, maintenance, and repair.

FACTORY ORGANIZATION and ADMINISTRATION. By H. Diemer. 5 ed. New York and London, McGraw-Hill Book Co., 1935. 412 p., illus., diagrs., charts, tables, 9x6 in., cloth, $4. A standard text revised to include the advances made during the past decade. Covers everything from factory location and building to department organization.

NATIONAL PHYSICAL LABORATORY, REPORT for the YEAR 1934. London, Dept. of Sci. and Ind. Res. His Majesty's Stationery Office, 1935. 260 p., illus., diagrs., charts, tables, 11x8 in., paper, 13s. A concise report on the work done in the fields of physics, electricity, radio, metrology, engineering, metallurgy, aerodynamics, and hydrodynamics. A list of papers published by the laboratory is included.

RISE of MODERN PHYSICS. By H. Crew. 2 ed. Baltimore, Williams & Wilkins Co., 1935. 434 p., illus., diagrs., tables, 8x5 in., cloth, $4. Makes no requirements of technical knowledge upon the reader. Intended for undergraduates, but will be equally useful to all those interested in the history of science. Three new chapters, dealing with the inertia of electricity, the rise of modern spectroscopy, and restricted relativity have been added.

SMITH'S COLLEGE CHEMISTRY. By J. Kendall. 3rd rev. ed. New York and London, D. Appleton-Century Co., 1935. 753 p., illus., diagrs., charts, tables, 9x6 in., cloth, $3.75. The new edition of this textbook of general chemistry exhibits few important changes from the preceding one, the chief being in the treatment of ionization and the discussion of recent advances in our knowledge of the atom.

STEAM TURBINES. By. E. F. Church. 2 ed. New York and London, McGraw-Hill Book Co., 1935. 327 p., illus., diagrs., charts, tables, 9x6 in., cloth, $3. For students in engineering colleges. The form and characteristics of turbines are presented, and thermodynamic principles are applied to the calculation of the flow of steam, and to the various factors that effect efficiency.

STRUCTURE of CRYSTALS, Supplement for 1930–34 to 2nd Ed. By R. W. G. Wyckoff. (Amer. Chem. Soc. Monograph Series No. 19-A). New York, Reinhold Publ. Corp., 1935. 240 p., diagrs. charts, tables, 9x6 in., cloth, $6. Brings up to date the second edition by reviewing the X ray determinations of structure which have been published during the last 4 years. The bibliography contains over 2,000 references.

IMPREGNATED PAPER INSULATION, the Inherent Electrical Properties. By J. B. Whitehead. N. Y., John Wiley & Sons, 1935. 221 p., illus., 9x6 in., cloth, $4.00. A comprehensive survey of researches at The Johns Hopkins University, the fourth of the series of monographs issued by the committee on electrical insulation of the National Research Council. While much of the content has appeared previously in periodicals, it is here united into a connected account.

INTRODUCTION to PHYSICAL SCIENCE. By C. W. Miller, 2 ed. N. Y., John Wiley & Sons, 1935. 409 p., illus., 9x6 in., cloth, $3.00. An introductory text using only common mathematics, and including late discoveries in nuclear physics.

PRINCIPLES of ELECTRIC POWER TRANSMISSION by Alternating Currents. By H. Waddicor. 3 ed. N. Y., John Wiley & Sons, 1935. 449 p., illus., 9x6 in., lea., $6.00. A text for students and also a handbook for designers and operators, supplying a systematic exposition of the principles that underlie the electrical design of transmission lines.

SIX-PLACE TABLES, with Explanatory Notes. By E. S. Allen. 5 ed. N. Y. and Lond., McGraw-Hill Book Co., 1935. 175 p. 7x4 in., lea., $1.50. A collection of tables, cubes, square and cube roots, circumferences and areas of circles, fifth roots and powers, common logarithms of numbers and trigonometric functions, natural trigonometric functions and logarithms, radians and degrees, and exponential, hyperbolic and gamma functions.

SPEKTROSKOPIE (Sammlung Göschen 1091). By K. W. Meissner. Berlin and Leipzig, Walter de Gruyter & Co., 1935. 180 p., illus., 6x4 in., cloth, 1.62 rm. A concise yet comprehensive introduction to spectroscopy.

THIS MODERN WORLD and the ENGINEER. Edinburgh, Royal Scottish Society of Arts, 1934. 140 p., illus., 9x6 in., cloth, 5s. Lectures discussing the achievements and problems of modern engineering in the fields of physics, civil, mechanical, electrical, and chemical engineering, and in mining.

Industrial Notes

New Name for Fansteel.—The Fansteel Products Co., North Chicago, Ill., has changed its name to the Fansteel Metallurgical Corporation. The new corporate name more clearly defines the nature of the business, that of refining rare metals such as tantalum, tungsten, molybdenum, electrical contacts, etc., and fabricating them to a wide variety of industrial uses.

Westinghouse Promotes T. I. Phillips.—Appointment of T. I. Phillips, as general works manager of the Westinghouse Electric & Mfg. Co., has been announced. In his new position Mr. Phillips will serve as central authority for all manufacturing operations of the company. He succeeds C. H. Champlain, who has been forced by prolonged illness to relinquish his activities with the company. Mr. Phillips has been with Westinghouse since 1915.

Steel and Tubes Transfers Personnel.—Due to increased business activity in the territory served from Philadelphia, Steel and Tubes, Inc., Cleveland, has recently created a new sales district, headed by C. J. Boyd, formerly of the Brooklyn, N. Y., sales organization. J. F. Keeler, formerly of the sales promotion department in Cleveland, has been transferred to the Brooklyn office, and J. S. Anderson, formerly of Detroit, has been transferred to the new Philadelphia office. J. D. Benfield and Robert Turrell, formerly with the electrical division of Steel and Tubes, have formed their own organization, with headquarters at Detroit, and are representing Electrunite Steeltubes and Fretz-Moon conduit products in the Michigan territory.

New Heating Element.—A new electrical resistance alloy, "Kanthal," with operating temperature of better than 2460° is being produced by the C. O. Jelliff Mfg. Corp., Southport, Conn. According to the manufacturer it not only operates at higher temperatures than other base metal or alloy without protective gases but also possesses remarkable oxidation resisting qualities. Three grades are available: maximum 2462° for electric furnaces; 2372° for domestic appliances; 1150° for most purposes where nickel chrome is now used. It also has the advantage of lower cost and longer life.

New Angle Clamp.—The Ohio Brass Co., Mansfield, O., has developed a new angle clamp designed to simplify angle construction in farm line design. According to the manufacturer, any angle from 30 to 120° can be turned with this clamp without the necessity of dead-ending the conductors and using jumpers. The new clamp, No. 80431, is $4^3/_{16}$ inches long and the radius of its seat curvature is 3 inches, enough to take conductors up to 2/0 A.C.S.R. The special "U"-bolt practise as incorporated in the design, permits the clamp to be attached directly to a 6-inch suspension unit, without the use of intermediate fittings, and it will fit 41 different sizes and types of conductors.

Underground Cable Mole.—The Burndy Mole or insulated multiple connector has been designed to provide a simple method of making joints in underground secondary distribution systems. The manufacturer, Burndy Engineering Co., Inc., 305 E. 45th St., New York, claims that it is especially advantageous for connections between rubber-leaded and rubber-covered cables to paper-insulated, oil-impregnated cables; also, with the Mole, an effective stop joint on oil-impregnated cables can readily be made, eliminating entirely the need for a stub joint. Cables are clamped by simple solderless devices which in turn are attached to a copper, rubber-insulated body. Each connector can accommodate a range of cable sizes, and combinations for various arrangements of mains and feeders can be easily made.

Trade Literature

Voltage Regulator.—Bulletin 2204, 8 pp. Describes new type BFR, low cost, automatic branch feeder voltage regulator, available in all standard commercial voltages from 2400 to 13,200, single and three phase. Allis-Chalmers Mfg. Co., Milwaukee, Wis.

Pole Anchoring.—Bulletin, 24 pp., "Correct Anchoring." A comprehensive, illustrated study of pole anchoring and guying by means of various types of patented anchors. Tables on anchor holding powers in different soils are included, as well as a chapter on figuring strains on guys. Chance Company, Centralia, Mo.

Synthetic Rubber.—Bulletin, 12 pp., "Koroseal." Describes a new synthetic rubber-like material, known as Koroseal. The properties of the product, the forms in which it is available and the uses to which it has been successfully adapted, are listed. B. F. Goodrich Co., Mechanical Rubber Goods Div., Akron, O.

Output Switching Panel.—Bulletin, 8 pp. Describes a new output switching panel, 271A, for speech input equipment. It provides facilities for dispatching programs from as many as 6 amplifier channels over 4 output circuits to line amplifiers. Western Electric Co., 195 Broadway, New York.

Fire Extinguishing System.—Bulletin, 12 pp., "Smothering Fire in Power Plants." Describes the Lux carbon dioxide system of protecting electrical equipment from fires. The gas is stored in cylinders and in case of fire is released, either by manual or automatic control. The principal advantage in the use of this system is its freedom from damaging after-effects. Walter Kidde & Co., Inc., 140 Cedar St., New York.

Thermostatic Metals.—Bulletin, 32 pp. A comprehensive review of specifications, properties and methods of application of modern thermostatic bi-metals. Ten full-page charts show deflection and force characteristics, along with design formulae, providing up-to-date engineering data for bi-metal applications in the field of temperature indication and thermal control. H. A. Wilson Co., 105 Chestnut St., Newark, N. J.

Phenolic Insulation.—Catalog, 48 pp. Describes "Dilecto," a laminated plastic, developed to meet the demands for a waterproof insulating material with great mechanical strength and adaptability to all machining operations. The material is supplied in sheet, tube and rod form, or is fabricated to specifications. Process of manufacture is outlined and illustrated, applications set forth and a table compares the properties of hard rubber, vulcanized fibre, etc. and Dilecto. Continental-Diamond Fibre Co., Newark, Del.

Transformer Protector.—Bulletin, 8 pp., "How Do You Forestall Faults?" Describes the Kidde transformer protector, a mechanical device for the protection of conservator type transformers. It warns of the occurrence of minor faults and in the case of major faults it disconnects the transformer from the line. The principle upon which the protector operates is based upon the fact that gases are given off when any fault occurs in a transformer. These gases actuate the protector. Walter Kidde & Co., Inc., 140 Cedar St., New York.

Time Switches.—Bulletin GEA-1427D, 8 pp. Describes general-purpose automatic time switches. Type T-17 will control almost any electric circuit on a schedule related to the time of day. It will perform any practical number of operations per day and skip a day or more if desired. Standard switches are designed for a-c circuits, 115 and 230 volts, 40 amperes per contact. This switch can be furnished with either a plain dial (for fixed-time settings) or astronomic dial (for sun schedules) and certain combinations of both. Type T-27 switch is similar to the T-17 except that it is only for indoor service and is not equipped with an omitting device for day cutouts. In both types, drive is direct from Telechron motor, through a compact spur-gear train. General Electric Co., Schenectady, N. Y.

Motor Maintenance Equipment.—Catalog, 64 pp. Lists a wide variety of motor maintenance devices, including commutator and slip-ring resurfacers, grinders, truing tools, undercutters, slotters, blowers, sprayers, etc. Detailed information on care and servicing of commutators and slip-rings is presented; chapters cover definitions of electrical terms; tables on current carrying capacities of wires; fusing, wiring and full load current data for all types of motors; formula for determining amperes, horsepower, kilowatts and kva; for calculating electric current, etc. Among new products listed are improved precision grinders, a complete line of electric cleaners, a new industrial thermometer, an improved coil winding head and new sizes of thread-on, solderless wire connectors and lugs. Ideal Commutator Dresser Co., Sycamore, Ill.